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REFORMULATION OF THE ACTION AND PHASE JUMP METHOD TO OBTAIN MAGNETIC ERRORS IN THE LHC IRS*

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Abstract

One of the major problems when doing the commissioning of an accelerator is to identify and correct the linear components of magnetic errors. The Action and Phase Jump Technique is one of the available methods to perform this task. For this method to work, it is necessary to have one BPM measurement at the Interaction Region (IR), the region where the magnetic error is evaluated. In some cases, this BPM measurement become the biggest source of uncertainty when the action and phase jump technique is used. In this paper, a new formulation based on this method is presented. This new formulation doesn't make any use of BPM measurements at the IR, thereby allowing more robust error estimations. Quadrupole errors in the LHC lattice are estimated with this new formulation, using both, simulated data and LHC experimental data. A comparison with the previous formulation is included. The results on simulated data show that the reformulation leads to a reduction in the uncertainty, while for the experimental case, the reduction is not so clear. Explanations for this behavior and possible remedies will also be discussed.

INTRODUCTION

One of the main goals during commissioning of accelerators like LHC is to reduce the magnetic errors. The Action and Phase Jump Analysis Technique, also called Action and Phase method, or just APJ, is one of the available methods to perform magnetic error corrections. This method is based on the theoretical principle of preservation of the Action and Phase variables in absence of a magnetic error. The corrections are made locally and specially at the Interaction Regions (IR) of an accelerator. Its theoretical development is presented in the first part of [1].

Experimental studies using this method have been already presented. Different tests were run at RHIC in Brookhaven, for example using closed-orbit [2] or first turn orbit [3] data. In LHC, preliminary analysis had been done using turn-by-turn (TBT) orbits, these are [4] and [5].

In this paper, a reformulation of the APJ technique for linear error corrections will be presented. First, theoretical expressions will be shown and then a comparison between the previous and the new formulation, using simulated orbits, will be discussed. Finally, the results of proposed corrections using LHC data are reported.

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THEORETICAL REFORMULATION FOR THE ACTION AND PHASE ANALYSIS

With the APJ method, the variables of Action (J) and Phase (δ) are measured from orbit data around the accelerator and then three regions are identified: the region which contains the magnetic error, a region (or subsection of the accelerator) before the error, and a region after the error. To calculate the magnetic errors the method uses the J and δ from the region before and after the error, and one transverse position from the error region.

The transverse position at the error region is inferred from one BPM measurement at the IR as described in [6]. In some cases, this BPM measurement can become the biggest source of uncertainty when the action and phase jump technique is used, because it is a single measurement while the other quantities involved to estimate the magnetic error are obtained from multiple measurements.

Keeping the theoretical framework of the Action and Phase method, given in [1], new equations are introduced that mainly start by changing equation (15) from that paper (the strength of the magnetic error), and ends in new equations for the magnetic errors estimations, which changes equations (22), (25) and (26).

The reformulation implies the following procedure. Equation (15) in [1] is

$$\sqrt{2J_0 + 2J_1 - 4\sqrt{J_0 J_1}\cos(\delta_1 - \delta_0)}/\sqrt{\beta_{z,i}(s_\theta)} = \theta_z \quad (1)$$

where J_n, δ_n are the average of the Action and Phase variables, for the region before $n = 0$ and after $n = 1$ the error, θ_z is the strength of the magnetic error, and $\beta_{z,i}(s_\theta)$ is the beta-function at the longitudinal position of the error. In addition, in terms of the multipolar components of the magnetic field, the strength of the magnetic error is written as equations (19) and (29) from [1], these are:

$$\begin{aligned} \theta_x &= \mathbf{B}_0 - \mathbf{B}_1 x(s^\varepsilon) + \mathbf{A}_1 y(s^\varepsilon) + 2\mathbf{A}_2 x(s^\varepsilon) y(s^\varepsilon) + \\ &\quad + \mathbf{B}_2[-x^2(s^\varepsilon) + y^2(s^\varepsilon)] + \ldots \\ \theta_y &= \mathbf{A}_0 - \mathbf{A}_1 x(s^\varepsilon) + \mathbf{B}_1 y(s^\varepsilon) + 2\mathbf{B}_2 x(s^\varepsilon) y(s^\varepsilon) + \\ &\quad + \mathbf{A}_2[x^2(s^\varepsilon) - y^2(s^\varepsilon)] + \ldots . \end{aligned} \quad (2)$$

where $\mathbf{B}_i$ and $\mathbf{A}_i$ are quantities related with the multipolar expansion of the magnetic field, and $x(s^\varepsilon)$, $y(s^\varepsilon)$ are the transverse coordinates of the orbit at the error position.

When several errors are present, equation (1) can be rewritten as:

$$\sqrt{2J_0 + 2J_1 - 4\sqrt{J_0 J_1}\cos(\delta_1 - \delta_0)} = \sum_i [\theta_{z,i}\sqrt{\beta_{z,i}(s_\theta)}] \quad (3)$$

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where the strength of the error is now taken as the sum of the individual magnetic error contribution weighted by its corresponding beta-function. In this case, the strength of the magnetic error is given by equations (2) but applied to every magnetic corrector i. Using the expression for the position at the error region, given by $z(s_j) = \sqrt{2J_{z,0}\beta_z(s_j)}\sin(\psi(s_j) - \delta_0)$ and defining $m_{j,z} = \sqrt{2J_{z,0}}\sin(\psi_{z(s_j)} - \delta_{z,0})$ so $z(s_j) = m_{j,z}\sqrt{\beta_{z,j}}$, the magnetic strength errors transforms to:

$$\begin{aligned}
\theta_{x,i} &= \mathbf{B}_0 - \mathbf{B}_1 m_{s_i^\varepsilon,x}\sqrt{\beta_{x,s_i^\varepsilon}} + \mathbf{A}_1 m_{s_i^\varepsilon,y}\sqrt{\beta_{y,s_i^\varepsilon}} + \\
&\quad + 2\mathbf{A}_2 m_{s_i^\varepsilon,x}\sqrt{\beta_{x,s_i^\varepsilon}} m_{s_i^\varepsilon,y}\sqrt{\beta_{y,s_i^\varepsilon}} + \ldots \\
\theta_{y,i} &= \mathbf{A}_0 - \mathbf{A}_1 m_{s_i^\varepsilon,x}\sqrt{\beta_{x,s_i^\varepsilon}} + \mathbf{B}_1 m_{s_i^\varepsilon,y}\sqrt{\beta_{y,s_i^\varepsilon}} + \\
&\quad + 2\mathbf{B}_2 m_{s_i^\varepsilon,x}\sqrt{\beta_{x,s_i^\varepsilon}} m_{s_i^\varepsilon,y}\sqrt{\beta_{y,s_i^\varepsilon}} + \ldots. \qquad (4)
\end{aligned}$$

Therefore, from equation (3) :

$$\begin{aligned}
&\sqrt{2J_{x,0} + 2J_{x,1} - 4\sqrt{J_{x,0}J_{x,1}}\cos(\delta_{x,1} - \delta_{x,0})} = \\
&\sum_i \Big[\mathbf{B}_0\sqrt{\beta_{x,s_i^\varepsilon}} - \mathbf{B}_1 m_{s_i^\varepsilon,x}\beta_{x,s_i^\varepsilon} + \mathbf{A}_1 m_{s_i^\varepsilon,y}\sqrt{\beta_{y,s_i^\varepsilon}\beta_{x,s_i^\varepsilon}} + \\
&\quad + 2\mathbf{A}_2 m_{s_i^\varepsilon,x}\beta_{x,s_i^\varepsilon} m_{s_i^\varepsilon,y}\sqrt{\beta_{y,s_i^\varepsilon}} + \ldots\Big] \\
&\sqrt{2J_{y,0} + 2J_{y,1} - 4\sqrt{J_{y,0}J_{y,1}}\cos(\delta_{y,1} - \delta_{y,0})} = \\
&\sum_i \Big[\mathbf{A}_0\sqrt{\beta_{y,s_i^\varepsilon}} - \mathbf{A}_1 m_{s_i^\varepsilon,x}\sqrt{\beta_{y,s_i^\varepsilon}\beta_{x,s_i^\varepsilon}} + \mathbf{B}_1 m_{s_i^\varepsilon,y}\beta_{y,s_i^\varepsilon} + \\
&\quad + 2\mathbf{B}_2 m_{s_i^\varepsilon,x}\sqrt{\beta_{x,s_i^\varepsilon}} m_{s_i^\varepsilon,y}\beta_{y,s_i^\varepsilon} + \ldots\Big] \qquad (5)
\end{aligned}$$

This equation doesn't include the measurement at the IR BPM as was sought. The left-hand-side depends totally in measured (experimental) quantities whose determination involve multiple measurements, while the right-hand-side depends on quantities taken from the model of the accelerator, with the exception of the unknowns quantities and δ_0 which is also obtained from multiple measurements.

Previous studies show that all linear errors at one IR with low $\beta*$ can be compensated by tweaking two normal quadrupoles and one skew quadrupole corrector (see [4] and [7]).

The reformulated equations for those two normal quadrupole ($\mathbf{B}_{1,I}, \mathbf{B}_{1,II}$) and the skew quadrupole corrector ($\mathbf{A}_1$) are:

$$\begin{aligned}
&\sqrt{2J_{x,0} + 2J_{x,1} - 4\sqrt{J_{x,0}J_{x,1}}\cos(\delta_{x,1} - \delta_{x,0})} = \\
&\hat{\beta}_{A_1} m_{A_1,y}\mathbf{A}_1 - \hat{\beta}_{x,B_{1,I}} m_{B_1 I,x}\mathbf{B}_{1,I} - \hat{\beta}_{x,B_{1,I}} m_{B_1 II,x}\mathbf{B}_{1,II} \\
&\sqrt{2J_{y,0} + 2J_{y,1} - 4\sqrt{J_{y,0}J_{y,1}}\cos(\delta_{y,1} - \delta_{y,0})} = \\
&\hat{\beta}_{A_1} m_{A_1,x}\mathbf{A}_1 + \hat{\beta}_{y,B_{1,I}} m_{B_1 I,y}\mathbf{B}_{1,I} + \hat{\beta}_{y,B_{1,I}} m_{B_1 II,y}\mathbf{B}_{1,II}
\end{aligned}$$

where $\hat{\beta}_{z,t} = \int \beta_z ds$ for $t = \mathbf{B}_{1,I}, \mathbf{B}_{1,II}$ and $\hat{\beta}_{A_1} = \int \beta_x\beta_y ds$.

To obtain the magnetic errors there are more unknowns ($\mathbf{B}_{1,I}, \mathbf{B}_{1,II}, \mathbf{A}_1$) than equations, therefore two orbits are required, and they are chosen from the 4-type orbits that depend on both transverse planes. These orbits are chosen with the condition of having a maximum (positive or negative) of amplitude at the position of the magnetic error as showed in [4]. The orbits are called *maxmax*, *minmax*, *maxmin* and *minmin*, and the three first letters correspond with the condition on the X-plane amplitude, while the last three are for the Y-plane; *max* is used to denoted a positive maximum, while *min* is for a negative maximum.

During our studies, two ways to solve the system were analyzed. First, assuming that the phase advances for the correctors are equal and consequently by solving the system explicitly. The second, performing the numerical solution. In this paper the results using the numerical solutions given by *linalg.solve* from PYTHON [8] are shown. The integrals of the beta functions of the quadrupoles were obtained using the Simpson rule.

COMPARISON WITH THE OLD FORMULATION

TBT orbits were simulated in MAD-x [9] and the module PTC [10] using the nominal lattice. Normal and skew quadrupole errors were placed at IR5 and they were recover with the techniques explained in the previous sections. Thin normal quadrupole errors could be recovered with up to seven significant figures, while thick normal quadrupole errors were recovered within 0.1% uncertainty. All these values were independent of the amplitude of the orbits as it happened with the old formulation.

In order to investigate the sensitivity to noise of the new formulation compared to the old formulation, Gaussian noise with a σ equal to 10% of the orbit amplitude in the arcs was added to the TBT simulated data still with magnetic errors at IR5. The magnetic errors recovered using the old and the new formulations are presented in Table 1. The simulation corresponds to the V6.5.seq for protons injection with low $\beta*$, with (thick) magnetic errors included at location of the quadrupoles (MQXB.A2.L5, MQXB.B2L5), (MQXB.A2R5, MQXB.B2R5) and MQSX.3L5, at the same time [11]. To simplify their names, in this paper, they are called **B1(Q1)**, **B1(Q2)** and **A1**, respectively. During the simulation, the accelerator had the same tunes as selected experimental data.

As presented, the new formulation reduces in half the amount of statistical uncertainty σ for each magnetic error, when compare to the old formulation. The analysis were done on the same orbits and with the same conditions. The results show that the new formulation gives a more robust approximation when estimating the magnetic errors.

The results of both formulations were obtained from the average of orbits as explained in [4], using the same con-

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Table 1: Comparison with the Old Formulation using Simulated Orbits

Sim LHC_B1	**B1(Q1)** 10^{-6} [m^{-2}]	σ **B1(Q1)** 10^{-7} [m^{-2}]	**B1(Q2)** 10^{-6} [m^{-2}]	σ **B1(Q2)** 10^{-7} [m^{-2}]	**A1** 10^{-4} [m^{-2}]	σ**A1** 10^{-5} [m^{-2}]
New	-9.93	1.55	-13.1	1.14	2.98	0.60
Old	-10.1	2.48	-12.9	1.58	3.00	1.32

Table 2: Results using Old and New Formulation in Experimental Orbits

Data LHC_B1 Apr13/10	**B1(Q1)** 10^{-6} [m^{-2}]	σ **B1(Q1)** 10^{-7} [m^{-2}]	**B1(Q2)** 10^{-6} [m^{-2}]	σ **B1(Q2)** 10^{-7} [m^{-2}]	**A1** 10^{-4} [m^{-2}]	σ**A1** 10^{-5} [m^{-2}]
New	-8.28	6.98	-15.4	9.22	3.06	3.76
Old	-9.15	3.36	-13.9	7.80	3.19	3.08

figuration parameters and δ differences of maximum 1.5 rads.

A practical advantage of the new formulation compare to the old one, besides the fact that the BPM measurement at the IR is not used, is that it is not necessary to pick up a position for the equivalent magnetic error.

On the other hand, a theoretical advantage of the new formulation, is that the system of equations is more flexible.

MEASUREMENTS USING LHC DATA

Table 2 shows results of the APJ analysis using both the new and the old formulation on 2010 experimental data. The same quadrupoles used in the simulations are analyzed in this table. The predicted values for each of the two quadrupoles proposed for correction are very close between the two formulations and also with the values obtained with the Segment-by-Segment method (see [12]), in contrast with the results presented in [4], where no agreement was found between the old formulation and the Segment-by-Segment technique. Later, it was found that the origin of the disagreement was that quadrupole Q2 was considered to be composed of only MQXB2, when in reality it was composed of MQXB2 and MQXA2.

As for the values of the uncertainties, the new formulation has almost equal or higher uncertainties when compared to the old formulation. This is not what is observed using the simulated data. One reason for the unexpected behavior might have inferred from the plots of the Action And Phase analysis. The regions before and after the errors show higher variations at the arcs, even higher than the expected ones, therefore the experimental data could have a considerable amount of different type of errors in the arcs of the LHC that is not take into account in our analysis.

CONCLUSION

A reformulation of the Action and Phase Jump Analysis Method was introduced where the dependency on the BPM measurement at the IR region was suppressed. From the errors analyzed on simulated orbits, it is concluded that the reformulation leads to a reduction of uncertainty for the measurement of magnetic errors at the triplets of a LHC IR.

More experimental data is needed to ratify the conclusion from the simulation analysis, but in here it was shown that the recovery of the magnetic errors using APJ technique using both the old and the new formulation leads to a similar values from what is obtained with the usual LHC technique, Segment-by-Segment, on the contrary of what was reported in [4].

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