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# RECTILINEAR COOLING SCHEME FOR BRIGHT MUON SOURCES*

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*Abstract*

A fast cooling technique is described that simultaneously reduces all six phase-space dimensions of a charged particle beam. In this process, cooling is accomplished by reducing the beam momentum through ionization energy loss in absorbers and replenishing the momentum loss only in the longitudinal direction rf cavities. In this work we review its main features and describe the main results.

## INTRODUCTION

The initial muon beam occupies a relatively large phase-space volume which must be compressed by several orders of magnitude to obtain high-luminosity collisions. Furthermore, this phase-space reduction must be done within a time that is not long compared to the muon lifetime (2 μs in rest frame). Ionization cooling is currently the only feasible option for cooling a muon beam [1]. This technique is not very practical for protons, which would have frequent nuclear interactions, or electrons, which would have bremsstrahlung, but is practical for muons, and cooling rates compatible with muon lifetimes are possible. The main goal of this paper is to develop a potential baseline cooling lattice for a muon related applications and evaluate its performance. We also discuss a possible variant of this channel.

## LATTICE DESIGN

Three different geometries for ionization cooling towards micron-scale emittances as required for a muon collider have been previously considered: (a) A ring, (b) a helix commonly known as a Guggenheim helix, and (c) a rectilinear channel. The common feature for all cases was that the solenoids were slightly tilted to generate upward dipole fields. In the third case, essentially the same cells from a ring or a Guggenheim, including their coil tilts and resulting upward dipole fields, are laid out in straight (rectilinear) geometry. The solenoid focusing is so strong, compared with the dipole deflections, that the closed orbits are merely displaced laterally, but continue down the now straight lattice. This rectilinear scheme was proposed for the first time by Balbekov [2] and is represented in Fig. 1. Despite its much simpler geometry, it was found [3] that its cooling performance was essentially the same as with rings or a helix. As a result, the rectilinear scheme will be considered our new baseline cooling lattice.

Figure 1(a) shows a cross section of a cell of the cooling rearly stage. The rf is at 325 MHz operating at 19 MV/m and a phase of approximately 41 degrees. The cell contains two coils of opposite polarity, yielding an approximate sinusoidal variation of the magnetic field in the channel with a peak on-axis value of 2.6 T. The coils are tilted in opposite directions by 0.9 degrees. Figure 1(b) shows a cell of a late cooling stage that is required to cool the beam transversely to ≤0.3 mm. The cell consists of six solenoids which surround four 650 MHz cavities in the cell center. As before, the geometry of the lattice is such that the absorber is located at beta minima (blue dashed curve).

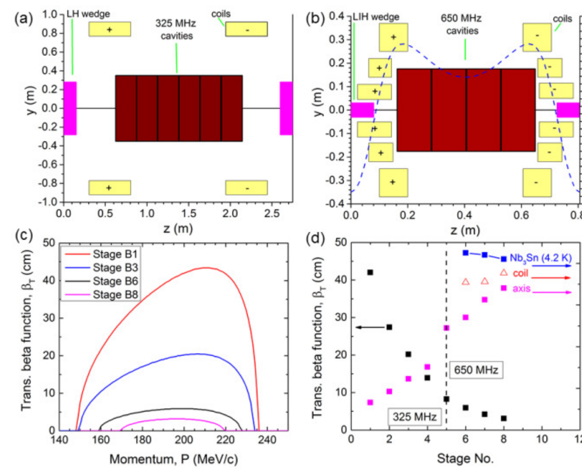


Figure 1: Lattice characteristics of the proposed rectilinear 6D cooling channel: (a) Early stage, (b) last cooling stage, (c) beta function for different stages, and (d) beta and maximum axial field per stage as well as allowable field limit for $Nb_3Sn$.

Figure 1(c) shows the transverse beta function at the absorber versus momentum for four stages out of the total 12 stages of the channel. Note that the transverse beta function becomes progressively smaller from stage to stage by scaling down the cell dimensions and raising the on-axis magnetic field [Fig. 1(d)]. As a result, the minimum beta function drops from 42.0 to 3.0 cm while the on-axis peak magnetic field increases from 2.6 to 13.6 T. This can become a challenge since the operating current in a superconducting magnet must be smaller than the critical current corresponding to the peak field in the coil. To highlight this last fact we also plot in Fig. 1(d) the maximum local fields in the coils for the last three stages (triangles) and compare them to the published maximum allowable field (squares) for the used coil current density, assuming a Nb3Sn conductor. Our findings indicate that the needed fields are consistent with the critical limits of existing conductor technology but the last stages are barely within the limits of Nb3Sn. A recent magnet feasibility study [4] revealed that for a more stable operation a 1.9 K operating magnet temperature is preferred for this stage. This would allow the Nb3Sn inner solenoids to operate at 85% of the load line at operational current.

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## SIMULATION AND THEORY

Figure 2 shows the overall cooling performance as simulated by ICOOL [5]. As noted in previous studies [3] the new born muon beam is first bunched and phase rotated (step #1) so that the initial single bunch with very large energy spread is converted into a train of bunches with much reduced energy spread of which we use 21. After phase rotation, the beam enters a multistage cooling channel (step #2). We found that the four-stage rectilinear channel described in Ref. 3 reduces the transverse emittance by a factor of ~11.5 and the longitudinal emittance by a factor ~19.5. The transmission is 52% including muon decays. After the bunch merging [6] (step #3), 15% of the beam is lost while the longitudinal and transverse emittance was found to increase to 10.0 and 5.1 mm, respectively. The now single is sent into another multistage channel for further cooling (step #4). We found [3] that the eight-stage rectilinear channel reduced the transverse emittance to 0.28 mm and the longitudinal emittance to 1.57 mm. At the end of the channel, the transmission was 40% including decays and the average momentum has dropped to 200 MeV/c.

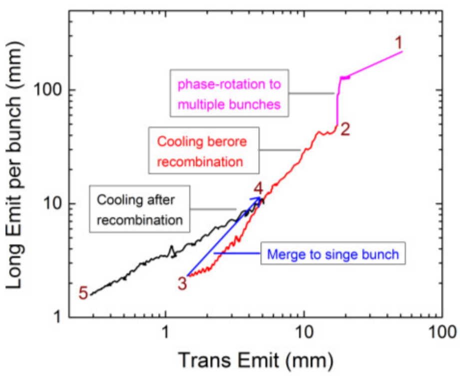


Figure 2: End-to-end simulation of a 6D cooling channel.

Based on our simulations, 12 cooling stages are sufficient to reduce the transverse and longitudinal emittances by a factor of ≈100 and factor of ≈10, respectively. As a result the 6D emittance has fallen by a factor of $10^5$ with a net transmission of ~20.8% while the overall distance needed to achieve this was ~0.96 km. While several parameters of the aforementioned scheme such as the rf gradient, window thickness, focusing field still need to be evaluated for practicality and cost the aforementioned facts suggest that the rectilinear channel has the potential to be a promising solution.

The mechanism of ionization cooling has some similarity with synchrotron damping: lose all 3 components of their momentum in the absorber while only the longitudinal component is restored by the rf field. The detailed expressions for the longitudinal and transverse emittance can be found elsewhere [7]. In Fig. 3 we plot the transverse and longitudinal emittances as a function of distance along the channel for the section after the bunch recombination. The solid lines depict the estimated emittances from ICOOL while the open circles depict the theoretical results from Ref [3].A comparison between theory and simulation is a key step since deviations are usually associated with poor dynamic acceptance, chromatic effects, or poor matching into the channel. A glance in Fig. 3 indicates that the theory predicts relatively well the cooling performance of our rectilinear channel. Quantitatively, the error between the theoretical values and numerical results is less than 18%. The most likely source of discrepancy at 263.5 m is mismatch caused by the rf frequency shift from 325 MHz to 650 MHz at that point.

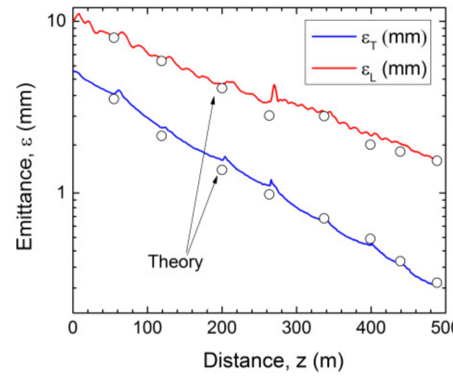


Figure 3: Cooling performance vs. distance. Solid lines are simulation results and circles are results from theory.

## SPACE-CHARGE EFFECTS

While there has been significant progress in the design and simulation of ionization cooling channels, little is known about the impact of particle-particle interactions in the whole design. In this section, we numerically examine [7] the influence of space-charge fields on the cooling process of muon beams with Warp, a well-established code for space-charge simulation that is properly modified so that it fully incorporates all basic particle-matter interaction physical processes such as energy loss, scattering, straggling and muon decay. Before examining space-charge effects, a study was carried out comparing ICOOL and Warp results without space charge, checking correctness of the models. We found that the difference on the above parameters is within a 3% error.

The evolutions of the transverse and longitudinal emittances as a function of distance and degree of beam intensity are shown in Fig. 4. At the early stages ($z < 100$ m) we see little difference in the emittances. However, at later stages, especially after $\sigma_z < 40$ mm, cooling becomes strongly correlated to the number of muons in the bunch. This indicates that $N_\mu$ is the key parameter that governs the final emittance in an ionization cooling channel. Our simulations also predict that space charge is accompanied with a substantial particle loss which becomes more noticeable within the intensity regime of a muon collider.

A salient feature of our study is that the transverse emittance is not affected by the bunch charge, suggesting that the deterioration of the cooling performance is primarily caused by the longitudinal space-charge force. This is not unexpected, since the force from the multi-Tesla magnetic field appears sufficient enough to balance the beam transversely. On the other hand, the space

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charge increases the length of the bunch, pushing more particles out of the rf bucket eventually leading to emittance growth and particle loss. Quantitatively, for a muon collider, longitudinal cooling below 1.3 mm is not viable under the current circumstances.

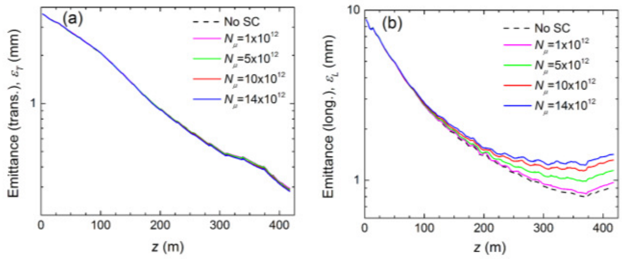


Figure 4: Emittance evolution and its dependence with the bunch charge.

One possible method to reduce this loss is to increase the rf gradient, which increases the depth of the bucket. In Fig. 5 we progressively raise the gradient from the initial design value of 15.5 MV/m and show results for one case: $N_\mu=12 \times 10^{12}$ initial muons. As the rf gradient grows, the final achieved longitudinal emittance decreases and approaches the optimum cooling emittance, which is shown by the red half-filled circle. In addition, we find that the transmission improves by ~5%, for both cases, which fully compensates the loss from space-charge effect. Conversely, the transverse emittance is unaffected and remains mostly flat and close to the optimum value which is depicted by the red half-filled square.

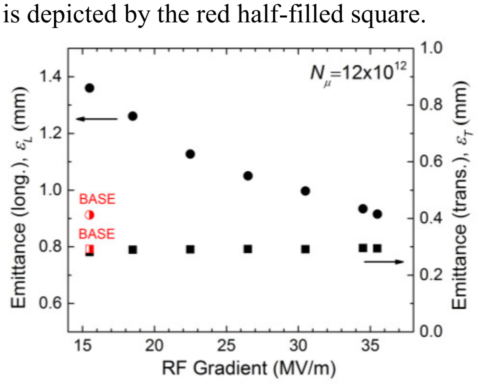


Figure 5: Final emittance achieved vs. gradient for a given space-charge intensity.

## ALTERNATIVE OPTIONS

A key requirement of that design is that it requires vacuum rf cavities to operate at 5 T or greater magnetic fields. This can become a technological challenge since the performance of a normal conducting cavity may degrade when the cavity is exposed in a strong axial magnetic field [9, 10]. To mitigate the possible problems in high magnetic fields we investigated [11] a hybrid approach so that to convert a vacuum rf 6D cooling channel to an high-pressure (HP) rf version. The scheme uses HP gas to avoid cavity breakdown, along with discrete LiH absorbers to provide the majority of the energy loss. Since our primary purpose is to avoid degradation of the cavity gradient to high magnetic field we use only enough gas to accomplish this task. The desired gradient of a 650 MHz cavity is 26-28 MV/m. We assumed that a pressure of 34 atm at room temperature will be enough to satisfy that goal.

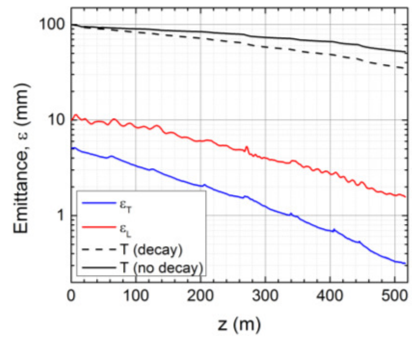


Figure 6: Emittance and transmission evolution in a hybrid cooling channel with gas filled cavities.

The transverse and longitudinal emittance and the transmission are shown as function of distance along the channel in Fig. 6. Note that the dashed curve shows the transmission of muons with the decay option disabled in the simulation. It is worth noting that after a distance of 515 m (9 Stages) the 6D emittance has fallen by a factor of 1000 with a transmission of 35% (52% with the decays disabled). Note that a transverse emittance ≈ 0.3 mm is the baseline requirement for a Muon Collider at the end of the 6D cooling sequence. We can conclude from the results in Fig. 4, that a hybrid channel would achieve very similar performance to an equivalent channel with gas free cavities [12].

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## RFERENCES

[1] D. Stratakis et al., Phys. Rev. ST Accel. Beams **17**, 071001 (2014).
[2] V. Balbekov, Muon Accelerator Program Document No. 4365 (2013) [http://map-docdb.fnal.gov].
[3] D. Stratakis and R. B. Palmer, Phys. Rev. ST Accel. Beams **18**, 031003 (2015).
[4] H. Witte, D. Stratakis, J. S. Berg, F. Borgnolutti, Proceedings of the 2014 IPAC, Dresden, Germany, p. 2740 (2014).
[5] R. C. Fernow, Proceedings of the 2005 PAC, Knoxville, TN, p. 2651 (2005).
[6] Y. Bao, G. Hanson, R. B. Palmer, and D. Stratakis, Proc. IPAC2015, Richmond, VA, TUPWI040 (2015).
[7] D. Stratakis, R. B. Palmer and R. B. Palmer, Phys. Rev. ST Accel. Beams **18**, 031003 (2015).
[8] E. Keil, Nucl. Instrum. Meth. A **532**, p. 249 (2004).
[9] D. Stratakis, Nucl. Instrum. Meth. A **709**, p. 1 (2013).
[10] D. Stratakis et al., Phys. Rev. ST Accel. Beams **14**, 011001 (2011).
[11] D. Stratakis, Proceedings of IPAC 2014, Dresden, Germany, p. 1401 (2014).
[12] D. Stratakis et al., Phys. Rev. ST Accel. Beams **16**, 091001 (2013).