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ANALYSIS OF FEL-BASED CEC AMPLIFICATION AT HIGH GAIN LIMIT*

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Abstract

An analysis of Coherent electron Cooling (CeC) amplifier based on 1D Free Electron Laser (FEL) theory was previously performed with exact solution of the dispersion relation, assuming electrons having Lorentzian energy distribution [1]. At high gain limit, the asymptotic behaviour of the FEL amplifier can be better understood by Taylor expanding the exact solution of the dispersion relation with respect to the detuning parameter [2, 3].

In this work, we make quadratic expansion of the dispersion relation for Lorentzian energy distribution [1, 4] and investigate how longitudinal space charge and electrons' energy spread affect the FEL amplification process.

INTRODUCTION

FEL instability plays a key role in a FEL-based CeC system: it determines the cooling force, the effective bandwidth, as well as the saturation level of the system[5]. An analytical analysis based on numerically solving the exact 1-D FEL dispersion relation had been previously performed for a warm electron beam with Lorentzian energy distribution [1]. Although the analysis in [1] is based on an exact solution of the FEL dispersion relation and takes into account both the warm beam and longitudinal space charge effects, the final solutions involve an inverse Fourier transformation that can only be evaluated numerically. During analysing and optimizing a CeC system, it is often beneficial to have analytical time-domain solutions, which provides scaling laws as well as more intuitive insights.

Recently, an analysis of CeC based on 1-D FEL at high gain limit has been performed [2] for cold electron beam. In the analysis, the solutions of 1-D FEL dispersion relation have been Taylor expanded up to the quadratic terms with respect to the detuning parameter. The approximation is valid if the FEL gain is large enough such that the beam spectrum is dominated by the components with small detuning parameter. The approximation significantly simplified the final results and allow for an analytical expression for the time-domain solution of the cooling force.

As an attempt to extend the analysis of [2], we start with the 1-D FEL dispersion relation of a warm electron beam as adopted in [1], and take the Taylor series of its solutions up to the quadratic terms in detuning as the approximate solutions. Similar to the results obtained in [2], the approximation leads to an electron density wave-packet with a Gaussian envelope in the time domain. Both the amplitude and the RMS width of the Gaussian envelope depend on the longitudinal space charge parameter and the energy spread parameter of the electron beam, which makes it possible to investigate how these beam parameters affect the cooling process.

The contents are organized as follows. In section II, we present the set of equations to be used to find the approximate solutions to the dispersion relation for arbitrary space charge and energy spread parameters. We then solve the equations for zero space charge parameter in section III, and investigate how energy spread parameter alone affects the gain and coherent length of the cooling force. For arbitrary space charge and energy-spread parameter, numerical method is required to solve the set of equations and we present our results in section IV. Section V consists our summary.

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SOLUTION OF 1-D FEL DISPERSION RELATION AT HIGH GAIN LIMIT

Assuming the following energy distribution of the electron beam*: $\hat{F}\left(\hat{P}\right)=\hat{q}\Big/\left[\pi\left(\hat{P}^2+\hat{q}^2\right)\right]$, the 1-D FEL dispersion relation reads

$$s\left[\left(s+\hat{q}+i\hat{C}\right)^2+\hat{\Lambda}_p^2\right]=i\ , \quad (1)$$

where s is the complex growth rate of 1-D FEL instability in unit of Γ, $\Gamma\equiv\left[\pi j_0\theta_s^2\omega\Big/\left(c\gamma_z^2\gamma I_A\right)\right]^{\frac{1}{3}}$ is the 1-D gain parameter, $\hat{\Lambda}_p\equiv\sqrt{4\pi j_0\Big/\left(\gamma_z^2\gamma I_A\right)}\Big/\Gamma$ is the normalized space charge parameter, $\hat{P}\equiv\left(E-E_0\right)/\left(\rho E_0\right)$ is the normalized energy deviation of an electron with energy E, ρ is the Pierce parameter, $\hat{q}$ is the normalized energy spread parameter, I_A is Alfven current, $\theta_s\equiv K/\gamma$ is the trajectory angle of electrons' motion, K is the undulator parameter, $\hat{C}\equiv\left[k_w-\omega\Big/\left(2c\gamma_z^2\right)\right]\Big/\Gamma$ is the normalized detuning parameter and $\gamma_z=\gamma\Big/\sqrt{1+K^2}$.

Let the growing root of eq. (1) to be

$$s_1=\lambda_0+\lambda_1\Delta\hat{C}+\lambda_2\Delta\hat{C}^2+\ldots \quad (2)$$

where $\Delta\hat{C}\equiv\hat{C}-\hat{C}_0$ and $\hat{C}_0$ is the detuning corresponding to the maximal growth rate, i.e.

$$\left.\frac{d\,\mathrm{Re}\left(s\right)}{d\hat{C}}\right|_{\hat{C}_0}=0\ . \quad (3)$$

Inserting eq. (2) into eq. (1) and using eq. (3) leads to

$$\lambda_0=\lambda_R+i\lambda_I\ , \quad (4)$$

* We adopt the formalism and definition of variables of [4] .

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where λ_R and λ_I are to be determined by the following two equations:

$$\lambda_R^2 = \lambda_I^2 \frac{3-4\hat{\Lambda}_p^2\lambda_I}{1-4\hat{\Lambda}_p^2\lambda_I}\ , \tag{5}$$

and

$$(\lambda_R+\hat{q})^2 = \frac{\left(3-4\hat{\Lambda}_p^2\lambda_I\right)\left(1-4\hat{\Lambda}_p^2\lambda_I\right)}{8\lambda_I\left(1-2\hat{\Lambda}_p^2\lambda_I\right)}\ . \tag{6}$$

Once λ_0 is solved from eqs. (5) and (6), the resonant detuning parameter, $\hat{C}_0$, as well as the linear and quadratic coefficients, λ_1 and λ_2, can be obtained from

$$\hat{C}_0 = \frac{\left(\lambda_R^2-\lambda_I^2\right)\left(\lambda_R+\hat{q}\right)}{2\lambda_I\lambda_R} - \lambda_I\ , \tag{7}$$

$$\lambda_1 = \frac{2\lambda_0^2\left(\lambda_0+\hat{q}+i\hat{C}_0\right)}{i2\lambda_0^2\left(\lambda_0+\hat{q}+i\hat{C}_0\right)-1}\ , \tag{8}$$

and

$$\lambda_2 = \lambda_0^2\left(\lambda_1+i\right)^3 - \frac{i\lambda_1^2}{\lambda_0}\left(\lambda_1+i\right)\cdot \tag{9}$$

ZERO SPACE CHARGE LIMIT

In the limit of $\hat{\Lambda}_p^2 \to 0$, eq. (4)-(9) yield $\hat{C}_0 = \hat{q}/\sqrt{3}$, $\lambda_0 = 2e^{i\frac{\pi}{6}}\lambda_R\big/\sqrt{3}$, $\lambda_1 = -2\lambda_R i/(3\lambda_R+\hat{q})$ and

$$\lambda_2 = \frac{4}{3}e^{-i\frac{\pi}{6}}\frac{\lambda_R\left(\lambda_R+\hat{q}\right)}{\left(3\lambda_R+\hat{q}\right)^3}\left[\lambda_R\left(\lambda_R+\hat{q}\right)^2 - \frac{3\sqrt{3}}{2}\right].$$

where

$$\lambda_R\left(\hat{q}\right) = \frac{1}{6}\left[A\left(\hat{q}\right) + \frac{4\hat{q}^2}{A\left(\hat{q}\right)} - 4\hat{q}\right],$$

and $A\left(\hat{q}\right) \equiv \left[9\left(9\sqrt{3}+\sqrt{243+32\sqrt{3}\hat{q}^3}\right)/2 + 8\hat{q}^3\right]^{\frac{1}{3}}$.

In the frequency domain, the electron density wave-packet is given by the following expression [1]:

$$\tilde{j}_1\left(\hat{z},\hat{C}\right) = \tilde{j}_1\left(0,\hat{C}\right)\sum_{i=1}^{3}B_i\left(\hat{C}\right)s_i\left(\hat{C}\right)e^{s_i\left(\hat{C}\right)\hat{z}}\ , \tag{10}$$
$$\approx B_1\lambda_0 e^{s_1\left(\hat{C}\right)\hat{z}}\,\tilde{j}_1\left(0,\hat{C}_0\right)$$

where

$$B_1 = \frac{-\left(s_2+s_3\right)-i\hat{q}^2 s_2 s_3 - i\hat{q}/\sqrt{3}}{\left(\lambda_0-s_2\right)\left(\lambda_0-s_3\right)}\ ,$$

$$s_2 = \frac{e^{i\frac{\pi}{6}}}{3\sqrt{3}}\left[e^{i\frac{2\pi}{3}}A\left(\hat{q}\right) - \frac{e^{i\frac{\pi}{3}}4\hat{q}^2}{A\left(\hat{q}\right)} - 4\hat{q}\right],$$

and

$$s_3 = \frac{e^{i\frac{\pi}{6}}}{3\sqrt{3}}\left[e^{-i\frac{2\pi}{3}}A\left(\hat{q}\right) - \frac{e^{-i\frac{\pi}{3}}4\hat{q}^2}{A\left(\hat{q}\right)} - 4\hat{q}\right].$$

For a delta-like electron current density modulation, $j_1\left(z,t\right) = -\frac{Z_i e}{S}\beta_z c\,\delta\left(z-\beta_z ct\right)$ and

$$\tilde{j}_1\left(0,\hat{C}_0\right) = -\frac{Z_i e}{S}\cdot \tag{11}$$

Inserting eq. (11) into eq. (10) and performing the inverse Fourier transformation leads to

$$j_1\left(z,t\right) = -\frac{c\Gamma\gamma_z^2}{\pi}e^{ik_w z}e^{i2\gamma_z^2 k_w(z-ct)}\int_{-\infty}^{\infty}\tilde{j}_1\left(z,\hat{C}\right)e^{-i2\gamma_z^2\Gamma(z-ct)\hat{C}}d\hat{C}\ ,$$
$$= \frac{Z_i eck_0}{S\sqrt{\pi}}B_1\left(\hat{C}_0\right)\lambda_0\frac{e^{2\lambda_0\rho k_w z}}{\sqrt{-\lambda_2}}\sqrt{\frac{\rho}{2k_w z}}e^{ik_w z}e^{ik_0(z-ct)}e^{-\frac{\left(t-t_p(z)\right)^2}{2\sigma_t^2}} \tag{12}$$

where $k_0 = 2\pi/\lambda_{opt}$, λ_{opt} is the optical wavelength of the FEL, $t_p\left(z\right) = z\left[1-\lambda_1/\left(i2\gamma_z^2\right)\right]/c$ and $\sigma_t^2 = -\lambda_2 k_w z/\omega_0^2\rho$. According to eq. (12), the wave-packet moves with the velocity $v_g \approx \eta c + (1-\eta)v_z$ with $\eta = (\lambda_R+\hat{q})/(3\lambda_R+\hat{q})$ and the RMS width of the wave-packet is

$$\sigma_{t,rms} = \frac{\left|\lambda_2\right|}{k_0 c}\sqrt{\frac{-k_w z}{\rho\,\mathrm{Re}\left(\lambda_2\right)}}\ . \tag{13}$$

As shown in Fig. 1, the coherent length of the wave-packet increase with the energy spread for the relative energy spread much smaller than the Pierce parameter. For energy spread larger than the Pierce parameter, the coherent length starts to decrease with increasing energy spread.

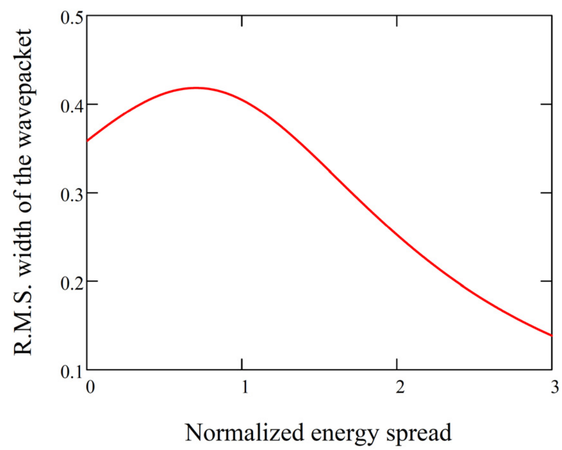

Figure 1: RMS width of the wave-packet as a function of the normalized energy spread parameter for zero space charge parameter. The abscissa is the normalized energy spread parameter, $\hat{q}$, and the ordinate is the RMS width of the wave-packet in unit of $\sqrt{k_w z}\big/\left(k_0 c\sqrt{\rho}\right)$.

The peak electric field due to the density modulation is $\left|E_{1,peak}\right| = \left|j_{1,peak}\right|/\left(\varepsilon_0 k_0 c\right)$ and the gain in electric field is

$$G = \frac{\left|E_{1,peak}\left(z,t_p\right)\right|}{E_0} = \frac{2\cdot 3^{\frac{1}{4}}}{\sqrt{\pi}}\rho\sqrt{\frac{L_g}{z}}\left|B_1\lambda_0\right|\frac{e^{\frac{\mathrm{Re}(\lambda_0)z}{\sqrt{3}L_g}}}{\sqrt{\left|-\lambda_2\right|}}\ , \tag{14}$$

with $L_g = 1/\left(\sqrt{3}\Gamma\right)$ and $E_0 = Ze/\left(2\varepsilon_0 S\right)\cdot$

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Figure 2 plots the gain in electric field as a function of the energy spread, suggesting a sharp decreasing in gain as the energy spread increasing from zero to ρ.

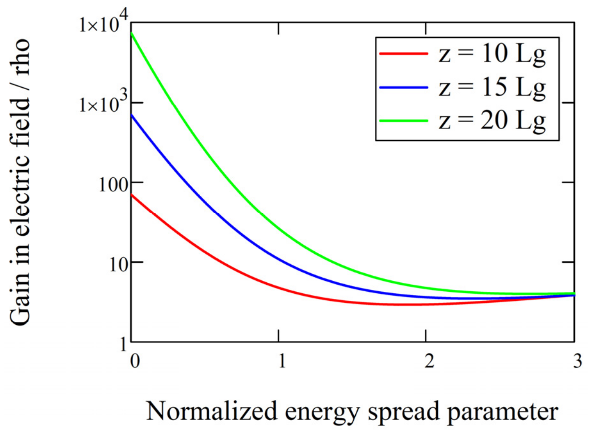

Figure 2: Electric field gain as a function of the energy spread parameter for zero space charge parameter. The abscissa is the normalized energy spread parameter, $\hat{q}$, and the ordinate is G/ρ as calculated from eq. (14).

NUMERICAL SOLUTION FOR NON-ZERO SPACE CHARGE

For arbitrary $\hat{\Lambda}_p$ and $\hat{q}$, solving eq. (5) and (6) requires finding the roots of the following expression:

$$\lambda_I \sqrt{\frac{3-4\hat{\Lambda}_p^2\lambda_I}{1-4\hat{\Lambda}_p^2\lambda_I}} = -\hat{q} + \sqrt{\frac{\left(3-4\hat{\Lambda}_p^2\lambda_I\right)\left(1-4\hat{\Lambda}_p^2\lambda_I\right)}{8\lambda_I\left(1-2\hat{\Lambda}_p^2\lambda_I\right)}} \,. \qquad (15)$$

Analytically finding the roots for arbitrary $\hat{\Lambda}_p$ looks difficult and we proceed with numerical approach.

The expansion coefficients, $\lambda_{0,1,2}$ and $\hat{C}_0$ are obtained from eqs. (5)-(9). The electron density modulation in the frequency domain is still given by eq. (10), but the coefficient B_1 become

$$B_1 = \frac{\lambda_0^2 + \left(i\hat{C}_0 + 2\hat{q}\right)\lambda_0 + \hat{\Lambda}_p^2 + \hat{q}^2}{2\lambda_0^3 + 2\lambda_0^2\left(i\hat{C}_0 + \hat{q}\right) + i} \,. \qquad (16)$$

Eqs. (12)-(14) remain valid for arbitrary $\hat{\Lambda}_p$ and $\hat{q}$.

Figures 3 and 4 show how the coherent length and electric field gain depends on the space charge and energy spread, which suggest that, for $\hat{\Lambda}_p \ll 1$, both the gain and the coherent length do not strongly depend on the longitudinal space charge effect.

SUMMARY

Taking the Taylor series of the solution to the FEL dispersion relation up to the quadratic term in detuning parameter leads to a time-domain electron density wave-packet with Gaussian envelope. For electron beam with Lorentzian energy distribution and zero space charge, we found close form expression for the amplitude and RMS width of the Gaussian envelope. For arbitrary space charge, analytical derivation is difficult and the solutions are found with numerical approach.

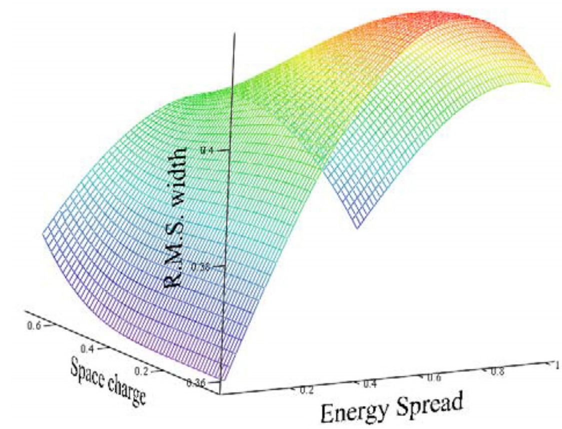

Figure 3: RMS width of electron density wave-packet as a function of the energy spread parameter, $\hat{q}$, and the space charge parameters, $\hat{\Lambda}_p$. The ordinate is the RMS width in unit of $\sqrt{k_w z}\Big/\left(k_0 c\sqrt{\rho}\right)$.

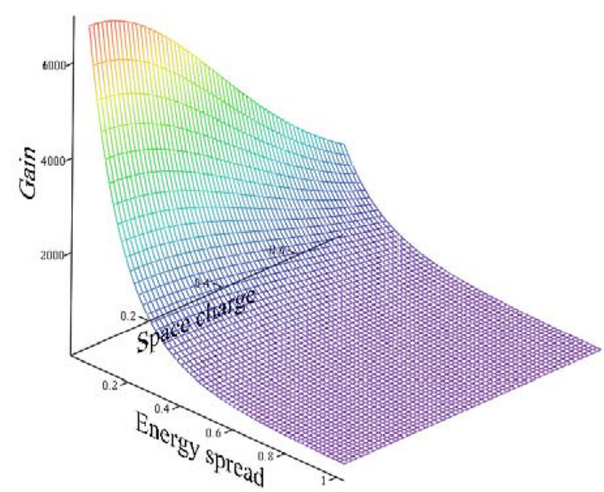

Figure 4: Electric field gain as a function of the energy spread parameter, $\hat{q}$, and space charge parameter $\hat{\Lambda}_p$.

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