

NON-INTERFEROMETRIC SPECTRAL ANALYSIS OF SYNCHROTRON RADIATION IN THE THz REGIME AT ANKA

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Abstract

Interferometry is the quasi-standard for spectral measurements in the THz- and IR-range. The frequency resolution, however, is limited by the travel range of the interferometer mirrors. Therefore, a resolution in the low megahertz range would require interferometer arms of about 100 m. As an alternative, heterodyne measurements provide a resolution in the Hertz range, an improvement of 6 orders of magnitude. Here we present measurements done at ANKA with a VDI WR3.4SAX, a mixer that can be tuned to frequencies from 220 GHz to 330 GHz and we show how the bunch filling pattern influences the amplitude of specific frequencies.

INTRODUCTION

The single particle synchrotron spectrum emitted by an electron in a bending magnet is broad-band and also continuous. In a storage ring where an electron bunch emits synchrotron radiation on every turn, the emitted electric field consists of equally spaced (T_0) series of short pulses. As an approximation we assume a Dirac delta function ($\delta(t)$) as bunch signal which leads to an infinite pulse train, known as Dirac comb or Shah distribution, that can be represented as a Fourier series [1]:

$$f(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} e^{j2\pi n \frac{t}{T_0}} \quad (1)$$

The importance of this lies in the fact that the Shah distribution is its own Fourier transform which gives us a discretized spectrum

$$\mathcal{F}(\omega) = \omega_0 \sum_{p=-\infty}^{\infty} \delta(\omega - p\omega_0) \quad (2)$$

The distance between the frequency combs is the revolution frequency ($\omega_0 = 2\pi f_0$, at ANKA: $f_0 = 2.71$ MHz). By having not only one bunch filled, but each RF-bucket (at ANKA: 184), amplified parts with harmonics of the accelerating radio frequency show up. This is because the buckets are not equally filled with electrons, but show some inhomogeneity as illustrated in Fig. 1.

The spectral components of a circulating electron beam have been described by Schott in 1912, long before synchrotrons were being thought of as light sources [2]. However, recent literature does not pay much attention to the spectral lines of synchrotron radiation, as they couldn't be resolved nor exploited by measurements [3, p. 807]. This has changed now.

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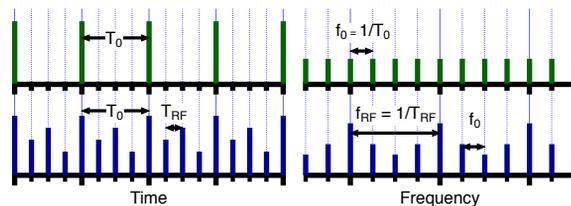


Figure 1: Theoretical spectrum of a series of short pulses as they happen in a storage ring. Each revolution the same pulse is seen, leading to a frequency comb spaced by the revolution frequency (top). By filling all buckets with (slightly) different bunch charges, we see the filling pattern modulated onto the frequency comb, showing extra amplification at the accelerating radio frequency (bottom).

The synchrotron radiation spectrum and the calculation of the frequencies are similar to the signal of a pick-up electrode. Also there, the short signal from the bunch is repeatedly coming by, creating a frequency comb whose analysis is a standard tool for longitudinal diagnostics in synchrotrons and therefore well studied [3, p. 627ff] [4, p. 165ff]. Taking the coherent synchrotron oscillation with frequency ω_s and amplitude $\hat{\tau}$ with bunch current $I = \frac{Ne\omega_0}{2\pi}$ and Gaussian bunch length σ_0 into account, we arrive at [5, p. 264ff]:

$$\mathcal{F}(\omega) = I \sum_{p,m=-\infty}^{\infty} j^{-m} J_m(p\omega_0\hat{\tau}) \delta(\Omega) e^{-\frac{(p\omega_0\sigma_0)^2}{2}} \quad (3)$$

$\Omega = \omega - p\omega_0 + m\omega_s$ leads to the known revolution harmonics. Additionally, every harmonic has satellites spaced by ω_s , where the m-th satellite has a spectral amplitude corresponding to the Bessel function of order m: $J_m(p\omega_0\hat{\tau})$. By having no coherent synchrotron oscillation ($\hat{\tau} = 0$), the satellites vanish. In a multi-bunch fill, due to non-uniform bunch currents, additional amplified harmonics of the bunch spacing appear.

The amplification of the frequencies corresponding to the repetition rate of the emitting particles is called superradiant and has been observed before [6]. Additionally its potential for spectroscopy has been shown recently [7, 8].

In this paper we present a method, how to deal with aliasing when analyzing heterodyne measurements, and how specific superradiant frequencies can be created by adjusting the filling of the bunch train.

MEASUREMENT SETUP

To increase the emitted THz radiation, ANKA is operated in the low-alpha mode, where the momentum com-

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raction factor α_c is reduced and the RF voltage is increased to shorten the bunches. This results in bunches of picosecond length leading to coherent synchrotron radiation at the observed frequencies around 260 GHz. At high bunch currents the so-called bursting threshold is crossed, above which micro-bunching instabilities arise and outbursts of THz radiation can be observed [9]. This radiation is measured at the Infrared1 beamline at ANKA [10]. Unless stated otherwise, the parameters in Table 1 are used in the measurements.

Table 1: Machine Parameters during Measurements

Beam Energy	$E = 1.3 \text{ GeV}$
Accelerating Frequency	$f_{RF} = 499.715 \text{ MHz}$
Revolution Frequency	$f_0 = 2.715 \text{ MHz}$
Synchrotron Frequency	$f_S = 7.5 \text{ kHz}$
RF Amplitude	$\hat{U} = 1.6 \text{ MV}$
Harmonic Number	$h = 184$
Momentum Compaction	$\alpha_c = 224(8) \times 10^{-6}$
Zero Current Bunch length	$\sigma_{z,0} = 2.48(5) \text{ ps}$
Avg. bunch length at 1 mA	$\langle \sigma_{z,1mA} \rangle = 20.0(1) \text{ ps}$

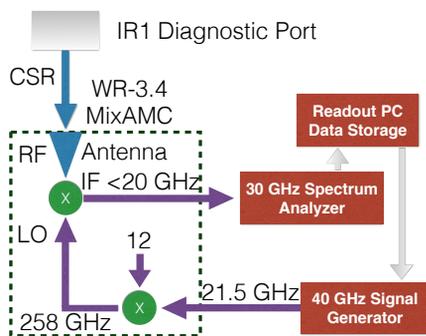


Figure 2: Setup of the measurement. The CSR is fed directly into the waveguide of the mixer by a horn antenna, and the output is detected by a spectrum analyzer.

MixAMC

A Mixer/Amplifier/Multiplier Chains (MixAMC) device from Virginia Diodes (VDI) for the WR3.4 waveguide standard (220 GHz to 330 GHz) is used. By mixing two signals, the output is the difference of the two frequencies. Thus, it is possible to convert a high frequency signal into the baseband and analyze it with standard RF equipment. Here the first signal, the Local Oscillator (LO) is generated by a signal generator (21.5 GHz) and then multiplied by 12 to reach 258 GHz. The second signal is the synchrotron radiation, that is coupled with a horn antenna into the waveguide where both signals are mixed by a sub-harmonic Schottky mixer. After the mixing process the output signal with a bandwidth of 20 GHz is coupled out and measured by a spectrum analyzer. The setup is sketched in Fig 2. The conversion loss is around 14 dB and not corrected in the shown measurements as absolute values were not necessary. If the device should be used as a power meter a calibration would have to be

done. By using a video bandwidth at the spectrum analyzer of a few Hertz, the sweep time is long enough to average out the bursting instabilities leading to a signal being stable in frequency and amplitude.

MEASUREMENT RESULTS

A single measurement is shown in Fig. 3. The mixer uses block downconversion and thus folds the frequencies above and below the LO into the same output. The shown frequency axis is only valid for the high band. On the bottom the number of the frequency harmonic is shown, again for the high band. At harmonic number 95312 of the 2.716 MHz revolution frequency there is an integer (518) harmonic of the 499.715 MHz RF frequency leading to increased power because of a non-uniform multi-bunch fill.

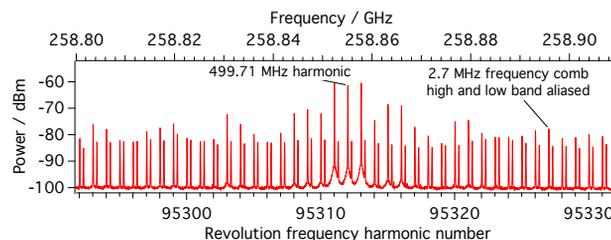


Figure 3: Mixer output measured with the spectrum analyzer. High and low band are aliased. Hence, two separate revolution frequency combs are visible. At the center is a RF frequency harmonic with increased power due to the non-uniform multi-bunch fill.

Separate Frequencies of High- and Low Band

Due to the discrete spectrum of the synchrotron radiation, the folded frequencies of both bands can be separated and thus the effective observable frequency band is doubled. In order to distinguish between both bands, the LO is shifted by an offset of -20 kHz which after the 12 times multiplier leads to a frequency shift of 240 kHz . Frequencies below the LO are shifted to lower frequencies because their offset to the LO decreases, while components above the LO will shift to higher frequencies. This measurement is shown in Fig. 4.

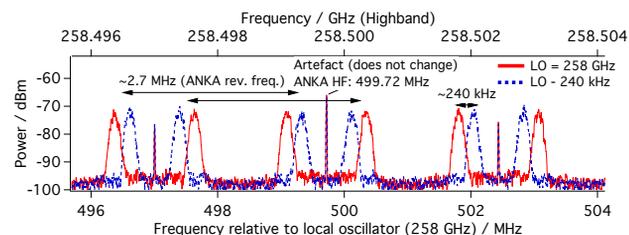


Figure 4: By shifting the local oscillator (LO) frequency, high band and low band can be separated because the difference from LO to the spectral line changes differently. The red, solid curve is the initial measurement, the blue dashed line is with a shifted LO.

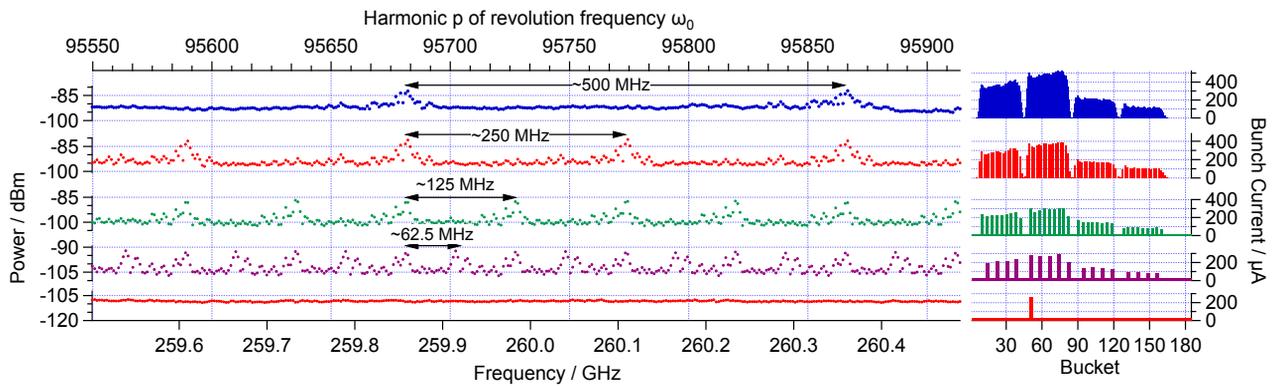


Figure 5: THz power spectrum for different filling patterns. On the left only the power of each harmonic at a given frequency is plotted, discarding the noise in between. The right panel shows the filling pattern. The top plot used the initial filling pattern of 4 trains with 33 bunches each. In the second plot from top every 2nd bunch is kicked out. In the 3rd plot only every 4th bunch is kept and below every 8th. In the bottom panel only a single bunch is left leading to a 2.71 MHz frequency comb with almost equal power.

The position of the frequencies is saved and in later data analysis both bands can be separated, because the origin of every frequency peak is known. Artefacts and spurious signals can be identified and masked during data processing. As long as the accelerating RF frequency is kept stable, frequencies that only depend on the revolution time are constant. The power of each harmonic is measured and sorted into high and low band. Due to the sorting it is easier to show a broad frequency band, as only the amplitudes are shown, neglecting the noise in between. So the amplitude change of all harmonics can be tracked easily.

Figure 5 shows data for different filling pattern structures. As the average noise level is at -115 dBm, all frequency peaks have a significant level. The filling pattern is measured by time-correlated-single-photon-counting (TCSPC) [11] and shown on the right. Initially it consisted of 4 trains with 33 bunches each and different bunch currents (top). Then every second bunch was kicked out by the bunch-by-bunch feedback system [12]. In the second line we see that additional multiples of 250 MHz separation arise. Even though the bunch current decayed and half of the electrons were removed, the power at the specific 250 MHz harmonics increased by almost a factor of 10. The same happens when only every 4th respective 8th bunch is kept and all others kicked out. In the bottom panel we can see that a single bunch leads to a 2.71 MHz frequency comb whose harmonics have almost identical intensity.

CONCLUSION AND OUTLOOK

We have shown that the harmonics of the revolution frequency that build up the synchrotron radiation spectrum in a storage ring, can be resolved and measured by heterodyne detection. The actual filling pattern of the storage ring has a crucial influence on the observed THz spectrum and the amplification of specific frequencies. By adjusting the filling pattern, superradiant frequencies can be created.

The output bandwidth is limited to 20 GHz but with our method to separate high and low band 40 GHz can be detected at once. By the use of a triplexer this could be extended in the future for ultra high resolution spectroscopy [13].

ACKNOWLEDGMENTS

We would like to thank Y.-L. Mathis and his team from the ANKA Infrared-Group for support during the beamtime. This work is supported by the Helmholtz International Research School for Teratronics (HIRST).

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