

SPACE CHARGE EFFECT ESTIMATION FOR SYNCHROTRONS WITH THIRD-ORDER RESONANT EXTRACTION*

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Abstract

In proton and ion storage rings using the third-order resonance extraction mechanism, beam particles are slowly extracted from the ring when reaching the resonance stop-band. Typically at beam injection, the horizontal tune is set to a value close to the resonance value. The tune is then moved towards the resonance value to trigger beam extraction in a controlled way. The tune shift generated by space charge forces needs to be taken into account. For this, the incoherent space-charge tune shift for protons of the MedAustron accelerator main ring has been evaluated. This has been performed by multi-particle tracking using an optics model based on MADX, considering a realistic Gaussian beam distribution and exact non-linear space charge electric field forces. The MedAustron accelerator is in the beam commissioning phase and is planned to start medical commissioning at the end of 2015.

INTRODUCTION

The recently constructed MedAustron synchrotron based accelerator in Austria is one of the novel therapy accelerators intended to use protons and light ions for clinical treatment, as already operating accelerators as e.g. CNAO in Italy and HIT in Germany. A collaboration was set up with CERN, TERA (CNAO), GSI in Germany and Onkologie Institute in Czech Republic. The design is based on the PIMMS study [1]. A report of the actual MedAustron beam commissioning can be found in [2].

For the clinical treatment a beam extracted in a slow controlled process over a couple of seconds is necessary to facilitate the measurement and control of the delivered radiation dose. The third-order resonance extraction method [1] can be used to extract particles from a synchrotron over a large number of turns and in a spill time period of 1-10 sec.

Third Order Resonance Extraction

In the slow extraction process, a sextupole field is turned on to excite the resonance. The extraction is activated by accelerating the beam into the resonance with a so called betatron core. By this mechanism, the horizontal tune is effectively moved towards the third order integer resonance tune $Q_x = 1.666$. Thus, a precise control of the beam tune at extraction is necessary.

Space charge forces, in particular for protons, may induce an incoherent tune shift that may perturb the extraction process.

Furthermore, the space charge tune shift at beam injection into the ring, when the beam energy is low, may induce resonance crossing and beam losses.

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Previous space charge estimations for the main ring have been performed as part of the design studies [1]. The estimation has been revisited with the assumption of a transverse Gaussian beam distribution, by modelling the space charge electric field and forces that are highly non-linear and to take into account recently updated machine design parameters, in particular an injected beam energy of 7 MeV [3, 4].

Possible space charge mitigations are vertical emittance dilution at injection and adjustment of the chromaticity to partially compensate for beam losses [1].

SPACE CHARGE EFFECT ESTIMATION

MedAustron Accelerator Parameters

During a cycle, the phases concerning the main ring are essentially four: injection, beam capture, acceleration using synchrotron radio frequency (RF) and slow beam extraction from the ring to the irradiation rooms. Main proton beam parameters assumed for the estimation of the effect in the MedAustron accelerator are listed in Table 1.

Table 1: MedAustron Ring Optics Parameters Assumed

Parameter	Value
Circumference (m)	77.65
Energy at injection/extraction (MeV)	7.0 / 250.3
Max. num. of particles per extracted spill	2×10^{10}
Normalized emittance ϵ_x, ϵ_y (π m rad)	0.519×10^{-6}
Q_x - Horizontal design tune inject./extract.	1.739-1.666
Q_y - Vertical design tune inject./extract.	1.79/1.79
Q_x' - Natural horizontal chromaticity at inj.	-0.6
Q_y' - Natural vertical chromaticity at inj.	-1.9
D_x - Horizontal dispersion maximum (m)	8.44

Analytic Estimation

The incoherent tune shift can be estimated analytically.

For this, one can use a linear approximation of the beam electric field, as given by

$$E_x = \frac{\lambda}{2\pi\epsilon_0} \frac{x}{\sigma_x(\sigma_x + \sigma_y)} \quad E_y = \frac{\lambda}{2\pi\epsilon_0} \frac{y}{\sigma_y(\sigma_x + \sigma_y)} \quad (1)$$

where λ is the linear charge distribution and using SI units. Using the linear electric field approximation and assuming a Gaussian charge distribution in the transverse plane, it is possible to derive the incoherent linear vertical

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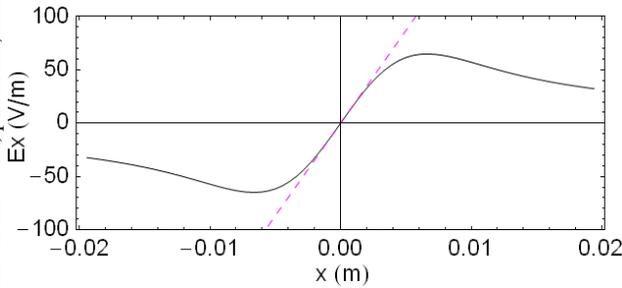


Figure 1: Electric field [6] along the horizontal axis ($y=0$) and the linear electric field approximation (dashed), for a Gaussian beam.

tune shift which is given by (see for example [5]),

$$\Delta v_y = -\frac{r_p N_0}{2\pi\beta^2\gamma^3\sqrt{2\pi}\sigma_z} \int_0^L \frac{\beta_y(s)}{\sigma_y(s)(\sigma_x(s)+\sigma_y(s))} ds \quad (2)$$

where β_y is the vertical beta function, r_p the proton radius, N_0 the number of particles, $\sqrt{2\pi}\sigma_z$ the total bunch length, s the longitudinal coordinate and the integral is performed over the whole ring circumference. Similar equation is obtained for the x direction. All the physical parameters on the right side of eq. (2) are positive and thus the space charge tune shift is always negative.

Furthermore, the tune shift scales linearly with the number of particles N_0 , at least in first approximation and for relatively low space charge forces.

In Figure 1, the linear electric field approximation eq. (1) is compared to the correct field for a Gaussian charge distribution [6], for a 7 MeV beam at injection with $(\sigma_x, \sigma_y) = (5.5\text{mm}, 3.2\text{mm})$, bunch population of 2×10^{10} and total bunch length of 67 m.

The linear electric field approximation represents well the electric field near the beam center, as can be seen in Fig. 1. Though, at large distances, it can significantly over-estimate the size of the space charge forces.

Thus, the analytic estimation eq. (2) is the tune shift of a particle near the beam center which represents also the largest tune shift within the beam particles.

We have used eq. (2) to estimate the linear tune shift as shown in Figure 2 and Figure 3 for a circulating beam immediately after RF capture and during the acceleration ramp to 250 MeV as shown in Figure 4 and Fig. 5.

The vertical tune shift is always larger than the horizontal tune shift, since the vertical beam size is on average smaller implying larger space charge forces.

Beam Tracking Simulations

To estimate the space-charge effect in more detail, we simulate a beam circulating in a real lattice as supplied by MADX. To compute the space charge forces one can consider the two dimensional Gaussian charge distribution in the x - y plane at the z location of the particle.

As mentioned above, the electric field of a transverse Gaussian charge distribution is given by the Bassetti-Erskine formula [6]. It is an analytical expression in terms of the complex error function which assumes that

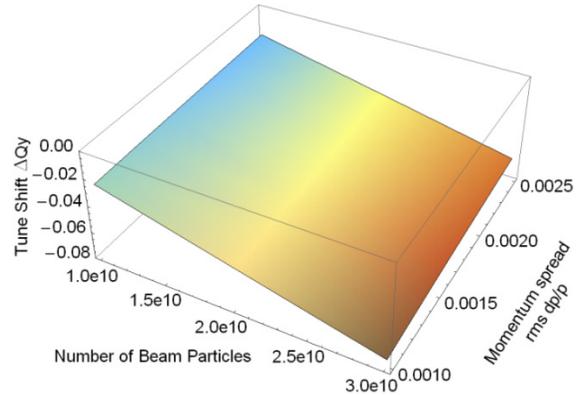


Figure 2: Vertical tune shift as a function of momentum spread and number of beam particles, just after RF beam capture, estimated by eq. (2).

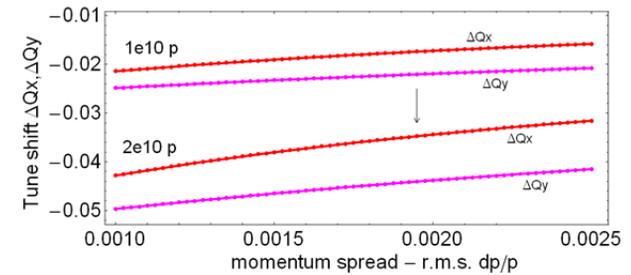


Figure 3: Tune shift immediately after beam RF capture as a function of momentum spread, for 1 and 2×10^{10} proton beams, estimated using eq. (2).

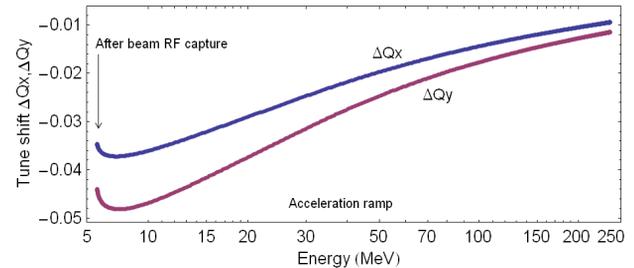


Figure 4: Estimated tune shifts during an acceleration ramp to 250MeV, assuming $dp/p \sim 2 \times 10^{-3}$, using eq. (2).

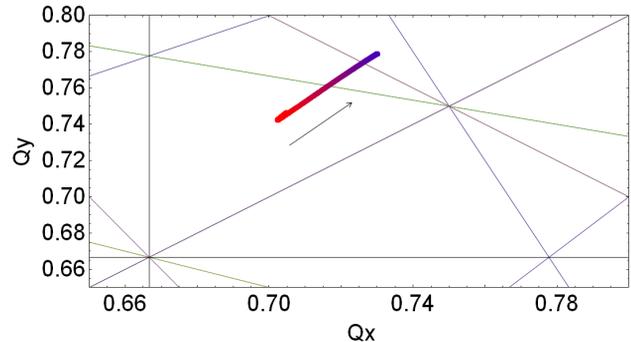


Figure 5: Estimated the tune shift evolution in time during acceleration, using eq. (2).

the two-dimensional charge distribution is Gaussian with r.m.s. beam size dimensions σ_x and σ_y and a 2D beam with a defined total charge per unit length. In our case, the bunch length is much larger than its transverse

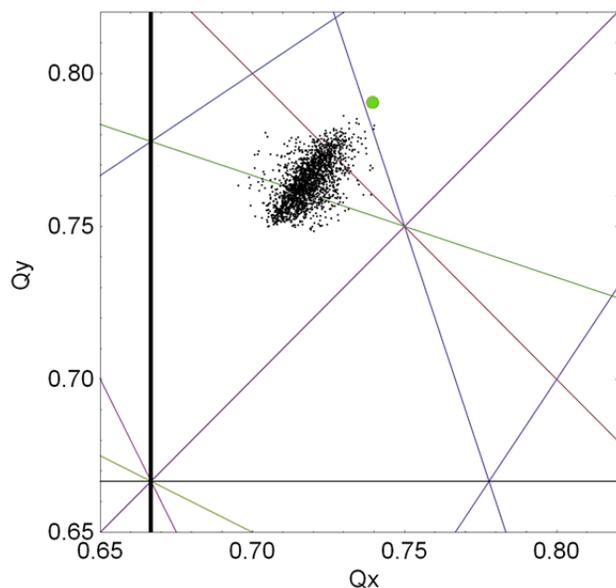


Figure 6: Fractional tune footprint of a beam at injection obtained by beam tracking simulations. Just the first 2000 macro-particles are shown for better resolution. The green dot indicates the tunes with no space charge. The third order integer fractional tune for extraction is $Q_x = 0.666$.

dimensions which justifies the use of the Bassetti-Erskine formulation.

The bunch is defined in the laboratory frame with a macro-particle distribution in the six-dimensional phase space (x, p_x, y, p_y, z, dp) with the phase space coordinates normalized with respect to the reference particle coordinates. Through the sectormap command, MADX supplies the 6×6 matrices R used for particle tracking. Thus, a linear lattice and natural chromaticity has been assumed. The linear matrices are typically symplectic.

Particles are transported along the ring from element to element. At each element, space charge forces are recalculated and beam particles are kicked according to the electric field experienced at their transverse position in the bunch. The kick is effectively applied by updating the particle momentum. From the equations of motion of a particle in a bunch one obtains the momentum update due to the beam electric field by

$$y'' = \frac{dp_y}{ds} = \frac{e E_y}{\beta^2 \gamma^3 m c^2} \quad (3)$$

where E_y is the beam electric field. A similar equation is obtained for the x direction.

To compute the particle tunes, the beam is tracked for six turns and the eigenvalues of the perturbed one turn matrix are calculated following the method in [7].

Alternatively, the tunes can be inferred by the fast Fourier transform of the particle position recorded for several hundred turns.

The result of the particle tracking simulations including the space charge effect is shown in Figure 6. Each particle experiences a different electric field depending on its transverse position, this results in an incoherent tune

spread within the bunch. The effect is additionally smeared by the betatron oscillations.

The fractional tune footprint shown in Figure 6 is for a 2×10^{10} proton beam as injected into the main ring at 7 MeV. The tune shift ranges between $\Delta v_{x,y} \sim -0.01$ and -0.04 for the beam particles with an average of -0.02 and -0.025 respectively for the horizontal and vertical planes. The largest tune shift -0.04 obtained by simulations is in good agreement with the analytic estimation eq. (2).

Finally, an estimate of the vacuum chamber effect on the tune shift *via* image charges can be obtained following [8]. In first order approximation, the vacuum chamber walls are approximated as parallel plates and considering MedAustron characteristic beam size, distance to plates and distance to dipole magnet iron plates, the vacuum chamber give a ~ 5 - 15% positive contribution to the total space charge tune shift.

Next studies should include the realistic particle distribution and tracking with Particle-in-Cell codes.

CONCLUSIONS

The tune shift should be taken into account to avoid beam losses at injection and at extraction when the tune is carefully moved towards the third order resonance tune to trigger the extraction mechanism. We have computed the tune shift using a macro-particle simulation approach that agrees well with analytic estimates. Furthermore, an analytic estimation indicates that following injection of a 2×10^{10} proton beam into the main ring, the largest expected tunes' shift is in the order of -0.04 with the vertical tune being larger. Following RF capture and during the first part of acceleration, the beam sizes decrease due to emittance adiabatic damping and the tunes shift reaches the largest value. During beam acceleration, the beam energy is the dominating term and the tune shifts decrease to stabilize at ~ -0.01 .

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