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MATHEMATICAL MODELING AND ANALYSIS OF A WIDE BANDWIDTH BIPOLAR POWER SUPPLY FOR THE FAST CORRECTORS IN THE APS UPGRADE CONTROLLER*

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Abstract

This paper presents the mathematical modeling and analysis of a wide bandwidth bipolar power supply for the fast correctors in the APS Upgrade controller. A wide bandwidth current regulator with a combined PI and phase-lead compensator has been newly proposed, analyzed, and simulated through both a mathematical model and a physical electronic circuit model using MATLAB and PLECS. The proposed regulator achieves a bandwidth with a -1.23dB attenuation and a 32.40° phase-delay at 10 kHz for 0.3% AC component. The mathematical modeling and design, simulation results of a fast corrector power supply control system are presented in this paper.*

INTRODUCTION

Argonne National Laboratory's Advanced Photon Source (APS) Upgrade requires a bipolar power supply with a 10 kHz and -3dB small signal bandwidth for the fast correction magnets. In addition, low output current offset and DC drift to achieve a few hundred parts per million (ppm) output current stability are required. These requirements present a technical challenge to the design because the magnet load may have a significant inductance, making it difficult to achieve a high bandwidth for the current, and the input DC bus voltage may be limited. Until now, there have been no known commercial-off-the-shelf (COTS) power supplies that meet the 10kHz bandwidth requirement.

In order to meet the requirements, different circuit topologies and regulators are being investigated. One of the candidates is a standard full-bridge power circuit topology with a high switching frequency to provide a ±15A DC output current with an AC component less than 1% of the full DC scale with the required bandwidth.

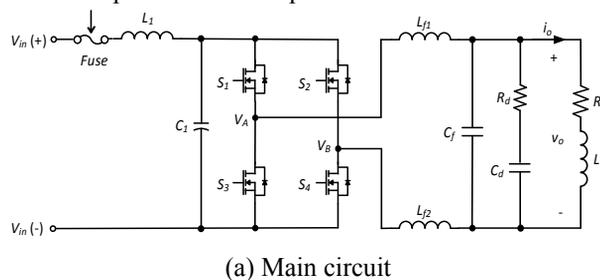
In this paper, a 200kHz interleaved pulse-width-modulation (PWM) power supply circuit with a proportional-and-integral (PI) and phase-lead current regulator is proposed for the fast corrector power supply. The proposed current regulator is mathematically analyzed and optimized, so that it achieves the performance requirement of the power supply by minimizing the attenuation and phase-shift of the compensated closed-loop control system. The performance evaluation of the current regulator has been conducted through Matlab and PLECS simulations.

A WIDE BANDWIDTH BIPOLAR POWER SUPPLY

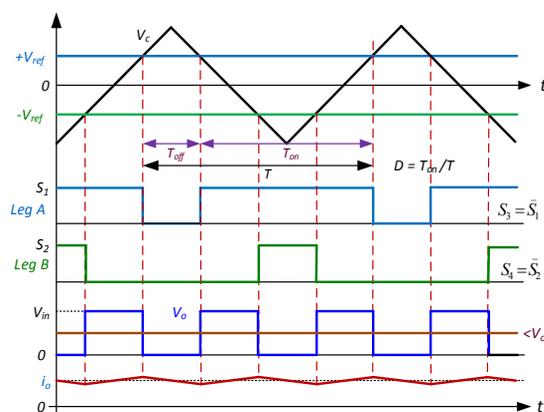
Main Circuit and Its PWM Operation

Figure 1(a) shows a bi-directional power circuit for fast corrector power supplies. The circuit consists of a 200kHz interleaved full-bridge MOSFET power circuit and a 50kHz low pass filter. The power circuit has four switches in two legs, S_1 and S_3 , and S_2 and S_4 .

Figure 1(b) shows the key PWM switching of the power circuit. Two references with the same amplitude but opposite sign, $+V_{ref}$ and $-V_{ref}$, are compared with a 100kHz triangular waveform, V_c , to generate two sets of PWM signals. The output voltage, V_o , is proportional to the duty cycle, $D^*(1-D)$, and input voltage, V_{in} . With this switching pattern, the output current, i_o , has a ripple frequency that is twice the PWM frequency. Thus, this low current ripple can reduce the size of magnetic components and capacitors of the low pass filter in the power circuit.



(a) Main circuit



(b) PWM switching

Figure 1: Main circuit and its PWM switching for a fast corrector power supply.

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MATHEMATICAL MODELING OF A CURRENT REGULATOR

Overall Power Supply Control System

Figure 2 shows the overall control block diagram of the proposed the circuit with the feedback control loop. The mathematical descriptions of the control system are defined as the following transfer functions:

$$\begin{aligned}
 G_{PI}(s) &= K_P + \frac{K_I}{s} \\
 G_{PL}(s) &= K \cdot \frac{1 + s/\omega_{z2}}{1 + s/\omega_p} \\
 G_{Conv} &= K_{PWM} \cdot V_{in} \cdot D(1 - D) \\
 G_{LPF}(s) &= \frac{1 + C_d R_d s}{1 + C_d R_d s + (L_f C_f + L_f C_d) s^2 + R_d C_f C_d L_f s^3} \\
 G_L(s) &= \frac{1}{R_m + L_m s}
 \end{aligned} \quad (1)$$

Where $G_{PI}(s)$ is the transfer function of a PI compensator, consisting of the proportional and integral gains, K_P and K_I , respectively. $G_{PL}(s)$ is the transfer function for the phase-lead compensator, $G_{LPF}(s)$ for the low pass filter, $G_L(s)$ for the fast corrector magnet, and $H_C(s)$ for the current feedback sensor. In addition, G_{Conv} is the gain of the power converter based on the PWM voltage level and duty cycle.

Using the transfer functions given above, the open-loop output transfer function, $Q(s)$, is derived by integrating the transfer functions of the overall control system as:

$$Q(s) = \frac{I_o(s)}{I_0^*(s)} = G_{PI}(s) \cdot G_{PL}(s) \cdot G_{Conv} \cdot G_{LPF}(s) \cdot G_L(s) \quad (2)$$

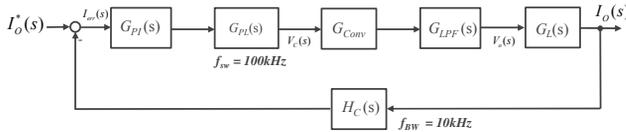


Figure 2: Overall control block diagram of a fast corrector power supply.

Wide Bandwidth Current Regulator Design

A wide bandwidth current regulator for fast corrector power supplies requires an excellent dynamic response over a wide range of operation. To reach this end, a typical PI current controller is considered for the power supply regulators.

PI controllers can be mainly designed with the zero that is placed at or near the frequency at which the signal magnitude has a -3 dB attenuation to cancel the pole of the load transfer function created by the corrector magnet [1]. Thus, the open-loop output transfer function, $Q(s)$, of the system yields the first-order system transfer function. This means that a high bandwidth regulator requires a high proportional gain K_P to increase the bandwidth.

For a 10 kHz small-signal -3dB bandwidth regulator, it may happen that the control variable reaches the voltage limits. When this happens, the feedback loop is broken and the regulator can cause instability problems in power supplies.

Combined PI and Phase-Lead Regulator

By adding a phase-lead compensator to the PI controller, the controller can increase the bandwidth of the closed-loop control system. The proposed regulator is designed with essentially two parts: a PI compensator at low frequencies and a phase-lead compensator at high frequencies. The transfer function, $G_C(s)$, of the current regulator with combined PI and phase-lead compensation network, as shown in Fig. 3, is defined as:

$$G_C(s) = G_{PI}(s) \cdot G_{PL}(s) = G_{C0} \cdot \frac{(1 + s/\omega_{z1})(1 + s/\omega_{z2})}{s(1 + s/\omega_p)} \quad (3)$$

Where G_{C0} is the gain of the current regulator to achieve the stable overall closed-loop control system, and ω_{z2} is the zero introduced by the lead compensator, and thus $\omega_{z2} < \omega_p$. In addition, while selecting ω_{z1} for low frequency compensation, it should be $\omega_{z1} < \omega_{z2}$. In the practical design, ω_{z2} should be 10 times greater than ω_{z1} , in order to separate the effects of the PI and phase-lead portions of the compensator.

Using the Bode plot, the value of G_{C0} can be calculated. Once G_{C0} is determined, based on the pole and zeros of the $G_C(s)$, the component values of the regulator circuit using operational (OP) amplifiers can be determined.

$$G_C(s) = \frac{V_{co}(s)}{V_{ci}(s)} = G_{C0} \cdot \frac{(1 + s/\omega_{z1})(1 + s/\omega_{z2})}{s(1 + s/\omega_p)} \quad (4)$$

Where $\omega_{z1}(s) = \frac{1}{R_1 \cdot C_1}$, $\omega_{z2}(s) = \frac{1}{R_2 \cdot C_2}$, and $\omega_p(s) = \frac{1}{R_3 \cdot C_3}$

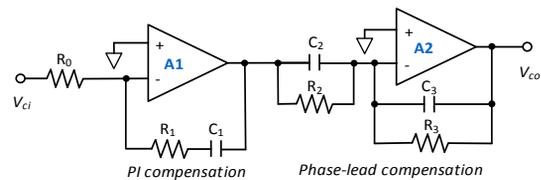


Figure 3: Current regulator circuit using OP amplifiers.

SIMULATION RESULTS

The design parameters of the bipolar power supply for the fast corrector magnet ($R_m = 0.19\Omega$ and $L_m = 16.5\text{mH}$) are given below:

- DAC (Digital to analog converter) output
 - o Input signal DC range : $\pm 10\text{V}$
 - o Input signal AC component : Up to 10 kHz
 - o AC component amplitude : $< 1\%$ of the DC
- DC source and output current
 - o Input voltage : 40VDC
 - o Max. output current : $\pm 15\text{A}$
 - o Closed-loop system gain : $\geq -3\text{dB}$ at 10kHz

In order to meet the design requirements, simulations have been conducted on the overall system model using Matlab and PLECS. In the regulator design, the key functional aspects are: 1) minimize steady-state error, 2) increase phase and gain margins, 3) minimize phase-delay, and 4) increase system bandwidth.

Frequency Responses of Current Control Systems

To predict the closed-loop performance of a current regulator, calculation of the magnitude and phase responses of the overall system through Bode plots is required. Figure 4 shows the frequency responses of the system with two different regulators: a PI regulator and a combined PI plus phase-lead regulator. It can be observed that the combined regulator has the -1.49dB small-signal bandwidth and 32.8° phase-delay at 10kHz, but -3.01dB magnitude attenuation and 51° phase-delay for a PI regulator.

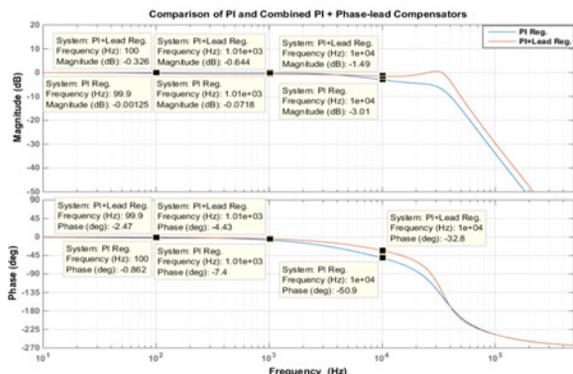


Figure 4: Frequency responses of a PI regulator and a combined PI plus phase-lead regulator.

Mathematical and Electronic Circuit Models

To design a practical current regulator, it is necessary to simulate the overall control system with physical electronic circuit models.

Figure 5(a) shows the block diagram of a mathematical control system model. Each block is identified with the transfer functions of the control system shown in Fig. 2. This mathematical model makes the design more feasible for the practical electronic circuit. Fig. 5(b) shows the electronic circuit model for the power supply, including a 50kHz low pass filter, a PWM generator, a current feedback circuit, and a regulator circuitry. The current reference includes a DC offset and a small amplitude AC sine wave signal. Table 1 summarizes the selected parameters of a combined current regulator.

Figure 6(a)-(d) show the simulation results of the overall system. The power supply with the proposed regulator achieved the 10 kHz bandwidth operation, while the load current, i_o , accurately tracked the current reference, i_{ref} . Table 2 summarizes the simulation results of the regulator with the AC signal amplitude of a 0.5% DC full-scale.

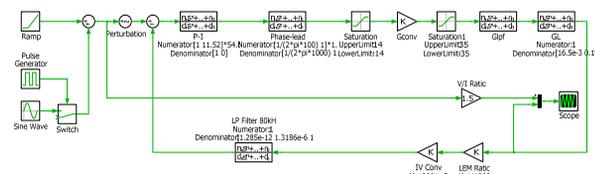
Table 1: Current Regulator Parameters

PI compensation	Phase-lead compensation
$R_0 = 12 \text{ k}\Omega$ $R_1 = 657 \text{ k}\Omega$	$R_2 = 10 \text{ k}\Omega$ $R_3 = 12 \text{ k}\Omega$
$C_1 = 0.1 \mu\text{F}$	$C_2 = 0.15 \mu\text{F}$ $C_3 = 13 \text{ nF}$

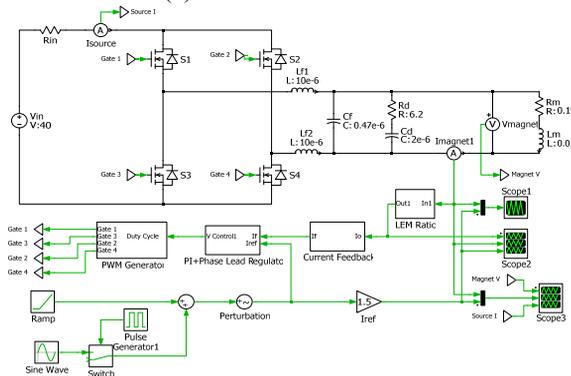
Table 2: Frequency Responses of a Current Regulator at 15A

	100Hz	1kHz	5kHz	10kHz
Mag. Atten. [dB]	-0.32	-0.71	-0.99	-1.23*
Phase-shift [°]	0.96	4.31	18.12	32.49

* Peak value of the AC component is 0.3% of the full DC scale.



(a) Mathematical model



(b) Electronic circuit model

Figure 5: Simulation models of fast corrector power supply system using PLECS.

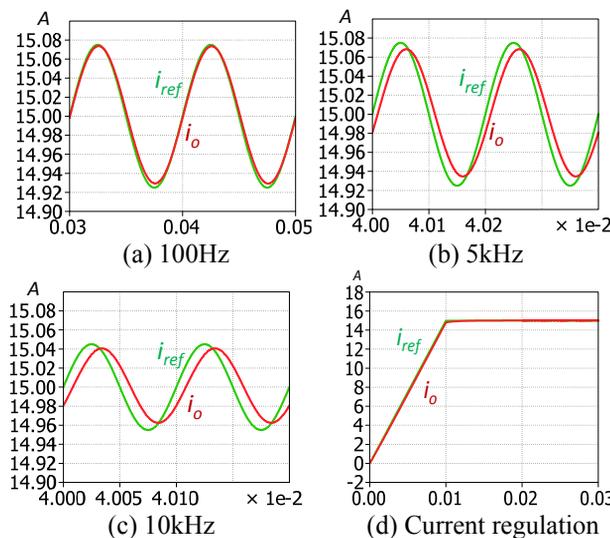


Figure 6: Simulation of a current regulator at 15A.

CONCLUSION

This paper has presented a current regulator for fast corrector power supplies with a 10kHz small signal bandwidth. The regulator, based on combined PI plus phase-lead compensator, was newly designed and analyzed using Matlab and PLECS. The simulation results with a physical electronic circuit model using PLECS, show a -1.23dB magnitude attenuation and 32.49° phase-delay at 10kHz for 0.3% AC component. The next step is to construct the power circuit with this controller to verify the performance.

REFERENCES

[1] P. Bellomo and A. de Lira, "SPEARS3 Intermediate DC Magnet Power Supplies," EPAC 2004, Lucerne, Switzerland, 2004.

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