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HIGHER ORDER MODE PROPAGATION AND DAMPING STUDIES ON AXISYMMETRIC SUPERCONDUCTING MULTICELL RF-RESONATORS*

B.D. Isbarn†, B. Riemann, M. Sommer, T. Weis
TU Dortmund University (DELTA), Center for Synchrotron Radiation, Dortmund, Germany

Abstract

Higher order mode (HOM) propagation and damping is a major concern in feasibility studies regarding the upcoming upgrade of BESSY II, named BESSY-VSR, which involves the utilization of superconducting multicell RF-resonators in a storage ring while maintaining a reasonably high beam current typical for third generation synchrotron radiation facilities. In addition to the computation of typical figures of merit, we focus on studies of the mode propagation in axisymmetric structures. Due to the focus on axisymmetric studies we are able to use 2D codes to investigate in eigenmodes with substantialy higher frequencies than usually considered with full 3D codes in parametric studies. In this work we present preliminary studies involving mode propagation in superconducting elliptical multicell cavities.

INTRODUCTION

The upcoming BESSY II upgrade BESSY-VSR aims to provide both short and long electron bunch lengths simultaneously [1], this can be achieved through a modulation of the rf-frequency. In order to fullfill the space restrictions and field requirements the use of superconducting multicell rf-cavities is inevitable. Due to the vast intrinsic quality factors of superconducting cavities and the high beam currents of typical third generation synchrotron radiation facilities, proper higher order mode damping techniques must be implemented. A major concern for designing appropriate damping techniques is the mode propagation resp. the power flow in beam direction. We already investigated the mode propagation of a single cell spline cavity [2] using Floquet periodic boundary conditions in an earlier work [3]. To incorporate evanescent coupling and the mode propagation of normal modes we took this approach one step further and investigated in the mode propagation of a multicell cavity with varying beam tube lengths attached to the end of the multicell structure. In the following we are using the nomenclature of [4] to refer to the modes corresponding to the single cell cavity as *cavity modes* and to the modes emerging from the coupling of the single cell cavitiy into an array of cavities as *normal modes*.

NUMERICAL STUDIES

All numerical studies were performed on the 2D axisymmetric eigenvalue problem with COMSOL Multiphysics 5.0 [5]. The base cell design consisted of elliptical shaped cavities as displayed in Fig. 1 with the geometry parameters

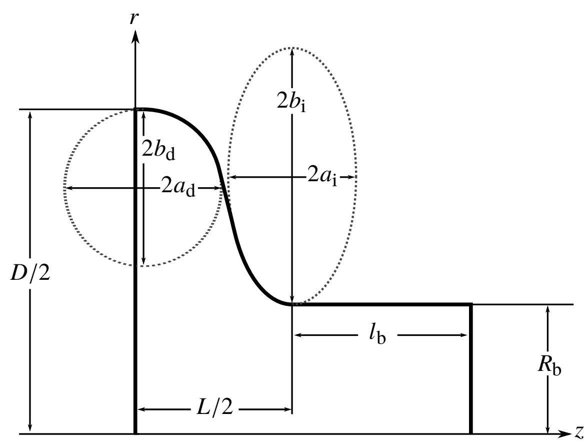

Figure 1: Geometry parameters of an elliptical cavity.

of the HZB layout [6]. These base cells were concatenated to form an array of 5 equally shaped cavities with beam tubes of length l_b attached to its ends. By restricting our scope of observation to equally shaped cells, without applying any end-cell tuning respectively optimization, we could increase the performance and therefore the observable parameter space.

Simulation Setup

In order to observe mode propagation we applied Floquet periodic boundary conditions at the ends of the attached beam tubes

$$\boldsymbol{E}_\mathrm{dst} = \boldsymbol{E}_\mathrm{src} e^{-i\boldsymbol{k}_z\cdot(\boldsymbol{r}_\mathrm{dst}-\boldsymbol{r}_\mathrm{src})}$$
$$\boldsymbol{H}_\mathrm{dst} = \boldsymbol{H}_\mathrm{src} e^{-i\boldsymbol{k}_z\cdot(\boldsymbol{r}_\mathrm{dst}-\boldsymbol{r}_\mathrm{src})} ,$$

where $\boldsymbol{E}_\mathrm{dst/src}$ and $\boldsymbol{H}_\mathrm{dst/src}$ represents the electric and magnetic field at the leftmost respectively rightmost port, and the wave vector $\boldsymbol{k}_z$ can be expressed as a function of the phase advance ψ and the overall length of the structure L_s

$$\boldsymbol{k}_z = \frac{\psi}{L_\mathrm{s}}\hat{z} .$$

All other boundaries were set to be perfect electric conductors (PEC) leading to loss-free solutions with real eigenvalues. We used the external quality factor as a figure of merit for the mode propagation, in analogy to the usual definition of the quality factor [4], described by

$$Q_\mathrm{ext} = \frac{\omega_0 U}{P_z} = \frac{\omega_0 \iiint_V \frac{1}{2}\varepsilon_0 |\boldsymbol{E}|^2 \,\mathrm{d}V}{\iint_A \rho_z \,\mathrm{d}A} .$$

* Work supported by the BMBF under contract no. 05K13PEB
† benjamin.isbarn@tu-dortmund.de

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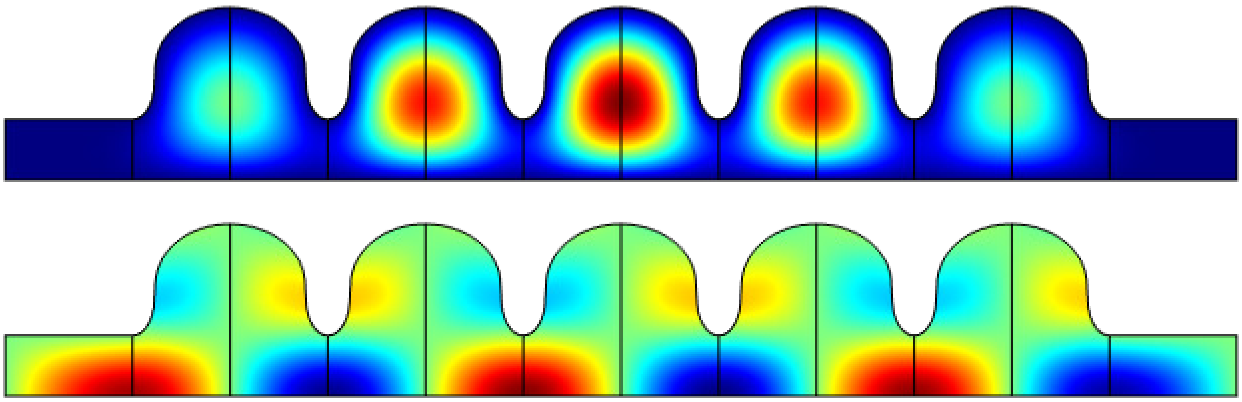

Figure 2: Cross section view of the electric field distribution of the TE_{011} (top, out-of-plane electric field) and TM_{021} mode (bottom, electric field in beam direction).

ρ_z is the power flow density in beam direction, which is integrated over the cavity cross section to obtain the overall power flow in the beam direction.

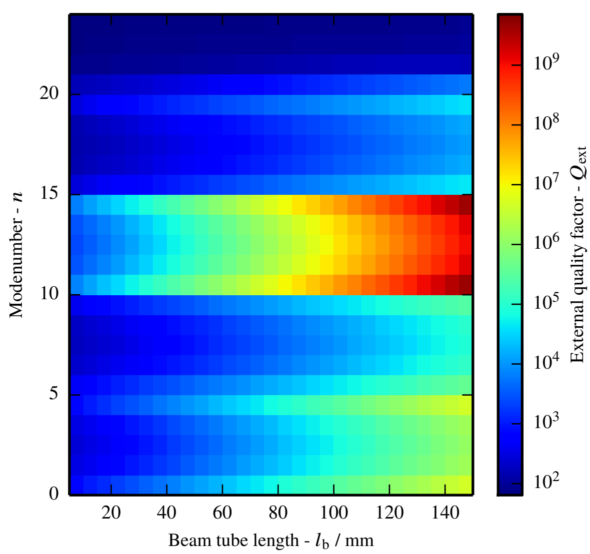

Figure 3: External quality factors Q_{ext} for the first 5 cavity modes and their corresponding normal modes as a function of the beam tube length l_b.

Parametric Sweep

We performed a parameter sweep of the phase advance between the two Floquet periodic boundary conditions between $\psi = 0$ and $\psi = \pi$ with 21 samples. In addition we varied the beam tube length between $l_b = 5$ mm and $l_b = 150$ mm in steps of 5 mm.

Results

The minimal external quality factors inside each individual normal mode passband corresponding to the first 5 cavity modes is displayed in Figs. 3 and 4. We could observe significantly higher external quality factors for the first transverse

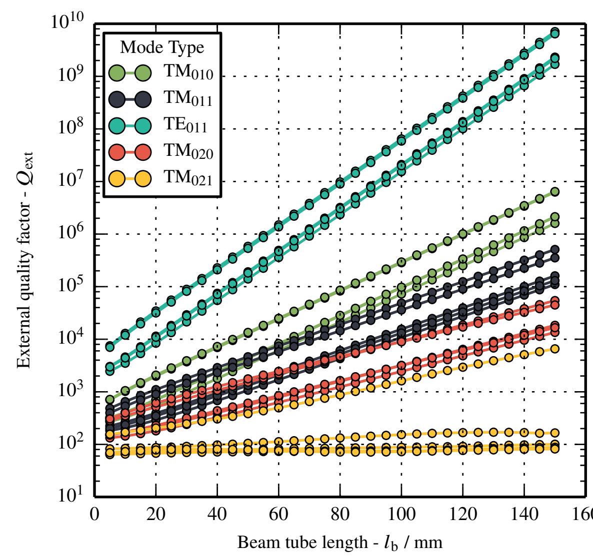

Figure 4: External quality factors Q_{ext} for the first 5 cavity modes and their corresponding normal modes as a function of the beam tube length l_b.

electric mode (see Fig. 2), namely the TE_{011}-mode, due to the weak cell to cell coupling. The external quality factors of cavity modes 1, 2 and 4 grow exponentially as a function of the beam tube length which is the expected behavior for evanescent coupled oscillators. In addition the 5th cavity mode is able to couple resonantly to the beam tube. This is due to the frequency of the mode $\nu = 3.2836$ GHz being above the cut-off frequency of the TM_{01} waveguide-mode of the beam tube

$$\nu_{TM_{01,c}} = c_0 \frac{x_{01}}{R_b} \approx 3.1350\,\text{GHz}\,,$$

where x_{01} corresponds to the first root of the 0th bessel function. All higher modes can therefore be partially or entirely overlapped by the beam tube waveguide modes (see Fig. 5) depending on the Floquet wave vector $\boldsymbol{k}_z$, thus complicating the seperation of cavity and waveguide modes. Nonetheless

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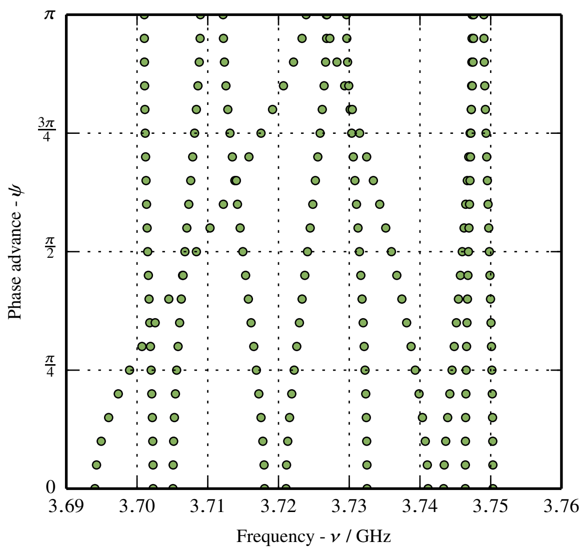

Figure 5: Dispersion relation of the cavity mode 6 and a waveguide mode. Both passbands overlap.

it is possible to compare the external quality factors for all calculated modes with a fixed phase advance of $\psi = \frac{\pi}{2}$ which is, according to expectations, close to the minimal external quality factor (see Fig. 6). Those external quality factors

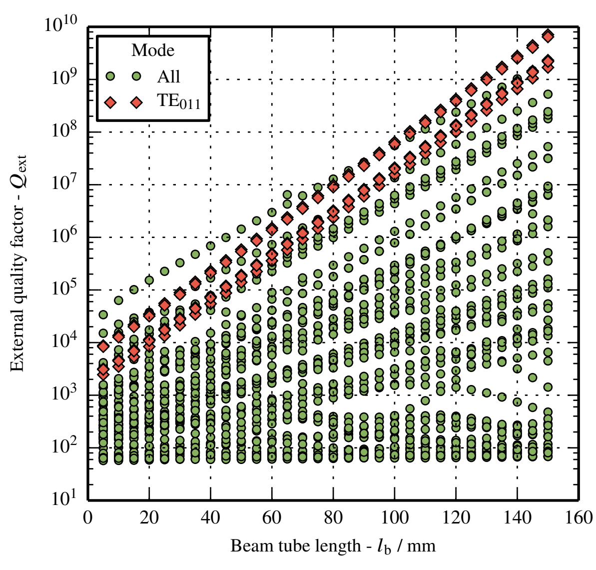

Figure 6: External quality factors of all cavity and normal modes with a fixed phase advance of $\psi = \frac{\pi}{2}$ as a function of the beam tube length l_b.

can be interpreted as upper limits for the minimal external quality factor of the corresponding normal modes. In addition our investigations show that the external quality factors of the modes calculated in our scope of observations are bounded by the external quality factors of the TE_{011} mode for higher beam tube lengths. This leads to the assumption that all monopole modes with a frequency above the cutoff-frequency of the corresponding waveguide mode can at least partially couple resonantly with the beam tube, leading to substantially lower external quality factors.

CONCLUSION

Using Floquet periodic boundary conditions, we were able to observe the mode propagation behavior of different monopole modes inside a five-cell elliptical superconducting cavity under varying beam tube lengths. These studies have shown that the external quality factor for the first transverse electric monopole cavity mode are significantly higher than those of the neighboring transverse magnetic cavity modes. Due to the passband overlap caused by waveguide modes for modes with frequencies above the first waveguide cut-off frequency of the beam tube, proper mode assignment was impeded. To circumvent this problem of mode assignment, we used a fixed phase advance for comparison of the external quality factors, instead of searching for the minimal external quality factors inside each normal mode passband. This study showed that the external quality factor as a function of the beam tube length of the formentioned TE-mode enclosed the external quality factors of all the other modes in our scope of observations, at least for higher beam tube lengths. This leads to the assumption that all other monopole modes are able to couple at least partially resonantly to the attached beam tube. To cover the whole mode spectra future studies with our inhouse-developed 2.5D capable FEM-solver [7] are planned, to investigate in the mode propagation of multipole modes.

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