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TRANSITION TO SPACE-CHARGE LIMITED FLOW IN CROSSED-FIELD DEVICES *

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Abstract

This work presents a fully kinetic description to model the electron flow in the electronic crossed-field configuration observed in a smooth-bore magnetron. Through this model, it has been observed that, according to the electromagnetic field, the injection temperature and the charge density, the electron flow can be classified in two different stationary modes: magnetic insulation mode where most of the electrons returning to the cathode after a transient time and Child-Langmuir mode where most of the electrons reach the anode after a transient time. Focusing on magnetic insulated mode, it has been found that charge density and injection temperature define whether electrons are accelerated (accelerating regime) or decelerated (space-charge limited regime) on the cathode. Besides, when the injection temperature is relatively low (high), a small charge increase causes (does not cause) an abrupt transition between accelerating and space-charge limited regime. Basing on the results, it was possible to identify a critical temperature that separates abrupt and continuous behavior. The results have been verified by using self-consistent computer simulations.

INTRODUCTION

Describing the electron flow in presence of crossed electric and magnetic fields is fundamental for the development of several advanced applications in areas ranging from microwave sources [1] to space propulsion [2]. The study of the electron dynamics in such field configuration was pioneered by Hull [3] who showed that a magnetic field might limit the particle flow from the cathode to the anode. This result was based on a single-particle model that assumes given external electromagnetic fields. Nowadays, a large number of papers have investigated the equilibrium and stability of these systems by explicitly taking into account the particles self-fields [4–6]. The self-fields may play a major role in the dynamics since they can also limit the particles flow from the cathode to the anode as the current density exceeds a certain threshold and a space-charge limited (SCL) regime emerges [7, 8]. However, given the complexity that long ranged self-fields add to the problem, the large majority of the theoretical analysis done so far are based on models that assume the electron flow is either completely cold or is a fluid with postulated equation of state. These fluid models might not properly take into account thermal effects. Because of it, recently we have developed a fully kinetic model to investigate thermal effects in crossed-field configuration observed in a smooth-bore magnetron [9]. Here, we review the theoretical framework and compare it against simulations with different injection distributions at the cathode.

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THE PHYSICAL MODEL

The model and field configuration of a smooth-bore magnetron is shown in Fig. 1. There are two plates separated by a distance L along the y axis. The one localized at $y = 0$ is a thermionic cathode kept at zero electrical potential and the one localized at $y = L$ is an anode kept at V_0 electrical potential. As consequence of the electrical potential difference in the gap region between the plates, there is an uniform electric field $\mathbf{E}_0 = -(V_0/L)\hat{\mathbf{y}}$. Besides, in gap region there is an uniform constant magnetic field $\mathbf{B}_0 = -B_0\hat{\mathbf{z}}$, consolidating the crossed-field configuration.

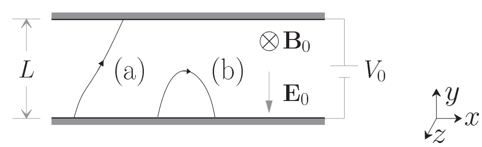

Figure 1: Model and field configuration of a smooth-bore magnetron. The curves correspond to the trajectories of test electrons emitted from the thermal cathode with vanishing velocities. In the case (a) $V_0 > V_H$ and in the case (b) $V_0 < V_H$.

At instant of time $t = 0$, the thermionic cathode starts to emit electrons, which enter in gap region. These are accelerated by the electric field $\mathbf{E}_0$ along the y direction while they are deflected along clockwise direction on $x-y$ plane by the magnetic field $\mathbf{B}_0$. In Fig. 1 is also shown two trajectories of test electron under external fields influence. Assuming the test electron enters the gap region with vanishing velocities, it is observed that, whether electrical potential satisfies the condition $V_0 > V_H$– where $V_H = eB_0^2L^2/2m$ is called potential Hull and m and e are the mass and electric charge of the electron– the test electron has enough energy to reach the anode, as illustrated in detail (a) of the Fig. 1. On the other hand, whether $V_0 < V_H$ the test electron has not enough energy to reach the anode and the magnetic field deflects the test electron to cathode, as illustrated in detail (b) of the Fig. 1. In the smooth-bore magnetron system, there is a spatial symmetry and it is assumed the thermal cathode emits infinite charges sheets parallel to $x-z$ plane instead of single particles. Consequently, the particle distribution function on phase space $f(\mathbf{r}, \mathbf{p}, t)$ only depends on the y spatial coordinate and its evolution is dictated by Vlasov equation [10]

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial H}{\partial p_y}\frac{\partial f}{\partial y} - \frac{\partial H}{\partial y}\frac{\partial f}{\partial p_y} = 0, \qquad (1)$$

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where $H = (\mathbf{p} + e\mathbf{A})^2/2m - e\phi(y)$ is the single particle Hamiltonian and $\mathbf{A}$ and ϕ are the vector and scalar electromagnetic potentials. The scalar potential is self-consistently determined from the particle distribution by the Poisson equation

$$\frac{\partial^2\phi}{\partial y^2} = \frac{e}{\epsilon_0} n(y), \qquad (2)$$

with the boundary conditions: $\phi(0) = 0$ and $\phi(L) = V_0$. In equation (2), $n(y)$ is the electron density and ϵ_0 is the vacuum permittivity. Assuming the external magnetic field is stronger than the self-consistent magnetic field– it is true when the gap region is sufficiently thin [6]– it is possible to use $\mathbf{A} = \mathbf{A}_0$, where $\mathbf{A}_0 = -B_0 y\hat{\mathbf{x}}$. After defining the vector and scalar potential, the single particle Hamiltonian can be written as:

$$H = \frac{1}{2m}\left[(p_x + eB_0 y)^2 + p_y^2 + p_z^2\right] - e\phi(y). \qquad (3)$$

The Hamiltonian (3) does not depend on x and z, thus p_x and p_z are constant of motion and their values are determined by initial conditions. For simplicity, it is supposed the electrons are injected in gap region only with orthogonal velocities to the cathode, such that $p_x = p_z = 0$. It does not imply the velocity parallel to cathode is always zero, in fact $\mathbf{v} = (\mathbf{p} + e\mathbf{A})/m$ and consequently $v_x = eB_0 y/m$.

After a transient time, it is expected the electron flow reach a stationary state. In that situation, the electric field value on cathode $E_c = -\partial\phi/\partial y|_{y=0}$ define whether the flow will be accelerated (accelerating regime– $E_c/E_0 > 0$) or decelerated (space-charge limited regime– $E_c/E_0 < 0$) when it enters in gap region.

In order to construct the theoretical model, it is considered the electrons are injected from cathode according to a waterbag velocity distribution

$$f(y{=}0, p_y) = \frac{n_0}{p_{0max} - p_{0min}}, \qquad (4)$$

for $p_{0min} \leq p_y \leq p_{0max}$ and zero elsewhere. In equation (4), n_0 is the electron density at the cathode and p_{0max} and p_{0min} are initial momentum of the most and the least energetic particle. Once the stationary state has been achieved, all quantities become time independent, even the single particle Hamiltonian (3) and it is possible to write the momentum for an electron as: $p_y(p_0, y) = \pm[p_0^2 + 2em\phi(y) - e^2B_0^2y^2]^{1/2}$ where p_0 is its momentum at the cathode and the plus (minus) signal refers to an electron that is moving toward the anode (cathode). In the theoretical model, Vlasov equation imposes the electron flow is incompressible in the phase space; it implies the distribution everywhere inside this region has the same density. Focusing on magnetic insulated cases– where $V_0 < V_H - p_{0max}^2/2m$ and all the particles ejected from the cathode eventually return to it– it is possible to write the particle density as:

$$n(y) = 2n_0 \frac{|p_y(p_{0max}, y)| - |p_y(p_{0min}, y)|}{p_{0max} - p_{0min}}, \qquad (5)$$

where the factor "2" accounts for the fact that there is an equal number of particles moving to and from the cathode and $p_y(p_{0max}, y)$ and $p_y(p_{0min}, y)$ are real functions to be considered zero whenever they become imaginary. Using the density given by (5) in electric potential equation given by (2), it is possible to obtain the stationary states of the electron flow and consequently the value to the electric field over the cathode.

RESULTS

In this section, it will be shown the results. In order to simplify, it is convenient define dimensionless parameter: $\nu_0 = V_0/V_H$, $\eta_0 = en_0L^2/\epsilon_0V_0$, $p_0 = (p_{0max} + p_{0min})/2eB_0L$ and $T_0 = (p_{0max} - p_{0min})^2/12e^2B_0^2L^2$ which measure, respectively, the accelerating potential, the electron density, the average momentum and the temperature (momentum spread) at injection.

In Fig. 2(a), it is plotted E_c/E_0 as function of electron density for $\nu_0 = 0.8$, $\bar{p}_0 = 0.2$ and $T_0 = 8.3 \times 10^{-4}$ (low temperature). Through theoretical model (solid line) is observed whether charge density is small ($\eta_0 \to 0$) the electrical potential solution approaches the vacuum solution $\phi(y) = V_0 y/L$ and consequently $E_c \approx E_0 \approx -1$. As η_0 increases, E_c/E_0 decrease because more charge is present in gap region, depleting the accelerating electric field. When $\eta_0 \approx 0.82$ the self-field produced at cathode by the charge density is stronger then the external field and a space-charge solution is found. Moreover, whether $\eta_0 < 0.82$ or $\eta_0 > 0.836$ there is only one possible solution, but whether $0.82 < \eta_0 < 0.836$ there are three different solutions predicted by the theoretical model.

To verify the theoretical model, it has been executed a N-particle self-consistent simulation. The dynamic of the ith charge sheet is derived from the Hamiltonian and is: $\ddot{y}_i = -\Omega_c^2 y_i - e(E_0 + E_s^i)/m$, where $\Omega_c = eB_0/m$ is the cyclotron frequency and E_s^i is the self-consistent electric field acting over the ith charge sheet. To model the charging process, it has been initialized the simulation with the gap region empty and it has been computed the electric field at the cathode E_c as the charge builds up in the system. After some transient time, E_c saturates and the system reaches a stationary state. Results of E_c obtained from N-particle simulations are shown by the symbols in Fig. 2 for two different initial velocity distribution: waterbag (points) and Gaussian (squares). It is observed a good agreement between the theory (solid line) and simulation (symbols). Moreover, it is possible to see the charge arrange to accelerate regime (it does not depend on the initial condition) and there is an abrupt transition from accelerating ($E_c/E_0 = 0.2$) to space-charge limited regime ($E_c/E_0 = -0.05$) when $\eta_0 = 0.835 \to 0.845$.

The phase space before and after the abrupt regime transition observed in Fig. 2(a) is shown in Fig. 3(a) and Fig. 3(b), respectively. In Fig. 3, solid lines represent theoretical solution for the most and the least energetic charge sheet and dots represent charge sheets in gap region when it is

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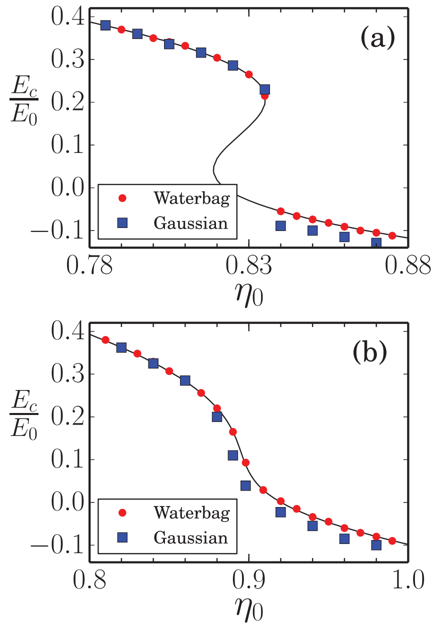

Figure 2: Normalized electric field at cathode as function of the electron density for (a) $T_0 = 8.3 \times 10^{-4}$ and (b) $T_0 = 3.3 \times 10^{-3}$. The solid line corresponds to results from the theory whereas the symbols from the simulation with waterbag (points) and Gaussian (squares) initial velocity distributions. Others parameters are: $v_0 = 0.8$ and $\bar{p}_0 = 0.2$.

used Gaussian initial velocity distribution. It is possible to conclude even using a different initial velocity distribution from theoretical model, the simulation electric field at cathode are almost the same.

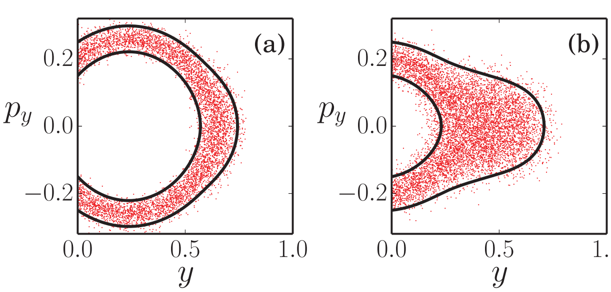

Figure 3: Phase space plots before (a) and after (b) the abrupt transition shown in Fig. 2 (a). These figures show a dramatic change in charge distribution as the system pass from accelerating to decelerating regime. In these figures, the parameters are: $T_0 = 8.3 \times 10^{-4}$, $v_0 = 0.8$, $\bar{p}_0 = 0.2$ and (a) $\eta_0 = 0.835$ and (b) $\eta_0 = 0.845$.

In Fig. 2(b), it is plotted E_0/E_0 as function of electron density for $v_0 = 0.8$, $\bar{p}_0 = 0.2$ and $T_0 = 3.3 \times 10^{-3}$ (high temperature). Through theoretical model (solid line) is observed whether charge density is small ($\eta_0 \to 0$) the electrical potential solution approaches the vacuum solution $\phi(y) = V_0 y/L$ and consequently $E_c \approx E_0 \approx -1$. As η_0 increases, E_c/E_0 decrease because more charge is present in gap region, depleting the accelerating electric field. When $\eta_0 \approx 0.9$ the self-field produced at cathode by the charge density is stronger then the external field and a space-charge solution is found. In this case, there is only one solution of E_c/E_0 for each charge density value. It means, when injection temperature is relatively high there is not abrupt transition between accelerate and decelerate regimes. In Fig. 2(b), the results were confirmed by simulation (symbols) for both Gaussian and waterbag initial velocity distribution.

These results indicate there is a critical temperature that separates the occurrence of abrupt and continuous regime transitions. In fact, the critical temperature is: $T_{0c} \approx 1.4 \times 10^{-3}$.

CONCLUSION

In this work, it was shown and analyzed some of electron flow properties in a crossed electromagnetic field configuration observed in smooth-bore magnetrons. Basing on the waterbag theoretical model, it was found that depending on the parameters of the system it may present either a single or multiple stationary solutions. However, through simulations using two different initial velocity conditions– waterbag and Gaussian– it was observed charge sheets arrange to accelerate regime and there is an abrupt transition from accelerating to decelerating regime when injection temperature is low. It was not occurred when the injection temperature was hight. Basing on these results, it was evaluated a critical temperature which separates abrupt from continuous transition behavior.

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