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NUMERICAL STUDY OF THREE DIMENSIONAL EFFECTS IN LONGITUDINAL SPACE-CHARGE IMPEDANCE*

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Abstract

Longitudinal space-charge (LSC) effects are generally considered as detrimental in free-electron lasers as they can seed instabilities. Such "microbunching instabilities" were recently shown to be potentially useful to support the generation of broadband coherent radiation pulses [1, 2]. Therefore there has been an increasing interest in devising accelerator beamlines capable of sustaining this LSC instability as a mechanism to produce a coherent light source. To date most of these studies have been carried out with a one-dimensional impedance model for the LSC. In this paper we use a N-body "Barnes-Hut" algorithm [3] to simulate the 3D space charge force in the beam combined with ELEGANT [4] and explore the limitation of the 1D model often used.

INTRODUCTION

Space-charge forces are essential to account for in realistic beam dynamics simulations. The nature of these forces lies in particle-to-particle Coulomb interaction. However, the numerical complexity of the problem grows as $O(N^2)$, where N is the number of particles. Therefore, it is not possible to exactly compute all space-charge contributions. Several approximation techniques can be used: mean-field on a grid approximation [5], space-charge impedance [6], analytical sub-beams or ensembles model [7]. All of those methods reduce the problem's complexity via some approximations which ultimately limits the maximum attainable spatial resolution.

Space-charge problem is very similar to the well-known N-body problem in celestial mechanics. One of the most effective algorithms for the gravitational N-body problem is the so called "tree" or Barnes-Hut (BH) algorithm [3], which scales as $O(N \log N)$. In this paper we present the results obtained using a modified version of the program available at [8]. Such a code was successfully employed to simulate early beam dynamics in photocathode [9]. Other more efficient algorithm have been recently developed [10, 11] and will be eventually used in further refinement of our work.

METHOD & VALIDATION

For the studies presented in this paper and our cascaded space charge amplifier study [17], we used the BH algorithm as an external script within the ELEGANT simulations. At a user-specified axial locations along the accelerator beam line, space charge kicks were applied. The distribution at the defined locations was saved and Lorentz transformation to the bunch rest frame was applied. The BH algorithm was used to obtain the 3D electrostatic field $\mathbf{E}'$. This field was then transformed in the laboratory frame and the obtained electromagnetic fields $(\mathbf{E}, \mathbf{B})$ were used to compute the Lorentz force on each of the macroparticles composing the beam. We used an impulse approximation so that only the momentum was altered by the space charge force. The distribution then was finally passed back to ELEGANT and tracked up to the next space-charge kick where the above process repeated. The main assumption in our calculations was that there was no magnetic field in the rest frame. This assumption although not strictly valid, was shown to hold for the beam with low energy spread typically produced in photoinjectors [12]. We henceforth refer to the combination of the BH algorithm with ELEGANT as "ELEGANT-BH".

To validate our simulations we both rely on analytical results and simulations carried out with the ASTRA program [5]. We first consider a 3D homogeneous ellipsoidal bunch with electric field linearly dependent on the position within the charge distribution as [13]

$$E_u(u) = \frac{C}{\gamma^2} \frac{(1-f)u}{r_u(r_x+r_y)r_z}, \text{ and } E_z(z) = \frac{Cf}{r_x r_y r_z} z, \qquad (1)$$

where $C \equiv 3Q/(4\pi\epsilon_0)$, $u \in [x, y]$, $r_{x,y,z}$ are the ellipsoid semiaxes, $f \approx \sqrt{r_x r_y}/3\gamma r_z$ and Q is the bunch charge. The simulated fields are in excellent agreement with the field given by Eq. 1 as shown on Fig. 1. To assess longer-term

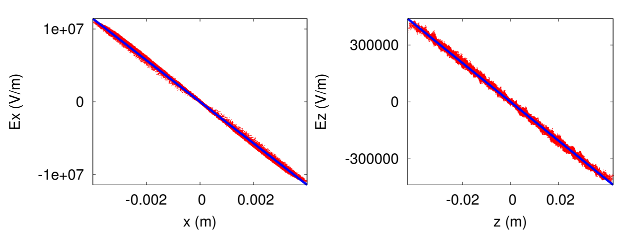

Figure 1: Transverse (left) and longitudinal (right) electric field experienced by the macropaticle simulated with ELEGANT-BH (symbols) and obtained from Eq. 1 (lines).

tracking, we compared the evolution of the beam envelope over a drift space. For a stationary uniform beam the transverse envelope evolution is governed by [14]

$$a''_{x,y} - \frac{\varepsilon^2_{rx,ry}}{a^3_{x,y}} - \frac{K}{2(a_{x,y}+a_{y,x})} = 0, \qquad (2)$$

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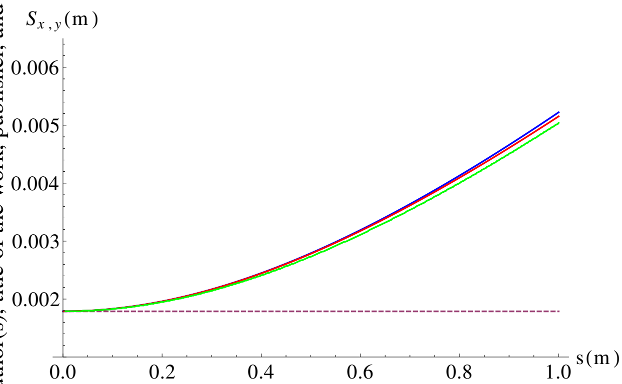

Figure 2: Comparison of the beam envelope evolution along a 1-m drift. Eq. 2 solution (blue), is compared against ASTRA (red), and ELEGANT-BH simulation (green). The dashed line corresponds to no space-charge case.

where $a_{x,y}$ is the rms beam size in x, y, $\varepsilon_{rx,ry}$ is the corresponding emittance and K is a 3D space charge parameter. Figure 2 compares the solution of Eq. 2 against the beam envelope simulated with ELEGANT-BH. The geometric emittance is very low ($\varepsilon_{x,y} = 1 \times 10^{-11}$ m) for these studies so that when space charge is turned off the beam envelope (dashed line) is quasi-constant.

IMPEDANCE CALCULATION

LSC is commonly investigated using analytical impedance models. For a transversely Gaussian cylindrical-symmetric beam the impedance is given by [6]

$$Z(k) = -i\frac{Z_0}{\pi\gamma\sigma}\frac{\xi_\sigma}{4}e^{\xi_\sigma^2/2}Ei(-\frac{\xi_\sigma^2}{2}), \quad (3)$$

where $Z_0 = 120\pi$ is the free-space impedance, $Ei(x) \equiv -\int_{-x}^{\infty} dte^{-t}/t$, σ is the beam rms size and $\xi_\sigma \equiv k\sigma/\gamma$. $Z(k)$ can be normalized by $\pi\gamma\sigma/Z_0 = \gamma\sigma/120$ to be dimensionless. Note, that although the Eq. 3 is effectively one-dimensional, it assumes the bunch has a Gaussian transverse distribution.

In order to benchmark Eq. 3 with ELEGANT-BH we considered initial bunch distribution with pre-modulated current profiles of the form $f(\mathbf{r}) = T(x,y)L_z(z)[1 + m\cos kz]$, where m is the amplitude of the modulation, k the modulation spatial wavenumber, and $L(z)$ and $T(x,y)$ are respectively the nominal longitudinal and transverse beam distributions. The axial modulation in z direction leads to an energy modulation due to the LSC impedance and eventually produces further current modulation depending on the longitudinal dispersion of the beamline. From the definition of the impedance, and given the Fourier-transformed longitudinal electric field $\widetilde{E}_z(k)$ and current distribution $\widetilde{I}(k)$, the longitudinal impedance can be recovered as $Z(k) = -\widetilde{E}_z(k)/\widetilde{I}(k)$. Note, that $\widetilde{E}_z(k)$ and $\widetilde{I}(k)$ have a $\pi/2$ phase shift; see Fig. 3 (bottom). We therefore use ELEGANT-BH and simulate the space-charge force effect on an initially modulated bunch over one kick; see Fig. 3 (top images). The generated longitudinal phase space can then be analyzed to provide information on $\widetilde{E}_z(k)$ and $\widetilde{I}(k)$.

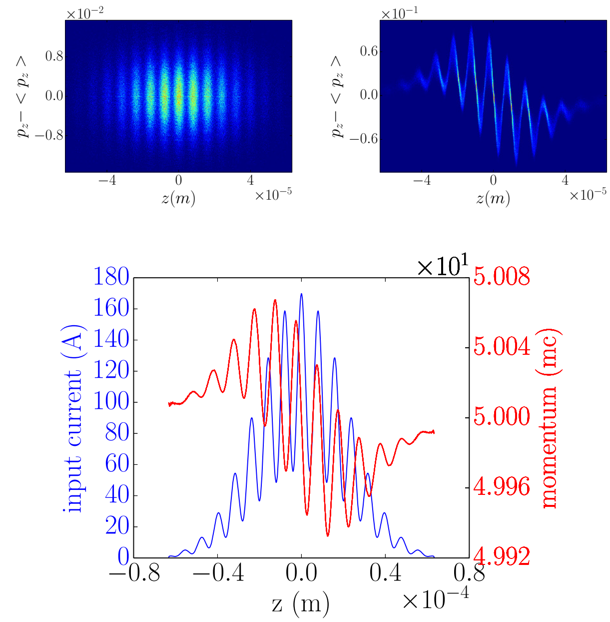

Figure 3: Modulated longitudinal phase space of a Gaussian beam before (top left) and after (top right) the application of one space-charge kick. Comparison of the induced energy modulation (bottom plot, red trace) computed from top-right image with the current distribution (bottom plot, blue trace).

The Fourier transform was carried out using a fast-Fourier transform (FFT) algorithm and the ELEGANT-BH and was performed over different values of the initial modulation wavenumber k. While spanning k, different number of macroparticles ($N = [1,4,6] \times 10^6$) was used. The number of FFT bins was also tuned to minimize discretization effects when altering the value of k. As the wavenumber k value was decreased, the bunch duration length was increased to ensure the number of macroparticles per bin was consistent with the large values of the wavenumber. In our simulations we set this ratio to be $N/n_b \approx 5000$. The resulting impedance evolution as a function of k is shown in Fig. 4 and is in a reasonable agreement with Eq. 3 and simulation using the 1-D `LSCdrift` model available within ELEGANT.

The Fourier images $\widetilde{E}_z(k)$ and $\widetilde{I}(k)$ were also found numerically via FFT. The longitudinal electric field and current functions analysis can be further enchanced by polynomial fit [15]. Our method overall demonstrates a good agreement with Eq. 3.

THREE-DIMENSIONAL EFFECTS

Our simulations consistently underestimate the impedance compared to Eq. 3 over the range of k values explored. Such an effect was previously recognized [16] and is attributed

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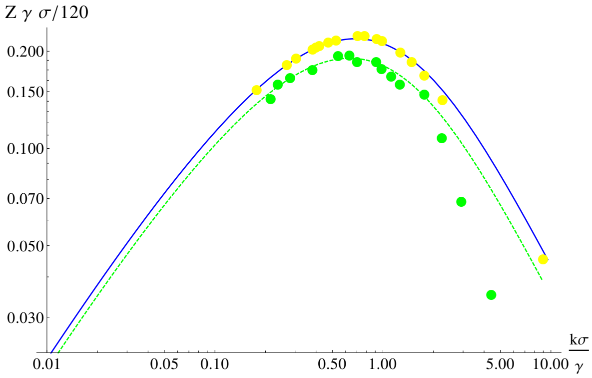

Figure 4: Space-charge impedance. ELEGANT-BH algorithm (green), analytical form Eq. 3 (blue), ELEGANT built-in LSCdrift element, very close to Eq. 3 (yellow).

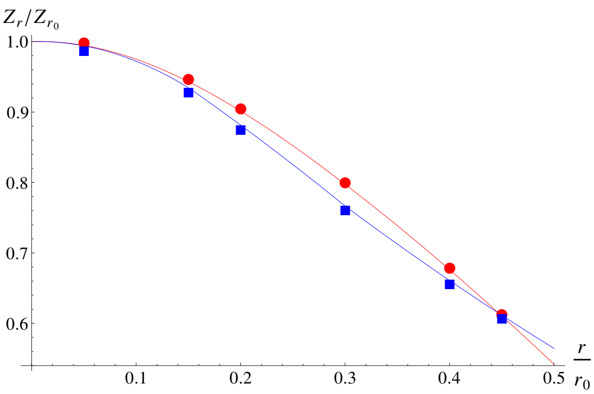

Figure 5: Radial dependence of $Z(k,r)$ for a fixed value of k. The red and blue traces respectively correspond to a Gaussian and uniform transverse distribution (the lines are splines to the simulated data symbols). For both case the longitudinal distribution is taken to be Gaussian. The parameter r_0 is the rms transverse size of the distribution.

to the radial dependence of the LSC field conferring a similar dependence on the impedance. To further explore this possibility we performed a similar analysis as was detailed in the previous section but over thin radial slices $[r, r + \delta r]$ where $\delta r \approx 0.1 r_0$. The results of such an analysis provide the radial dependence of the LSC impedance Fig. 5.

Additionally, one can show that for a parabolic $f(\mathbf{r}) = f_0(a^2 - r^2)\theta(r - a)$ and uniform $f(\mathbf{r}) = f_0\theta(r - a)$ distribution an analytical form of the impedance can be retrieved [here a, f_0, and $\theta(r)$ are respectively the radius, normalization factor, and Heaviside function]. It can especially be shown that the parabolic distribution yield an impedance with weak dependence on the radius.

CONCLUSIONS AND FUTURE WORK

Using a gridless code adapted from Astrophysics we have investigated three-dimensional effects in the LSC impedance and found that the one-dimensional often used LSC impedance model is a good approximation.

We will use the developed method in our further numerical studies of the Cascaded Longitudinal Space-Charge Amplifier at the Fermilab's Advanced Superconducting Test Accelerator [17].

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