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AN IMPROVED ANALYTIC MODEL OF ELECTRON BACK-BOMBARDMENT IN THERMIONIC-CATHODE RF GUNS

J. P. Edelen[#], S. G. Biedron, J. R. Harris, S.V. Milton, Colorado State University, Fort Collins, CO
J. W. Lewellen, LANL, Los Alamos, NM

Abstract

This paper describes work done at Colorado State University to improve upon the recent theory developed to predict the back-bombardment power in single-cell thermionic-cathode electron guns. The previous theory used a square-wave approximation of the time varying field to solve for the total kinetic energy deposited on the cathode due to the back-bombarded electrons. In addition the transit time factor was added as a correction to compensate for the non-sinusoidal field. By solving for the back-bombardment power using a sinusoidal field, the transit time factor can be removed and therefore a better overall model is produced. These alterations continue to accurately predict how back-bombardment varies as a function of the gun parameters and provides improvement when compared to the existing theory.

INTRODUCTION

Recent work has successfully developed a first principles model that accurately predicts the back-bombardment power for single-cell thermionic-cathode electron guns as a function of the design parameters, Equation 1 [1]:

$$P_{ave} = \frac{3E_0 I c^2}{4\alpha^2 f v_{eff}} TK \quad (1)$$

Here E_0 is the peak RF field in the gap, α is the RF wavelength normalized to the gap length ($\alpha = \lambda_{rf}/L_{gap}$), f is the RF frequency, I is the average beam current, v_{eff} is the effective velocity of electrons in the gap defined by Equation 2, T is the transit time factor defined by Equation 3, and K is the field normalization factor defined by Equation 4.

$$v_{eff} = c\sqrt{1-\left(1+\frac{qE_0\lambda}{2m_0c^2\alpha}\right)^{-2}} \quad (2)$$

$$T = \frac{\sin\left(\pi c/\alpha v_{eff}\right)}{\pi c/(\alpha v_{eff})} \quad (3)$$

$$K = \frac{\int_0^{L_{gap}} E(z)dz}{E_0\lambda/\alpha} \quad (4)$$

In the previous work we used a square wave in order to reveal exact solutions to the equations of motion and solve for the back-bombardment power. Follow-on studies indicated that a sinusoidal field could also be used to solve for the back-bombardment power, contingent on the constant velocity principle used in the previous work.

This paper presents a modification to the theory that utilizes a sinusoidal time varying field, which makes for a better a-priori model and allows for the omission of the transit time factor from the back-bombardment power equation. We begin with an overview of the modification to the theory and compare these results with both the previous model and the numerical solutions to the relativistic equations of motion. The new model is then compared with simulations performed using SPIFFE [2]. Finally we provide a quantitative comparison between the new model and the existing model over a wide range of parameters.

ADDITION OF A SINUSOIDAL FIELD TO THE BACK-BOMBARDMENT MODEL

To solve for the back-bombardment power, the effective kinetic energy as a function of the electron emission time is calculated using Equation 5.

$$K_{eff}(t_0) = v_{eff}\int_{t_0}^{t_0+t_{transit}} E_0\sin(\omega t)dt \quad (5)$$

Here $t_{transit}$ is the particle transit time. The particle transit time is different for particles that exit the gun and those that are back-bombarded. For particles that exit the gun the transit time is $t_{transit}^{fw} = \lambda/(\alpha v_{eff})$. For particles that are back-bombarded the transit time is a function of the emission time given by $t_{transit}^{bb}(t_0) = 4(\tau/2 - t_0)$ [1]. Solving Equation 5 for both the output case and the back-bombardment case, gives the effective kinetic energy of all the particles as a function of their emission time (Green line in Figure 1). This was compared with the result of numerically integrating the relativistic equations of motion of the same representative geometry, which gives the blue line in Figure 1. Additionally the model derived in previous work by using a square wave field is given in black.

This shows that there is still a fairly poor agreement for the output beam, however the new model is much better than the previous model. However, when computing the back-bombardment power only the area under the curve to the right of the discontinuity is of interest. Inspection of both the blue and green curve shows that this is indeed a reasonable approximation.

jedelen@engr.colostate.edu

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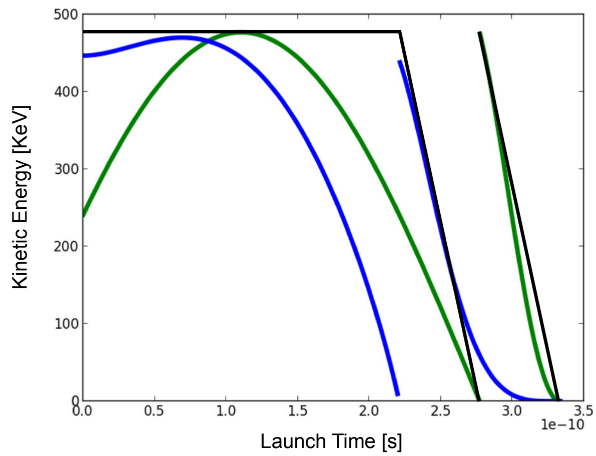

Figure 1: Comparison of numerical solutions to the equations of motion for a relativistic particle in a pillbox cavity (blue) with the approximation derived using Equation 5 for a particle traveling at a constant effective velocity (green) and linear model presented in the previous work (black).

To solve for the back-bombardment power, a mapping of the emission time to the deposition time as seen by the cathode is required. This change of coordinates is given by $t(t_0) = -3t_0 + 3\tau/2$. Integrating the result of Equation 5 over the time domain of the back-bombarded particles, and multiplying by the RF frequency and the beam current, gives the average back-bombardment power.

$$P_{ave} = \frac{IE_0 v_{eff}}{4\pi^2 f}\left(3\sin\left(\frac{c\pi}{v_{eff}\alpha}\right) - \sin\left(\frac{3c\pi}{v_{eff}\alpha}\right)\right)K \quad (6)$$

COMPARISON WITH SIMULATIONS

Equation 6 was used to compute the back-bombardment power for a wide range of cases and compared with simulation results. Figure 2 shows the spatial field profiles used for the two series of simulation runs.

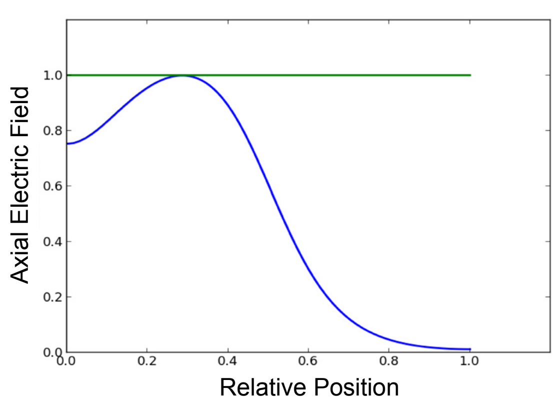

Figure 2: Field profiles used in the back-bombardment simulations normalized to the gap length and peak field. Green is a standard pillbox field and blue is an axial field map from a representative short gap single-cell gun design [3].

Figure 3 shows the back-bombardment power as a function of alpha for three frequencies and a peak field of 20 MV/m, while Figure 4 shows the back-bombardment power as a function of frequency for three values of alpha and the same peak field. Both the RF frequency and the fractional gap length have geometrical implications, which is why they are considered first.

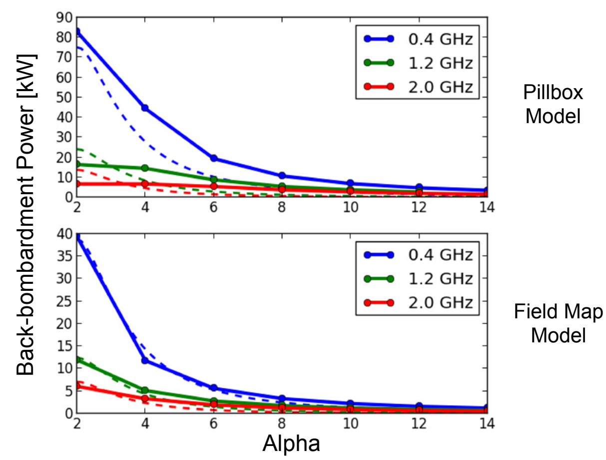

Figure 3: Comparison of Equation 6 and simulation results for both the field map case and the pillbox case. The back-bombardment power is given as a function of alpha for three values of frequency with a peak field of 20MV/m.

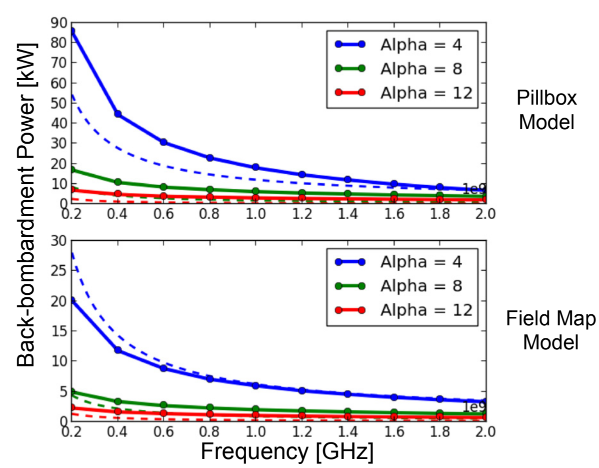

Figure 4: Comparison of Equation 6 and simulation results for both the field map case and the pillbox case. The back-bombardment power is given as a function of frequency for three values of alpha with a peak field of 20MV/m.

This shows that Equation 6 accurately predicts the trends with RF frequency and with alpha for both the field map case and for the pillbox cavity case. Figure 5 shows the back-bombardment power as a function of the peak

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field for three values of alpha and a RF frequency of 1GHz.

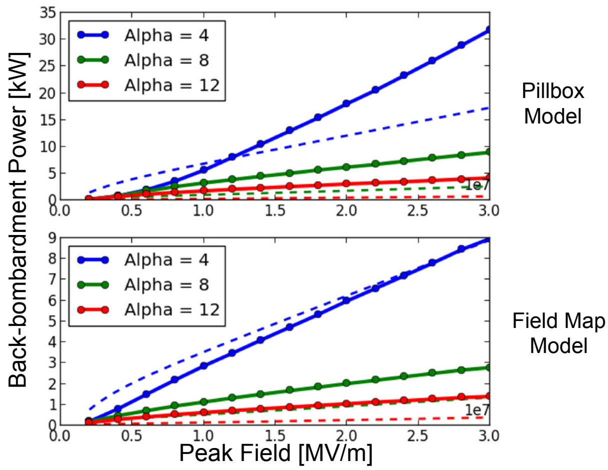

Figure 5: Comparison of Equation 6 and simulation results for both the field map case and the pillbox case. The back-bombardment power is given as a function of peak field for three values of alpha with a RF frequency of 1GHz.

This shows that both the model and the simulations predict a linear trend with the peak field, however the slopes do not agree as well. Note that the difference in variation with alpha appears to be the reason why the slopes do not agree. Unlike the RF frequency and peak field, Equation 6 shows a very different relationship than Equation 1 for how the back-bombardment power varies with alpha.

NUMERICAL ANALYSIS

In order to quantify the improvement of the new model from the old model, the absolute difference between the model and the simulation was computed. Table 1 provides some statistics for this difference function.

Table 1: Comparison of square wave model results [1] with sinusoidal results developed in this paper. The Root Mean Squared (RMS) difference, normalized RMS difference, average difference, and peak difference are shown for both the field map and pillbox model for both cases.

	D_{rms}	D_{rmsn}	D_{ave}	D_{peak}
Pillbox Cavity Case				
Equation 1	100	0.0082	1700	13000
Equation 6	100	0.0083	2300	9100
Field Map Case				
Equation 1	30	0.0053	420	5000
Equation 6	16	0.0050	330	2700

This shows that there is no improvement in the RMS difference and the normalized RMS difference by the addition of the sine wave for the pillbox model. However there is a significant reduction in the peak error indicating that the updated model is doing a better job at predicting the regions where the previous model was not as accurate. For the field map case, the addition of the sinusoidal field greatly improves the performance of the back-bombardment prediction showing a 47% reduction in the RMS difference and the peak difference, with a 20% reduction in the average difference.

Note that the data used to fill Table 1 required that the gap voltage be greater than 1.5MV. This was determined to be the cut-off of the model validity in previous work [1].

CONCLUSIONS

This paper has provided a simple extension to our previous theory of electron back-bombardment in single-cell thermionic-cathode RF guns. Through the addition of a sinusoidal time varying field for calculating the effective kinetic energy the dependence on the transit time factor was removed. Comparison of the new model with simulations showed that in general the theory predicts the physics. Numerical analysis of the model over a broader range of simulation data, and comparing with the previous results, showed a 30% reduction in the peak error for the pillbox case and a 46% reduction in the peak error for the field map case. This indicates that the new model performs better in regions where the previous model was less accurate. Additionally the RMS error for the field map case was reduced by 47%, indicating that the new model is more accurately predicting the back-bombardment power in these cases.

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