

# TRANSVERSE FIELD PERTURBATION FOR PIP-II SRF CAVITIES\*

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## Abstract

Proton Improvement Plan II (PIP-II) consists in a plan for upgrading the Fermilab proton accelerator complex to a beam power capability of at least 1 MW delivered to the neutrino production target. A room temperature section accelerates H<sup>-</sup> ions to 2.1 MeV and creates the desired bunch structure for injection into the superconducting (SC) linac. Five cavity types, operating at three different frequencies 162.5, 325 and 650 MHz, provide acceleration to 800 MeV. This paper presents the studies on transverse field perturbation on particle dynamic for all the superconducting cavities in the linac. The effects studied include quadrupole defocusing for coaxial resonators, and dipole kick due to couplers for elliptical cavities. A multipole expansion has been performed for each of the cavity designs including effects up to octupole.

## INTRODUCTION

PIP-II stands for Proton Improvement Plan-II [1] which is Fermilab plan for future improvements to the accelerator complex. The upgrade is aimed at providing LBNE (Long Base Neutrino Experiment) operations with a beam power of at least 1 MW on target. The central element of the PIP-II is a new 800 MeV superconducting linac, injecting into the existing Booster. The PIP-II 800 MeV linac is a derivative of the Project X Stage 1 design. The room temperature (RT) section includes a Low Energy Beam Transport (LEBT), RFQ and Medium Energy Beam Transport (MEBT), accelerating H<sup>-</sup> ions to 2.1 MeV and it creates the desired bunch structure for injection into the superconducting (SC) linac. This article focuses on the SC linac and its five cavity types, half wave resonator (HWR), single spoke resonator (SSR1 and SSR2), low beta (LB) and high beta (HB) elliptical cavities. The technology map of the PIP-II linac, Fig. 1, shows the transition energies between accelerating structures, and the transition in frequency, HWR operates at 162.5 MHz (same as RT section), SSR1 and SSR2 at 325 MHz, LB and HB elliptical cavities at 650 MHz.

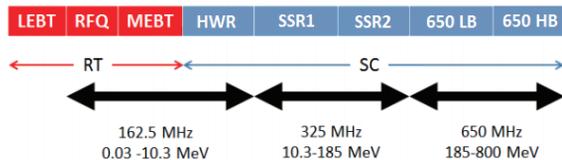


Figure 1: PIP-II linac technology map.

The three families of coaxial resonators show quadrupole perturbation to the transverse fields [2], this is due to the inner electrode which breaks the azimuthal

symmetry of the cavity. Elliptical cavities do not have inner posts, but the transverse symmetry is broken by the coupler port on top of the beam tube, leading to a dipole perturbation to the transverse fields. All the results presented are based on EM fields simulated with Comsol multi-physics, the simulation technique requires local mesh refinement around the beam axis to improve transverse field quality. In this article transverse field perturbation is calculated and analyzed for all the cavities in the SC linac of PIP-II, multipole expansion has been performed to show if any higher order effect on the beam dynamic can be present, e.g. sextupole and octupole.

## TRANSVERSE KICK

The transverse momentum gain is calculated by direct integration of the electric and magnetic fields transverse to the beam direction. Considering the z axis as the longitudinal (beam) direction, electric and magnetic fields will have x and y components. Assuming the particle velocity constant along z axis,  $\beta$  is constant, one can write:

$$\Delta p_x(r, \alpha)c = \int_{z_i}^{z_f} \left( \frac{E_x(r, \alpha)}{\beta} - \mathbf{Z}_0 i H_y(r, \alpha) \right) e^{i \frac{kz}{\beta}} dz \quad (1)$$

$$\Delta p_y(r, \alpha)c = \int_{z_i}^{z_f} \left( \frac{E_y(r, \alpha)}{\beta} + \mathbf{Z}_0 i H_x(r, \alpha) \right) e^{i \frac{kz}{\beta}} dz \quad (2)$$

$\Delta p_x(r, \alpha)c$  and  $\Delta p_y(r, \alpha)c$  are functions of the radius and the angle  $\alpha$  on the x-y plane,  $\alpha$  is taken with respect to the x axis:  $\alpha=0$  corresponds to x axis and  $\alpha=\pi/2$  refers to y axis. For low-beta structures, the radial component variation will be maximum between the kick on x and on y axis; to have an estimation of the asymmetry one can define a parameter called  $Q$ :

$$Q = \frac{\Delta p_x(r, 0)c - \Delta p_y(r, \pi/2)c}{(\Delta p_x(r, 0)c + \Delta p_y(r, \pi/2)c)/2} \quad (3)$$

Asymmetry parameter  $Q$  gives partial information on the transverse kick, evaluating  $\Delta p_R c$  on the whole x-y plane, it is possible to calculate the multipole expansion of the transverse momentum gain:

$$\Delta p_R c(r, \alpha) = A_0(r) + \sum_{n=1}^{\infty} A_n(r) \cos(n\alpha) + B_n(r) \sin(n\alpha) \quad (4)$$

where  $A_n(r) \propto nr^{n-1}$  and  $B_n(r) \propto nr^{n-1}$  are the Fourier series coefficient of  $\Delta p_R c$  while  $A_0(r) \propto r$  and it is the mean value. One coefficient set out of two has non zero values and the other vanishes. Normal or skew components are due to the symmetry of the problem and to the choice frame of reference to define x-y plane. The transverse kick has been expanded up to octupole, corresponding to  $n = 4$ .

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## LOW BETA RESONATORS

Spoke and half wave resonator geometries have a central electrode that lies on one of the axes perpendicular to the particles motion, breaking axial symmetry of the cavity. Fig. 2 shows a spoke resonator RF volume with a sketch of the EM field lines. These structures have a fundamental mode which is a quasi-TEM: the electric field is perpendicular to the central conductor, Fig. 2(a), while the magnetic field lines surround the spoke spinning around it, Fig. 2(b).

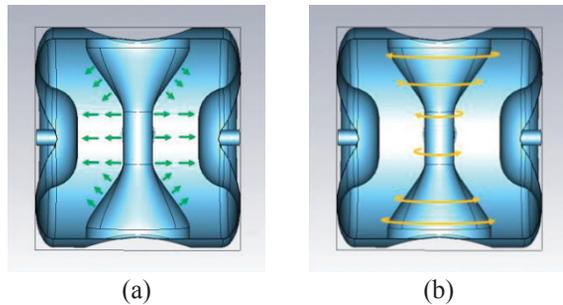


Figure 2: Single spoke resonator electric (a) and magnetic (b) field lines.

Both electric and magnetic transverse field can contribute to the quadrupolar perturbation to the beam dynamic and different solutions can be adopted. Since the focusing is provided with solenoids in the HWR, SSR1 and SSR2 cryomodules, a corrector has been added for quadrupole. A good estimation of the perturbation helps in assessing what kind and how strong the correction should be. The HWR will be placed horizontally in the cryomodule [3] this will give a normal quadrupole, while SSR1 and SSR2 inner electrodes will be rotated by 45 degrees in the transverse plane [4], and that will bring to a skew quadrupole.

### HWR and SSR1 Field Asymmetry

The initial design of the HWR for PIP-II, and PXIE, showed transverse field asymmetry that was affecting the particle dynamic, which was corrected by modifying the shape of the inner post [5], [6]. SSR1 cavities transverse field asymmetry does not perturb the beam significantly. The Q parameter for both HWR and SSR1 is very low for the whole beta range, resulting in a very little transverse perturbation to the beam. Table 1 reports Q parameter values at initial and final particle  $\beta$  for the PIP-II lattice of HWR and SSR1. Figure 3 shows the transverse momentum gain for particles traveling through the HWR, Fig. 3(a), and the SSR1 section, Fig. 3(b). In Fig. 3 the dependence on particle beta is calculated considering the cavity operating at maximum gradient.

Table 1: Q parameter of HWR and SSR1, calculated at  $\beta$  lowest and highest values

	$Q_{HWR}$	$Q_{SSR1}$
$\beta_{initial}$	0.07298	0.09075
$\beta_{final}$	-0.02531	-0.01718

### SSR2 Transverse Field Asymmetry

SSR2 resonators for PIP-II affect the particle dynamic with a quadrupole perturbation due in major part to the magnetic field components. Two approaches can be used to achieve correction: the first implies the use of correctors within the focusing lens in the cryomodule, the other requires reshaping the inner electrode as an X or a Y, to force magnetic field transverse symmetry in the cavity structure [7]. The Q parameter ranges from 0.39433 at  $\beta_{initial}$  to 0.0462 at  $\beta_{final}$ . Figure 4 shows the electric and the magnetic transverse momentum gain for the whole beta range of SSR2 cavity.

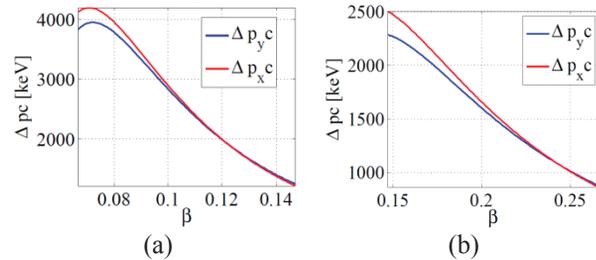


Figure 3: Transverse momentum gain comparison vs beta for HWR (a) and SSR1 (b).

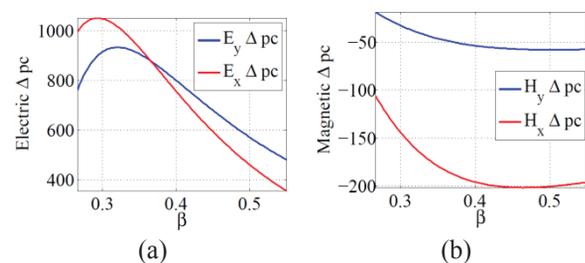


Figure 4: Transverse momentum gain vs particle beta for SSR2 cavity, (a) electric (b) magnetic component.

### Low Beta Cavities Multipoles

In order to understand if any higher order effects are relevant for these three coaxial resonators, one can perform a full multipole expansion of the transverse momentum gain. Table 2 reports the values of multipoles up to the octupole, for HWR, SSR1 and SSR2. It is important to notice that for  $n=2$  (quadrupole) amplitude is the highest for all the cavities, odd harmonics have zero amplitude, and no higher order effects are relevant (for  $n=4$   $A_n/r^{n-1}$  drops at least by three order of magnitude).

Table 2: multipoles amplitude calculated at 10 mm radial offset and at beta optimal for HWR, SSR1 and SSR2, quadrupole has been highlighted

$A_n/r^{n-1}$	HWR	SSR1	SSR2
1 <sup>st</sup> [keV]	7.474e-4	1.096e-13	3.427e-14
2 <sup>nd</sup> [keV/mm]	1.3003	1.979	2.745
3 <sup>rd</sup> [keV/mm <sup>2</sup> ]	7.474e-06	1.188e-15	5.484e-16
4 <sup>th</sup> [keV/mm <sup>3</sup> ]	5.906e-4	1.032e-3	4.568e-4

Since each cavity of PIP-II linac front end will be used in a wide range of energy it is helpful to investigate the behaviour of the Q parameter in the whole  $\beta$  range. Figure

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5 shows  $Q$  vs  $\beta/\beta_{opt}$  in the whole range of usage for all the coaxial resonators. SSR1 and HWR field asymmetry are the least significant:  $Q$  is smaller than 0.1 in the whole range of energy. SSR2 shows a much higher quadrupole asymmetry. Overall these are still small perturbations to the beam dynamic, and they will be compensated by correctors placed within the focusing solenoid in all cryomodules.

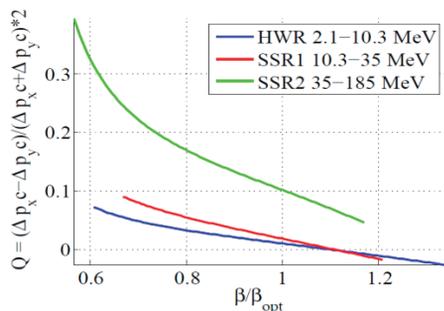


Figure 5:  $Q$  parameter vs  $\beta/\beta_{opt}$  for HWR, SSR1 and SSR2.

### ELLIPTICAL CAVITIES 650 MHz

The geometry of multi-cell cavities is intrinsically quadrupole free, because of the absence of inner electrodes. The only element perturbing the cavity symmetry is the coupler which needs to be inserted into the geometry trough a port perpendicular to the beam axis. The cross section of a 5 cell 650 MHz is shown in Fig. 6, one can see the coupler port on top of the first beam tube.

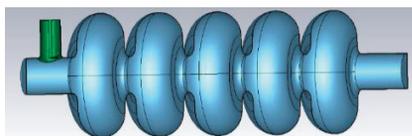


Figure 6: PIP-II 650 MHz HB (geometrical beta=0.92) 5 cell cavity cross section, coupler port is highlighted in green.

The presence on the coupler port breaks the symmetry on the X axis, since the port lies only on one side of the beam tube it will induce a dipole perturbation. Transverse fields on axis have very small amplitude: in order to get reliable transverse fields on axis from simulations, the mesh had to be refined properly on the beam axis to guarantee convergence on the EM solver. The perturbation to the beam is shown in Fig.7: dipole kick on X axis vs particle beta, for LB and HB 650 MHz operating at maximum gradient.

The  $Q$  parameter has been plotted in Fig. 8 to show that the quadrupole perturbation is almost negligible for elliptical cavities of PIP-II. Quadrupole correction is easily achieved since the focusing is provided by quadrupole magnets within the cryomodules. Finally to

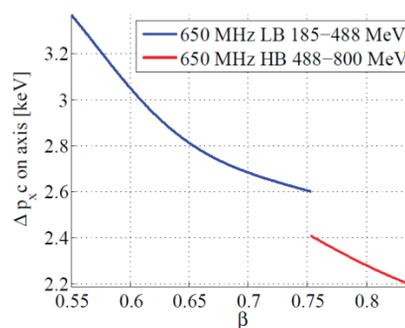


Figure 7: Dipole kick absolute value on axis, as a function of particle beta for LB and HB 650 MHz cavities.

verify that higher order effects are not relevant for the elliptical cavities of PIP-II, a multipole expansion was carried out. Table 3 shows the multipole amplitudes for both low and high beta design: small dipole and quadrupole components, sextupole and octupole are negligible.

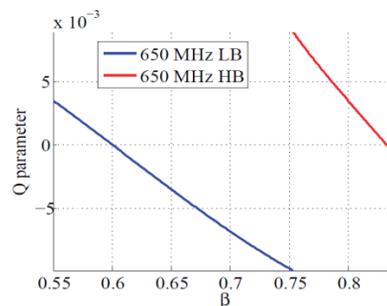


Figure 8:  $Q$  parameter vs  $\beta$  for 650 MHz cavities of PIP-II.

Table 3: Multipoles amplitude calculated at 10 mm radial offset and at beta geometrical for 650 MHz cavities of PIP-II, dipole is highlighted

$A_n/r^{n-1}$	LB 650	HB 650
1 <sup>st</sup> [keV]	0.183	1.733
2 <sup>nd</sup> [keV/mm]	0.023	0.053
3 <sup>rd</sup> [keV/mm <sup>2</sup> ]	4.235e-4	1.145e-3
4 <sup>th</sup> [keV/mm <sup>3</sup> ]	2.033e-05	1.140e-05

### CONCLUSIONS

Summarizing the study of field asymmetry for the PIP-II low-beta cavities showed that quadrupole perturbation is either normal (HWR) or skew (SSR1 and SSR2). It is desirable to have quadrupole correctors within cryomodules, especially for SSR2 cavities which have strong magnetic field components. The dipole perturbation due to the input coupler for the 650 MHz elliptical cavities is very small even for maximum field amplitude throughout the whole particle beta range. Neither low-beta nor elliptical cavities show higher order multipoles relevant to the beam dynamic.

## REFERENCES

- [1] V. Lebedev, PIP-II RDR, web link: [http://www-bdnew.fnal.gov/pxie/PIP-II\\_RDR/PIP-II\\_RDR.pdf](http://www-bdnew.fnal.gov/pxie/PIP-II_RDR/PIP-II_RDR.pdf)
- [2] P. Berrutti et al., WEPPC044, Proc. of IPAC'12, New Orleans, Louisiana USA (2012), <http://www.JACoW.org>
- [3] P. N. Ostroumov, MOP066, Proc. of SRF'13, Paris, France (2013), <http://www.JACoW.org>
- [4] T. Nicol et al., SSR1 cryomodule design for PXIE, THPMA09, Proc. of NA-PAC'13, Pasadena, California, USA (2013), <http://www.JACoW.org>
- [5] P. Berrutti et al., WEPPC047, Proc. of IPAC'12 New Orleans, Louisiana, USA (2012), <http://www.JACoW.org>
- [6] P. N. Ostroumov et al., WEPPC039, Proc. of IPAC'12, New Orleans, Louisiana, USA, <http://www.JACoW.org>
- [7] P. Berrutti et al., WEPAC022, Proc. of NA-PAC'13, Pasadena, California, USA, <http://www.JACoW.org>