

MODE TRANSFORMATION IN WAVEGUIDE WITH TRANSVERSAL BOUNDARY BETWEEN VACUUM AND PARTIALLY DIELECTRIC AREA*

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Abstract

We consider the mode transformation in a circular waveguide with a transversal boundary between a vacuum part and a part with a cylindrical dielectric layer and a vacuum channel. It is assumed that an incident mode can be both propagating and evanescent. Analysis is carried out with the using the mode decomposition technique. Numerical algorithm for calculating the mode transformation at an arbitrary channel radius is also developed. Typical dependences for the reflection and transmission coefficients on the channel radius are presented and discussed.

INTRODUCTION

We study the electromagnetic field in a circular waveguide which consists of two semi infinite parts: one of them has a cylindrical dielectric layer and a vacuum channel, and the other does not have any filling. Such a problem is of interest, for example, for new perspective method of generation of terahertz radiation from an electron bunch in a dielectric loaded structure [1–3]. It is important to consider the field of the bunch flying into the vacuum part of the waveguide. Earlier this problem was considered for the case when the vacuum channel is absent [4, 5]. It is obvious that the principal modifications in the reflected and transmitted fields take place in the presence of the vacuum channel due to the effect of the mode transformation.

Another sample is the wakefield acceleration technique in the dielectric waveguide structures [6, 7]. The generation of the wakefield occurs at some distance after a driver flying into the dielectric part of waveguide. Analysis of process of the wakefield formation is essential for this application. This process is also interesting as well for development of the method of measurement of particle energy with applying the dielectric loaded waveguide [8, 9].

These examples show that both of the problem, when the bunch enters into the partially dielectric area, and the problem, when the bunch flies out of this, are of interest for applications. Such problems can be analyzed by means of an expansion of the incident field into a series of waveguide modes. Here we consider the case of a single incident mode. Note that such a problem statement is conventional in the waveguide theory and has an essential independent importance.

THE CASE OF THE MODE FALLING FROM THE VACUUM PART

First, we assume that the left part of the waveguide ($z < 0$) is filled up with a medium having permittivity ϵ_c and permeability μ_c . The right part of the waveguide consists of a channel and a dielectric layer described by ϵ_c, μ_c and ϵ_d, μ_d respectively (Fig. 1). Initially we assume that media are dissipative that is $\text{Im } \epsilon_{c,d} > 0$ for positive frequencies (these values will be tended to zero at numerical calculations). Both of the media are isotropic, homogeneous and nondispersive. We suppose that the z -axis coincides with the waveguide axis, and the transversal boundary is placed at $z = 0$. The incident field is one of the TM_{0i} mode which can be both propagating and evanescent.

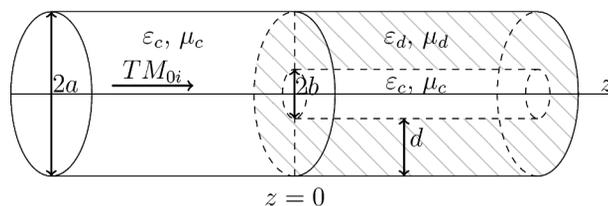


Figure 1: The problem geometry in the case of the mode falling from the vacuum part.

The solution is carried out by cross-linking method. This method presupposes that reflected and transmitted fields are presented as a series of eigenmodes. In order to obtain the equation for the excitation coefficients of the modes of reflected and transmitted fields, we use the continuity conditions for the tangential components of the field at the transversal boundary. After some transformation we obtain a system containing an infinite number of linear algebraic equations. For calculations, we consider the system of finite size. It is possible because the most of the modes are evanescent and they exponentially decrease with the distance from the boarder.

In general case, this matrix system is analytically unsolvable. However, in two dedicated cases (a narrow channel, $b \ll a$, and a thin dielectric layer, $d = (a - b) \ll a$) some approximate results can be obtained.

A Narrow Channel

The approximation for eigenvalues in the dielectric waveguide with a narrow channel ($b \ll a$) can be obtained using the iteration technique. One can see that eigenvalues don't have linear terms. In a similar way, the system of

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the equations for the excitation coefficients can be approximately solved.

As can be shown, for the reflection $R_n = E_{nz}^{(r)}/E_z^{(i)}|_{z=\rho=0}$ and transmission $T_n = E_{nz}^{(t)}/E_z^{(i)}|_{z=\rho=0}$ coefficients, we have the following expressions:

$$R_n \approx R_n^{(0)} \delta_{in} + R_n^{(2)} \cdot \frac{b^2}{a^2}, T_n \approx T_n^{(0)} \delta_{in} + T_n^{(2)} \cdot \frac{b^2}{a^2} \quad (1)$$

where $R_n^{(0)}, R_n^{(2)}, T_n^{(0)}, T_n^{(2)}$ are some values which do not depend on b/a , and δ_{in} is a Kronecker symbol where the index i is an incident mode number. It can be seen that the mode transformation is weak for the mode number $n \neq i$, when all of the coefficients are the values of the second infinitesimal order. Obviously, this is explained by the fact that the ratio of the channel cross-section square to the waveguide cross-section is proportional to b^2/a^2 . Furthermore, increasing the mode number results only in small correction of order of b^4/a^4 .

A Thin Dielectric Layer

Another situation takes place in the case of thin dielectric layer ($d \ll a$), when the eigenvalues are presented as a sum of terms of orders of $(d/a)^0$ and d/a . In this case, the reflection and transmission coefficients are written in the form

$$R_n \approx R_n^{(1)} \cdot d/a, T_n \approx \delta_{in} + T_n^{(1)} \cdot d/a \quad (2)$$

where $R_n^{(1)}, T_n^{(1)}$ are some constants. As expected the main transmitted mode has the same number i as the incident mode. All reflected modes have first order of small parameter. Thus, the mode transformation is the effect of the first order of the small parameter d/a , in contrast to the case of a narrow channel where this effect has the second order of smallness. Obviously, this is explained by the fact that the square of the dielectric layer cross-section (the ring square) is the value of order d/a with respect to the waveguide cross-section.

THE CASE OF THE MODE FALLING FROM THE DIELECTRIC PART

We consider as well the “mirror” problem where the transversal magnetic TM_{0i} mode falls from the partially dielectric part of waveguide. Note some results for this case.

If the channel is narrow, the reflection and transmission coefficients are written in the form:

$$\tilde{R}_n \approx \tilde{R}_n^{(0)} \delta_{in} + \tilde{R}_n^{(2)} \cdot b^2/a^2, \tilde{T}_n \approx \tilde{T}_n^{(0)} \delta_{in} + \tilde{T}_n^{(2)} \cdot b^2/a^2 \quad (3)$$

At solving the system, increasing in the mode number results only in small correction of order of b^4/a^4 .

In the case of thin layer these coefficients have the form

$$\tilde{T}_n \approx \delta_{in} + \tilde{T}_n^{(1)} \cdot d/a, \tilde{R}_i \approx \tilde{R}_i^{(2)} \cdot d^2/a^2, \tilde{R}_n \approx \tilde{R}_n^{(1)} \cdot d/a \quad (4)$$

It is interesting that, in this situation, the reflected mode with the number i is excited much weakly than the modes with other numbers, because the coefficients \tilde{R}_i has the second order of smallness.

NUMERICAL RESULTS

To analyze the fields’ components in the general case, a numerical algorithm is developed. The problem is reduced to the solution of an infinite number of linear algebraic equations. We use the method of successive approximations: at first we set system which has some finite size and further we increase system’s size by a unit at every step till relative difference of obtained results for main excited mode is much than some given value (usually we take 1%).

For the case of the mode falling from the vacuum part of waveguide into the dielectric one, the behaviour of the absolute values of the reflection R_n and transmission T_n coefficients is presented in Fig. 2. One can see that the behaviour of the reflection and transmission coefficients is nonmonotonic, they have several local extremums.

The graphics in Fig. 2 present the interaction of the 3^d evanescent incident mode with the boundary. Note that bold lines in figures correspond to the propagating modes, and narrow lines correspond to the evanescent modes. One can see that such mode can excite propagating modes in reflected and transmitted field. In reflected field the 1st mode is propagating but has very small amplitude. In transmitted field the 1st, 2nd and 3^d modes are propagating at certain values of the channel radius. Note that for the relatively narrow channel radius, the transmission coefficient of the 3^d propagating mode is not small.

Figure 3 shows the behaviour of the absolute values of reflection and transmission coefficients in the case when the mode falls from the dielectric part of waveguide into the vacuum one. In this case the incident mode at the same frequency can be both propagating and evanescent depending on the channel radius. One can see that the interaction of evanescent mode with the boundary can lead to the excitation of certain propagating modes as well as earlier. One can show that behaviour of coefficients in the cases of a narrow channel and a thin dielectric layer provides a good agreement with the results obtained from approximate formulas.

CONCLUSION

We have considered the mode transformation in a waveguide with a transversal boundary between a vacuum part and the part with a cylindrical dielectric layer and a vacuum channel. Analysis has been carried out by crosslinking method, when the reflected and transmitted fields are presented as a series of eigenmodes. The problem has been reduced to solving certain system of infinite number of linear algebraic equations. Approximate solution for the mode excitation coefficients has been obtained in two particular cases: for a narrow channel and a thin dielectric layer. Numerical algorithm for calculating the mode transformation has been built based on analytical investigation. Typical dependences of the reflection and transmission coefficients on the channel radius have been presented.

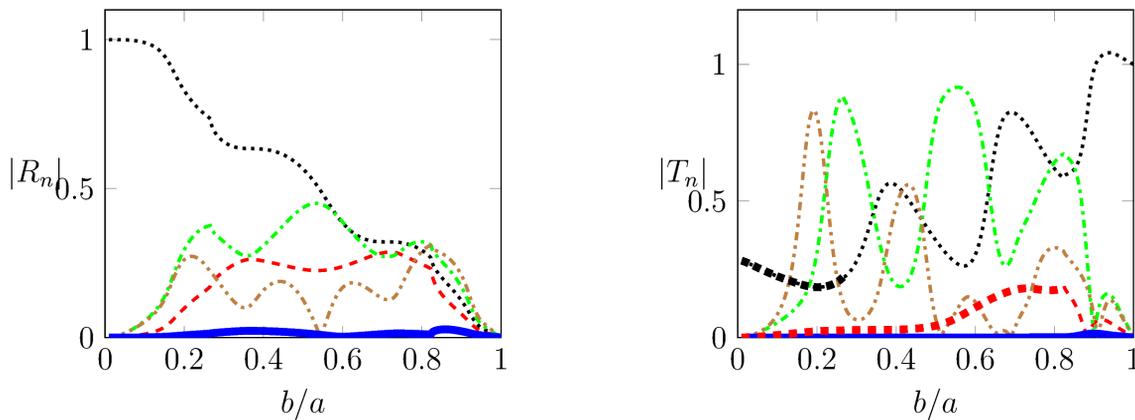


Figure 2: Dependences of absolute values of reflection coefficients $|R_n|$ (left) and transmission coefficient $|T_n|$ (right) on the relative channel radius b/a in the case of the mode falling from empty part of waveguide. $\epsilon_d = 7$, the 3^d incident evanescent mode, $c/\omega = 0.25$ cm. Bold line – the propagating modes, narrow line – the evanescent modes. Solid blue line - the 1st mode, dashed red line - the 2nd mode, dotted black line - the 3^d mode, green dash-dotted line - the 4th mode, brown double-dotted line - the 5th mode.

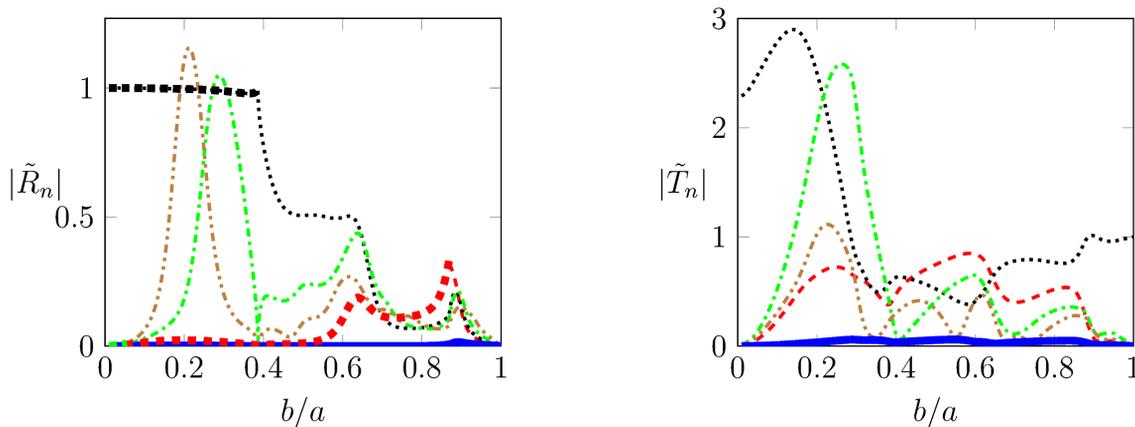


Figure 3: The same as in Fig. 2 for the case of the mode falling from the dielectric part; $\epsilon_d = 7$, $a = 1$ cm, the 3^d incident mode, $c/\omega = 0.22$ cm.

REFERENCES

- [1] G. Andonian, O. Williams, X. Wei, P. Niknejadi, E. Hemming, J. B. Rosenzweig, P. Muggli, M. Babzien, M. Fedurin, K. Kusche, R. Malone, and V. Yakimenko, "Resonant excitation of coherent Cherenkov radiation in dielectric lined waveguides", *Appl. Phys. Lett.*, v.98, 202901 (2011).
- [2] S. Antipov, C. Jing, A. Kanareykin, J. E. Butler, V. Yakimenko, M. Fedurin, K. Kusche, and W. Gai, "Experimental demonstration of wakefield effects in a THz planar diamond accelerating structure", *Appl. Phys. Lett.*, v.100, 132910 (2012).
- [3] S. N. Galyamin, A. V. Tyukhtin, S. Antipov, and S. S. Baturin, "Terahertz radiation from an ultra-relativistic charge exiting the open end of a waveguide with a dielectric layer", *Optics Express*, v.22, p.8902-8907 (2014).
- [4] T.Yu. Alekhina and A.V. Tyukhtin, "Cherenkov-transition radiation in a waveguide with a dielectric-vacuum boundary", *Phys. Rev. STAB*, v.15, 091302 (2012).
- [5] T.Yu. Alekhina and A.V. Tyukhtin, "Self-acceleration of a charge intersecting a boundary surface in a waveguide", *Phys. Rev. STAB*, v.16, 081301 (2013).
- [6] J. G. Power, M. E. Conde, W. Gai, R. Konecny, P. Schoessow, and A. D. Kanareykin, "Measurements of the longitudinal wakefields in a multimode, dielectric wakefield accelerator driven by a train of electron bunches", *Phys. Rev. STAB*, 3, 101302 (2000).
- [7] C. Jing, A. Kanareykin, J. G. Power, M. Conde, W. Liu, S. Antipov, P. Schoessow, and W. Gai, "Experimental demonstration of wakefield acceleration in a tunable dielectric loaded accelerating structure", *Phys. Rev. Lett.*, v.106, 164802 (2011).
- [8] V.V. Poliektov, A. A. Vetrov, K.A. Trukhanov, and V. I. Shvedunov, "Determining the energy spectrum of a charged particle beam using Cherenkov radiation in the deceleration system", *Instruments and Experimental Techniques*, v.51, p.191-199 (2008).
- [9] A.V. Tyukhtin, "Determination of the particle energy in a waveguide with a thin dielectric layer", *Phys. Rev. STAB*, v.15, 102801 (2012).