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BEAM-DRIVEN TERAHERTZ SOURCE BASED ON OPEN ENDED WAVEGUIDE WITH A DIELECTRIC LAYER: RIGOROUS APPROACH*

Sergey N. Galyamin†, Andrey V. Tyukhtin, Aleksandra A. Grigoreva, Viktor V. Vorobev
St. Petersburg State University, Physical Faculty, St. Petersburg, 198504, Russia

Abstract

Terahertz frequency radiation (0.1-10 THz) is a promising tool for a number of scientific and practical applications. One promising scheme to obtain powerful and efficient THz emission is usage of beam-driven dielectric loaded structures [1]. Recently we have considered the problem where the microbunched ultrarelativistic charge exits the open end of a cylindrical waveguide with a dielectric layer and produces THz waves in a form of Cherenkov radiation [2]. To investigate the applicability of utilized approximations, we analyze here the case of orthogonal end of a waveguide with continuous filling. However, presented rigorous approach can be generalized for waveguide with vacuum channel. We use the combination of Wiener-Hopf technique and tailoring technique. The infinite linear system for magnitudes of reflected waveguide modes is obtained and solved numerically. We present typical field distributions over the aperture and typical radiation patterns in the Fraunhofer zone.

THEORY AND ANALYTICAL RESULTS

Convenient rigorous method for investigation of radiation from open-ended plane dielectrically loaded waveguides has been developed several decades ago [3]. Here we generalize this approach for the case of cylindrical geometry. At the current stage, we consider in detail the case of continuous filling, which is relatively simple. In the sequel, we plan to apply the developed technique for layered waveguide.

Consider a semi-infinite cylindrical waveguide with radius a filled with a dielectric ($\varepsilon > 1$) (Fig. 1). We suppose that single TM_{0l} waveguide mode incidents the orthogonal open end (cylindrical frame ρ, φ, z is used):

$$\left\{H_{\omega\varphi}^{(i)}, E_{\omega\rho}^{(i)}\right\} = \left\{1, k_{zl} c(\omega\varepsilon)^{-1}\right\} J_1(\rho j_{0l}/a) e^{ik_{zl}z}, \quad (1)$$

$$E_{\omega z}^{(i)} = N J_0(\rho j_{0l}/a) e^{ik_{zl}z}, \quad N = ic(\omega\varepsilon)^{-1} j_{0l}/a, \quad (2)$$

where $J_0(j_{0l}) = 0$, $k_{zl} = \sqrt{k_0^2\varepsilon - j_{0l}^2 a^{-2}}$, $\operatorname{Im} k_{zl} > 0$, $k_0 = \omega/c$. The reflected field in the area $z < 0$, $\sqrt{x^2 + y^2} = \rho < a$ is decomposed into a series of waveguide modes propagating in opposite direction:

$$E_{\omega z}^{(r)} = \sum_{m=1}^{\infty} N_m J_0(\rho j_{0m}/a) e^{-ik_{zm}z}, \quad (3)$$

where N_m are unknown "reflection coefficients" that should be determined. The vacuum area is divided into two subareas (1) and (2) (see Fig. 1), where the field is described by Helmholtz equation:

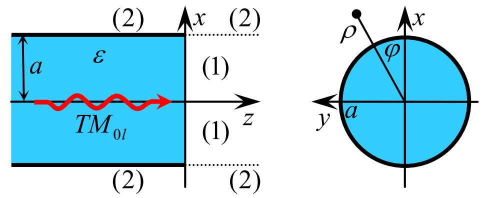

Figure 1: Geometry of the problem.

$$\left(\partial^2/\partial z^2 + \partial^2/\partial\rho^2 + \rho^{-1}\partial/\partial\rho + k_0^2\right) E_{\omega z}^{(1,2)} = 0. \quad (4)$$

We introduce functions $\Psi_{\pm}(\rho, \alpha)$ (hereafter subscripts $\pm$ mean that function is holomorphic and have no zeros for $\operatorname{Im}\alpha > 0$ and $\operatorname{Im}\alpha < 0$, correspondingly):

$$\Psi_{+}^{(1,2)}(\rho, \alpha) = (2\pi)^{-1} \int_0^{\infty} dz E_{\omega z}^{(1,2)}(\rho, z) e^{i\alpha z}, \quad (5)$$

$$\Psi_{-}^{(2)}(\rho, \alpha) = (2\pi)^{-1} \int_{-\infty}^{0} dz E_{\omega z}^{(2)}(\rho, z) e^{i\alpha z}. \quad (6)$$

From (4) we obtain

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \kappa^2\right) \begin{Bmatrix} \Psi_{+}^{(1)} \\ \Psi_{-}^{(2)} + \Psi_{+}^{(2)} \end{Bmatrix} = \begin{Bmatrix} F^{(1)} \\ 0 \end{Bmatrix}, \quad (7)$$

$$F^{(1)} = (2\pi)^{-1} \partial E_{\omega z}^{(1)}/\partial z\Big|_{z=+0} - (2\pi)^{-1} i\alpha E_{\omega z}^{(1)}\Big|_{z=+0}, \quad (8)$$

where $\kappa = \sqrt{k_0^2 - \alpha^2}$, $\operatorname{Im}\kappa > 0$. Function $F^{(1)}$ is determined using continuity of $\partial E_{\omega\rho}/\partial\rho \sim \partial E_{\omega z}/\partial z$ and jump of $E_{\omega z}$ at $z = 0$, in the issue we obtain:

$$\Psi_{+}^{(1)} = C_1 J_0(\rho\kappa) + \Psi_p^{(1)}, \quad \Psi_{-}^{(2)} + \Psi_{+}^{(2)} = C_2 H_0^{(1)}(\rho\kappa), \quad (9)$$

$$\Psi_p^{(1)}(\rho, \alpha) = \frac{i}{2\pi}\left[\frac{k_{zl} - \alpha\varepsilon}{\kappa^2 - j_{0l}^2 a^{-2}} J_0\left(\frac{\rho j_{0l}}{a}\right) - \right.$$
$$\left. - \sum_{m=1}^{\infty} N_m \frac{k_{zm} + \alpha\varepsilon}{\kappa^2 - j_{0m}^2 a^{-2}} J_0\left(\frac{\rho j_{0m}}{a}\right)\right], \quad (10)$$

where $C_{1,2}$ are unknown coefficients. Utilizing $E_{\omega z} = 0$ for $\rho = a$, $z < 0$ and continuity of $E_{\omega z}$ and $\partial E_{\omega z}/\partial\rho \sim (\partial^2/\partial z^2 + k_0^2) H_{\omega\varphi}$ for $\rho = a$, $z > 0$, we obtain the following relation

$$\frac{\partial\Psi_{+}^{(2)}(a, \alpha)}{\partial a} = \frac{-\kappa J_1(a\kappa)}{J_0(a\kappa)} \Psi_{+}^{(2)}(a, \alpha) + \frac{\partial\Psi_p^{(1)}(a, \alpha)}{\partial a} \quad (11)$$

and Wiener-Hopf equation for $\Psi_{+}^{(2)}(a, \alpha)$:

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† s.galyamin@spbu.ru

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$$\frac{2}{i}\frac{\Psi_+^{(2)}(a,\alpha)\kappa}{G(\alpha)}+\frac{\partial\Psi_-^{(2)}(a,\alpha)}{\partial a}+\frac{\partial\Psi_p^{(1)}(a,\alpha)}{\partial a}=0, \qquad (12)$$

where $G(\alpha)=\pi a\kappa J_0(a\kappa)H_0^{(1)}(a\kappa)$. Since the right-hand side of (11) should be holomorphic for $\mathrm{Im}\,\alpha>0$, including points $\alpha=\alpha_p=\kappa(j_{0p}/a)$ (note that $J_0(a\kappa(\alpha_p))=0$), we get for $p=1,2,\ldots$:

$$\Psi_+^{(2)}(a,\alpha_p)=ia^2J_1(j_{0p})\Big/\left(4\pi j_{0p}\right)\times \\ \times\left[\delta_{lp}(k_{zp}-\alpha_p\varepsilon)-N_p(k_{zp}+\alpha_p\varepsilon)\right], \qquad (13)$$

where δ_{lp} is the Kronecker symbol. Equation (12) is solved in a common way:

$$-2i\Psi_+^{(2)}(a,\alpha)\kappa_+G_+^{-1}(\alpha)+T_+=P(\alpha), \qquad (14)$$

where factorization $G=G_+G_-$, $\kappa=\kappa_+\kappa_-$ and subsequent decomposition

$$T=G_-\kappa_-^{-1}\partial\Psi_p^{(1)}(a,\alpha)\Big/\partial a=T_++T_- \qquad (15)$$

is performed using standard formulas [4]. Unknown polynomial function $P(\alpha)$ is determined using Meixner edge condition [3,4]. Since for $|\alpha|\to\infty$, $\mathrm{Im}\,\alpha>0$

$$\Psi_+^{(2)}(a,\alpha)\kappa_+G_+^{-1}(\alpha)\sim\alpha^{\tau},\; T_+\sim\alpha^{-1}, \qquad (16)$$

where $\tau=\pi^{-1}\sin^{-1}[(\varepsilon-1)/(2\varepsilon+2)]$, therefore one should put $P(\alpha)=const=T_+(-k_0)$. Calculating $\Psi_+^{(2)}(a,\alpha_p)$ and substituting it into (13), we obtain the linear system for N_m:

$$\sum_{m=1}^{\infty}N_mW_{mp}=w_p\;, \qquad (17)$$

where

$$W_{mp}=ia^2\delta_{mp}j_{0p}^{-1}J_1(j_{0p})(k_{zp}+\alpha_p\varepsilon)+ \\ +\frac{G_+(\alpha_p)G_+(\alpha_m)J_1(j_{0m})(k_{zm}-\alpha_m\varepsilon)\kappa_+(\alpha_p)}{2\alpha_m\kappa_-(\alpha_m)(\alpha_p+\alpha_m)}, \qquad (18)$$

$$w_p=ia^2\delta_{lp}j_{0p}^{-1}J_1(j_{0p})(k_{zp}-\alpha_p\varepsilon)+ \\ +\frac{G_+(\alpha_p)G_+(\alpha_m)J_1(j_{0l})(k_{zl}+\alpha_l\varepsilon)\kappa_+(\alpha_p)}{2\alpha_l\kappa_-(\alpha_l)(\alpha_p+\alpha_l)}. \qquad (19)$$

It can be shown that for $\varepsilon=1$ this system is analytically solved and the solution coincides with known result for vacuum waveguide [5]. For $\varepsilon\neq1$ system (17) can be solved numerically using the reduction technique.

NUMERICAL RESULTS

For the case of $\varepsilon\neq1$, we solve (17) by reducing it to the finite system of $N_{\max}$ equations, where $N_{\max}$ was chosen around two times as much as the total number of propagating modes in the waveguide at given frequency. After N_m are found, we can determine the total field for $\rho<a$, $z=-0$. Then, using continuity of $E_{\omega\rho}$ and $H_{\omega\varphi}$, we determine the tangential field at the outer surface of the aperture ($\rho<a$, $z=+0$). Figure 2 shows the behaviour of the exact $H_{\omega\varphi}$ field component over the outer surface of aperture (solid line). The mode frequency was chosen to be equal to the frequency of mode of Cherenkov radiation with number $m=20$ produced by a charge moving with relative velocity $\beta=\sqrt{1-\gamma^{-2}}$ (γ is Lorentz factor) in regular waveguide [2, 6]:

$$\omega_m=c\beta j_{0m}\Big/\left(a\sqrt{\varepsilon\beta^2-1}\right). \qquad (20)$$

We also show here the behaviour of this field component calculated via approximate technique used in [2] for dielectric aperture (dashed line). Recall, that in accordance with this technique, we approximately decompose a waveguide mode into two quasi plane waves and describe the refraction of each wave through the interface using formalism of Fresnel coefficients. One can see that for these curves maxima and minima are well correlated. Certain improvement in field magnitude is required for small area near $\rho=0$ and $\rho=a$ (edge).

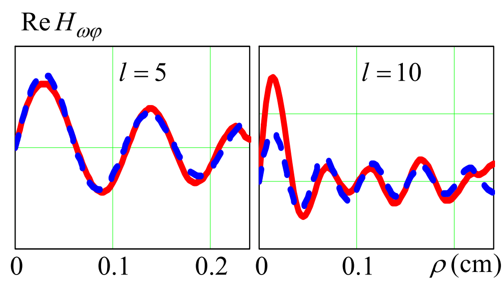

Figure 2: The behaviour of $H_{\omega\varphi}$ component over the waveguide aperture $\rho<a$, $z=+0$. Solid (red) line is calculated using solution of (17), dashed (blue) line is calculated using approximate technique from [2]. Mode frequency is $\omega_{20}=2\pi\cdot412$ GHz, $a=0.24$ cm, $\varepsilon=10$, $\gamma=7$. Waveguide supports 21 propagating modes.

Since the tangential field for $\rho<a$, $z=+0$ is known, it is convenient to utilize the Stratton-Chu formulas [2] to calculate the field in the far-field (Fraunhofer) zone (spherical frame R, θ, φ associated with cylindrical frame ρ, φ, z is utilized). The far field zone is determined by inequalities:

$$R>>1/k_0,\quad R>>a,\quad R>>a^2/\lambda\quad(\lambda=2\pi/k_0). \qquad (21)$$

Figure 3 shows typical radiation patterns in the Fraunhofer zone (21). Here the mode frequency was chosen to be equal to the frequency of l-th Cherenkov mode (20). For $\varepsilon=10$ and $\gamma=7$ waveguide supports l propagating modes. The low number modes ($l=1,2$) provide maximum radiation in orthogonal direction ($\pm90^\circ$) and radiation patterns possess weak directivity. With an increase in mode number, orthogonal radiation disappears, the directivity of radiation pattern increases, radiation mostly goes in forward direction and angle of pattern maxima decreases (it is around 30° for 5-th mode and 15° for 10-th mode).

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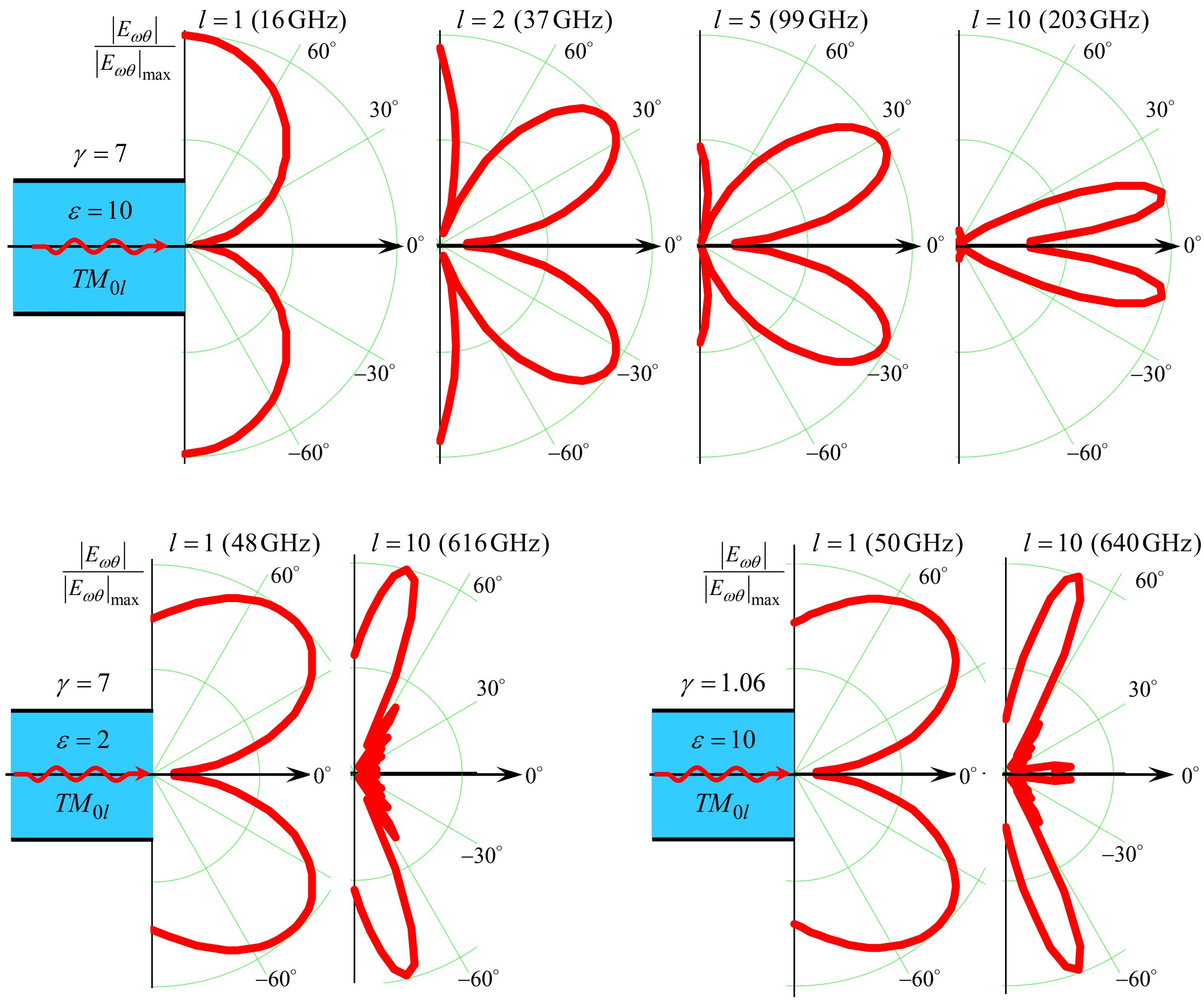

Figure 3: Radiation patterns (normalized $|E_{\omega\theta}|$ versus θ) in the Fraunhofer zone (21) produced by TM_{0l} mode at the open end of dielectrically loaded waveguide with $a = 0.24\,\text{cm}$. Frequency ω (shown near each pattern) is chosen to be equal to the frequency of l-th mode of Cherenkov radiation (20) produced by the charge with Lorentz factor γ.

The cases $\varepsilon = 2$, $\gamma = 7$ and $\varepsilon = 10$, $\gamma = 1.06$ are similar to each other by differ slightly from the case of $\varepsilon = 10$, $\gamma = 7$. The mode with number $l = 1$ gives the essential radiation in orthogonal direction, but pattern maximum is at approximately $\pm 45^\circ$. With an increase in mode number, the angle of main lobe increases. The 10-th mode produces two narrow main lobes at approximately $\pm 70^\circ$ and a number of weak secondary lobes. The presence of secondary lobes is connected with the fact that waveguide supports a lot of propagating modes (14 for $\varepsilon = 2$, $\gamma = 7$ and 32 for $\varepsilon = 10$, $\gamma = 1.06$).

In conclusion, we note that earlier we analyzed radiation from the open end of waveguide with a dielectric layer and a vacuum channel [2] where certain approximate analytical approaches applicable for modes with large numbers only were utilized. Now we will investigate this problem using the rigorous approach described above.

REFERENCES

[1] S. Antipov et al., Appl. Phys. Lett. 100, 132910 (2012).

[2] S.N. Galyamin et al., Opt. Express. 22(8), 8902 (2014).

[3] G.V. Voskresenskii, S.M. Zhurav, Radiotekhnika i Electronika. 4, 703 (1978).

[4] R. Mittra and S. W. Lee, *Analytical techniques in the theory of guided waves* (Macmillan, 1971).

[5] L. Weinstein, *The theory of diffraction and the factorization method: Generalized Wiener-Hopf technique* (Golem Press, 1969).

[6] B.M. Bolotovskii, Sov. Phys. Usp. 4, 781 (1962).