

EFFECT OF NUMBER OF MACRO PARTICLES ON RESOLUTION IN PHASE SPACE DISTRIBUTION

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Abstract

In order to analyze charged particle beam in an accelerator, a beam model is used to reduce number of degrees of freedom, e.g. charged disk model, charged cylinder model and macro-particle model. In numerical simulation, the macro-particle model, which has same mass-to-charge ratio, is widely used, since it does not require any symmetry of beam shape. However, the estimation of proper number of macro-particles is one of the important issues. In order to study the effect of the number of macro-particles for the numerical model, we defined a simple transformation to generate reduced distribution. The transformation was applied for one dimensional and two dimensional particle distributions. The static electric fields due to the transformed distributions were calculated. As a result, we confirmed the effectiveness of the transformation.

INTRODUCTION

In an accelerator, the motion of a charged particle beam which consists of N particles can be described by a trajectory on $6N$ dimension phase space. Since an actual beam contains enormous number of particles, for example $N \sim 4.8 \times 10^8$ electrons in an Energy Recovery Linac with 77 pC operation, degree of freedom in a simulation and a theoretical analysis is reduced using an approximation. In order to analyze the charged particle beam, a beam model is used to reduce number of degrees of freedom, e.g. charged disk model, charged cylinder model and macro-particle model [1]. The macro-particle model, which does not depend on the symmetry in the beam, is versatile method to describe it.

In numerical simulation, the macro-particle model, which has same mass-to-charge ratio, is widely used, since it does not require any symmetry of beam shape. However, the estimation of proper number of macro-particles is one of the important issues, since it affects the resolution of the phase space distribution of the beam. Then, we define a transformation to reduce macro-particles, and study the relation between the number of macro-particles and the resolution in the phase space. In this paper, we report the transformation and static electric fields calculated by the transformed distribution about an electron beam for one and two dimensional distributions.

MACRO PARTICLE MODEL

The equation of motion of an electron in electro-magnetic field, \mathbf{E} and \mathbf{B} , is

$$c \frac{m_e}{e} \frac{d(\gamma\boldsymbol{\beta})}{dt} = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \quad (1)$$

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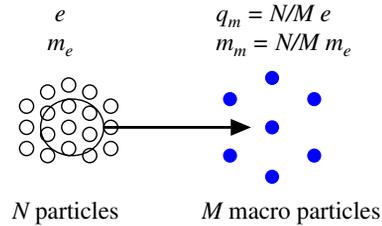


Figure 1: Description of charged particle beam by macro-particle model. The mass-to-charge ratio is a conserved quantity.

where, m_e and e are the mass and the charge of an electron, c is the speed of light, v is the speed of the electron, $\beta = v/c$ and $\gamma = 1/\sqrt{\beta^2 - 1}$. Here, the electron beam consists of N electrons. In order to describe the electron beam by the macro-particle model with M macro-particles, we have to preserve the mass-to-charge ratio. As shown in Fig. 1, the macro-particle contains $a (= N/M)$ electrons, and the mass and charge are $m_m = am_e$, $q_m = ae$, respectively. The equation of motion of the single macro-particle is

$$c \frac{m_m}{q_m} \frac{d(\gamma\boldsymbol{\beta})}{dt} = \mathbf{E} + \mathbf{v} \times \mathbf{B}. \quad (2)$$

It is the same equation as the single electron, Eq. (1), because $m_e/e = m_m/q_m$. Therefore, the description of the beam motion by macro-particle model corresponds to an approximation by M charged particles with the mass, am_e , and the charge, ae . However, we have to select proper number of macro-particles, which preserves the property of the original beam, because the replacement by the macro-particles causes the loss of information about the beam. In order to study this mechanism, we introduce a transformation, particle pair transformation, to describe the replacement by the macro-particle in the next section.

PARTICLE PAIR TRANSFORMATION

In this section, we define a particle pair transformation to describe the replacement of an particle distribution by macro-particles. The original distribution consists of n_0 macro-particles with the mass, m_{m0} , and the charge, q_{m0} . The procedure of the transformation has the following five steps.

1. Calculate the center of the original particle distribution.
2. Choose the most distant particle from the center, and the nearest neighbor particle from it.
3. Calculate the average position about the above two particles.
4. Replace the two particles by a new macro-particle with $m_{m1} = 2m_{m0}$ and $q_{m1} = 2q_{m0}$ on the average position.

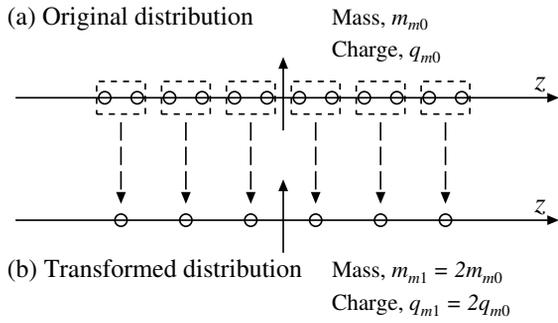


Figure 2: Particle pair transformation for one dimensional particle distribution.

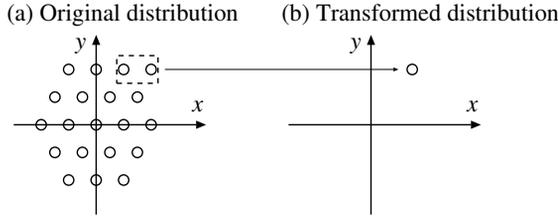


Figure 3: Particle pair transformation for two dimensional particle distribution.

5. Repeat step. 2 to step. 4 until all the original particles are replaced.

Figure 2 shows the particle pair transformation for one dimensional particle distribution. After the transformation, the number of transformed particles is reduced to $n_1 = n_0/2$. After i -th transformation, the number of transformed particles is reduced to $n_i = n_0/i$. The particle pair transformation for two dimensional particle distribution as shown in Fig. 3. The transformation for two dimensional case is almost same as the one dimensional case.

STATIC ELECTRIC FIELD

In the this section, we apply the transformation to simple distributions, and calculate the static electric field caused by the transformed particle distribution.

1D Particle Distribution

We think an one dimensional uniform particle distribution on z axis. The initial distribution consists of n_0 macro-particles with $m_{m0} = a_0 m_e$ and $q_{m0} = a_0 e$, and the particle distance is the same. The full width and the total charge of the distribution are 1.0 mm and -100 pC, respectively. The number of macro-particles is $n_0 = 16384$, and a_0 is 38333 electrons.

The static electric field of the one dimensional distribution is calculated by point-to-point model. The electric field at the j -th particle after the i -th transformation is

$$E_z(z_j) = \frac{q_{mi}}{4\pi\epsilon_0} \sum_{k \neq j} \frac{z_j - z_k}{|z_j - z_k|^3}. \quad (3)$$

Figure 4 shows the static electric fields with the number of transformations, $i = 0, 2, 4$ and 6 . As shown in Fig. 4,

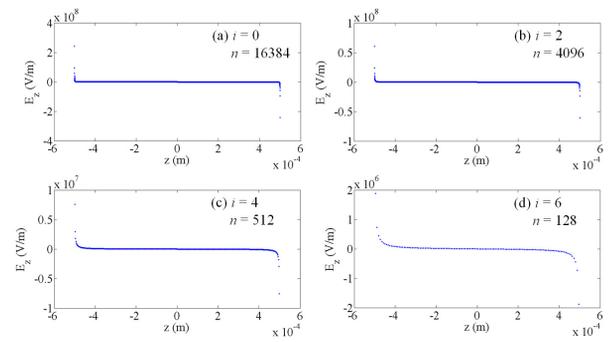


Figure 4: Static electric field of one dimensional particle distribution before and after particle pair transformations. The numbers of particles are 16384, 4096, 512, and 128.

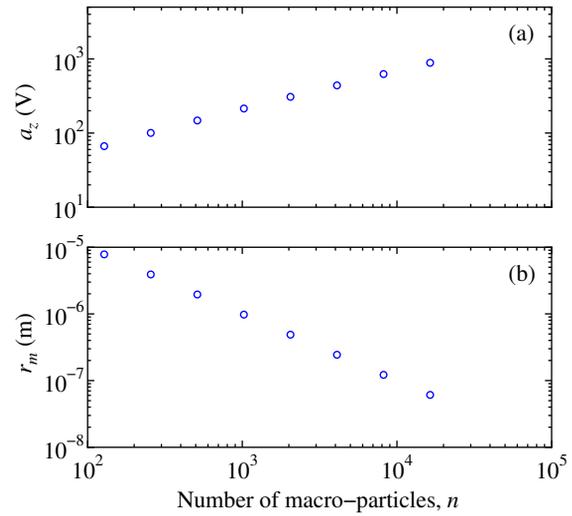


Figure 5: Area on z - E_z space, a_z , and average particle distance between nearest neighbor particles, r_m , for one dimensional particle distribution in Fig. 4.

the strengths of the electric field at the head and the tail decrease, when the number of macro-particles is reduced. After the transformation, the particle distance is stretched, and it reduces the electric field at the head and the tail.

In order to quantitatively analyze the effect of the transformation, we define the following two quantities. The first quantity is an area on the z - E_z space, which is calculated by

$$a_z = \sqrt{\langle z_c \rangle^2 \langle E_{zc} \rangle^2 - \langle z_c E_{zc} \rangle^2}, \quad (4)$$

where

$$z_c = z - \langle z \rangle, \quad (5)$$

$$E_{zc} = E_z - \langle E_z \rangle, \quad (6)$$

and $\langle \rangle$ indicates the distribution average. The second quantity is the average particle distance between nearest neighbor particles, r_m . Figure 5 shows a_z and r_m for the one dimensional distributions. It shows that a_z decreases and r_m increases, when the number of macro-particles is reduced by the transformation. As shown in the above, we can formulate the procedure to reduce the number of macro-particles by the

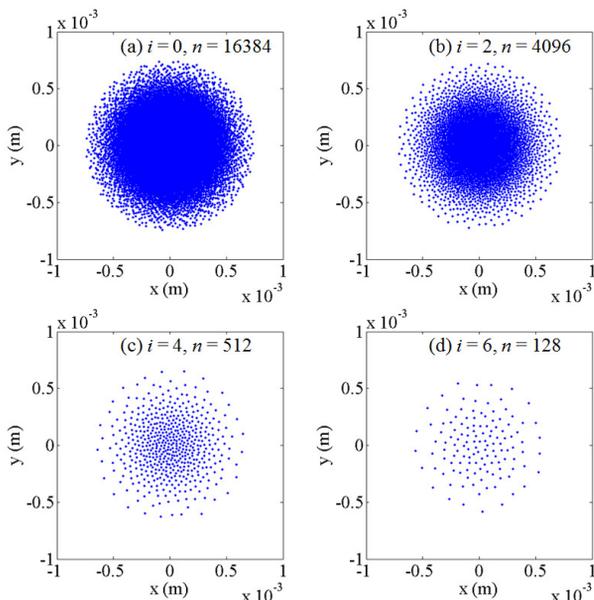


Figure 6: Two dimensional particle distribution on x - y plane before and after two pair particle transformation. The numbers of particles are 16384, 4096, 512, and 128.

simple transformation. Moreover, we can easily expand the transformation to higher dimensional particle distribution.

2D particle Distribution

We apply the particle pair transformation for two dimensional particle distribution on x - y plane. The transformation for two dimensional distribution is almost same as the one dimensional case, and is shown in Fig. 3.

Here, we think a circular gaussian distribution on x - y plane. The initial distribution consists of n_0 macro-particles with $m_{m0} = a_0 m_e$ and $q_{m0} = a_0 e$. The initial distribution was generated by General Particle Tracer (GPT) and quasi-random routine. The rms size and the total charge of the distribution are 0.25 mm and -100 pC, respectively. The number of macro-particles is $n_0 = 16384$, and a_0 is 38333 electrons.

The static electric field is calculated by point-to-point model. The electric field at the j -th particle is calculated by

$$\mathbf{E}(\mathbf{r}_j) = \frac{q_{mi}}{4\pi\epsilon_0} \sum_{k \neq j} \frac{\mathbf{r}_j - \mathbf{r}_k}{|\mathbf{r}_j - \mathbf{r}_k|^3}. \quad (7)$$

Figure 6 shows the particle distribution on the x - y plane with the number of transformations, $i = 0, 2, 4$ and 6 . After the transformation, the particle distance is stretched. Figure 7 shows the particle distribution on the x - E_x space. As shown in Fig. 7 (a), there are higher electric fields around the center for the initial distribution with $i = 0$. After the transformation, the higher electric field decreases. Figure 8 shows areas in the x - E_x and y - E_y spaces. For $i = 0$, a_x and a_y have larger values. After the transformation, these values converge for $i > 4$. As shown in the above, we confirmed the effectiveness of the transformation for two dimensional case.

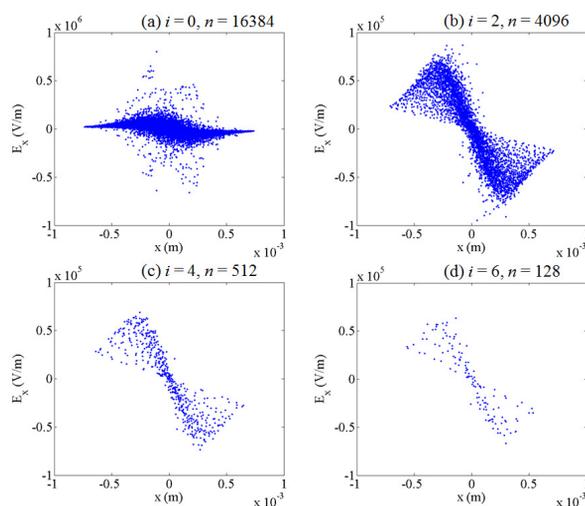


Figure 7: Particle distribution on x - E_x space for two dimensional distribution before and after particle pair transformations. The numbers of particles are 16384, 4096, 512, and 128.

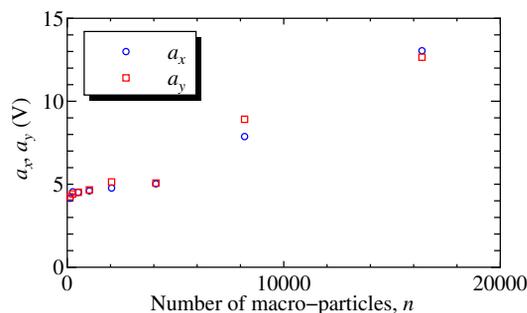


Figure 8: Areas on x - E_x and y - E_y spaces, a_x and a_y , for two dimensional particle distribution in Fig. 7.

SUMMARY

In order to study the effect of the number of macro-particles for charged particle beam, we defined a simple transformation to generate reduced distribution. The transformation was applied for one dimensional and two dimensional particle distributions. The static electric fields due to the transformed distributions were calculated. As a result, we confirmed the effectiveness of the transformation. As the next study, we plan to apply the transformation for three dimensional distribution, and to calculate the electro-magnetic field due to the movement of beam.

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REFERENCES

- [1] T. Miyajima, "Description of charged particle beam", in Proc. of the 9th Annual Meeting of Particle Accelerator Society of Japan, pp. 134-137 (2012).