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IONIZATION COOLING CHANNELS IN COSY INFINITY

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Abstract

Ionization cooling is a method to reduce the emittance of a beam through the use of absorbers, rf cavities, and strong solenoids for focusing, arranged into a condensed lattice. By tuning lattice parameters, it is possible to construct a staged cooling channel in which the beam emittance is always considerably greater than the minimum value. In the late stages of the cooling channel, space charge effects can become a significant obstacle to further emittance reduction once the beam becomes sufficiently condensed. A method has been implemented in COSY Infinity, a beam dynamics simulation and analysis code, which efficiently and accurately calculates the self-fields of all particles on each other based on a variant of the Fast Multipole Method (FMM). In this paper, we present simulations of a muon ionization cooling channel performed in COSY, utilizing the FMM, benchmarked against G4beamline, a standard code for muon beam analysis, in order to investigate the significance of space charge effects.

OVERVIEW OF IONIZATION COOLING

In the area of high energy colliders, using leptons has a significant advantage over hadrons due to hadron collisions being inefficient and complicated by secondary quark interactions. A muon collider could be used for high energy studies of lepton collisions without the limitations on energy due to synchrotron radiation. The muon beam is produced by sending protons through a target, producing pions which in turn decay into muons with a large momentum spread. In order to be accelerated, the six-dimensional (6D) phase space volume of the muon beam must be reduced for injection into a storage ring. Ionization cooling is currently the only feasible method for cooling the beam within a muon lifetime of 2.2 μs. In order for a full 6D ionization cooling experiment to be constructed, a baseline lattice design has to be studied and selected based on detailed simulations [1].

Emittance Reduction

For the reduction of transverse emittance, the beam is strongly focused with magnetic fields and subsequently passed through an absorber material to reduce overall momentum. The beam's longitudinal momentum is restored in RF cavities which results in a net reduction in transverse emittance. The change in the transverse emittance is described by:

$$\frac{d\varepsilon_n}{ds} = -\frac{1}{\beta^2}\frac{dE_\mu}{ds}\frac{\varepsilon_n}{E_\mu} + \frac{1}{\beta^3}\frac{\beta_T E_s^2}{2E_\mu m_\mu c^2 L_R},$$

$$\varepsilon_{n,min} = \frac{\beta_T E_s^2}{2\beta m_\mu c^2 L_R \left|\frac{dE_\mu}{ds}\right|},$$

where ε_n is the normalized transverse emittance, c is the speed of light, and $\beta = v/c$, E_μ, m_μ are the muon velocity, energy, and mass. The transverse betatron function within the absorber is β_T, dE_μ/ds is the energy loss per unit length in the absorber, L_R is the radiation length of the absorber material, and E_s is the characteristic scattering energy. The top equation describes the effects of cooling (first term) and heating (second term), and when the cooling rate equals the heating rate, we can find the minimum normalized transverse (or equilibrium) emittance ($\varepsilon_{n,min}$) for a given absorber material and focusing field.

To maximize cooling, materials with low Z and large radiation length (such as LH_2, LiH) are used in order to minimize multiple scattering. A high gradient RF field and high absorber density, the combination of which determines the energy loss per unit length dE_μ/ds, are used along with solenoids arranged such that there is strong focusing at absorbers, thus small β_T. Cooling typically occurs around a muon momentum of 200 MeV/c, where the curvature of the Bethe-Bloch function (to determine absorber dE_μ/ds) is favorable. The cooling channel is tapered into stages in order to vary the solenoid focusing, RF frequency and gradient, and geometry such that the minimum normalized emittance decreases with the beam [2–4].

The longitudinal momentum spread is reduced using the concept of emittance exchange. This process involves applying a magnetic field to create dispersion in the beam at wedge shaped absorbers, such that particles with higher energy pass through more material than those with lower energy, resulting in an overall reduction in the energy spread of the beam. The cooling process is essentially a transfer of emittance from the longitudinal to transverse direction combined with transverse ionization cooling for emittance reduction in all six dimensions.

SPACE CHARGE

One of the challenges presented in muon cooing channels is that as the size of the beam is reduced, Coulomb repulsion in the beam limits further emittance reduction. To investigate the effect of space charge, a method has been implemented in COSY INFINITY [5] to achieve efficient and accurate calculation of the interparticle Coulomb forces based on variants of the Fast Multipole Method (FMM). This method divides an arbitrary charge distribution into small boxes with a hierarchical structure. It then computes the multipole expansions and local expansions of charges far from the observer to achieve a computational efficiency that scales with the number of particles, N, and computational errors scaling with a high power of the expansion order. The FMM algorithm is especially suited for beam dynamics sim-

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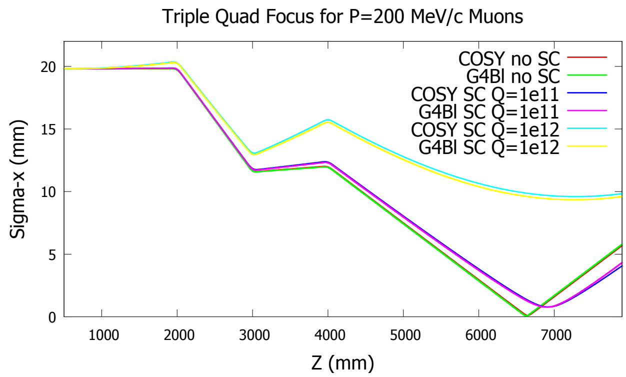

Figure 1: This plot compares the simulated deviation in x of a particle bunch through three quadrupole magnets tuned to focus the muons to a point. For varying amounts of total charge, COSY and G4Beamline are in good agreement.

ulations because of the efficiency and low computational error compared to other space charge algorithms.

Other methods, such as the particle-particle interaction (PPI) and the particle in cell (PIC), while computationally efficient, incur excess error due to modeling and statistics. The PPI method uses macroparticles and assumes a particular distribution. The PIC method places the charge distribution onto a mesh, solves the Poisson equation on mesh points and interpolates between mesh points to find the field on each particle. Both of these methods suffer from an inability to precisely handle complicated charge distributions. This difficulty is overcome in the FMM by decomposing the charge distribution into boxes according to the charge density such that there are a pre-specified number of particles in each box to efficiently and accurately compute the multipole expansions [6, 7].

In order to benchmark the effect of space charge, we simulated a bunch of 5000 muons through a triplet quadrupole magnetic focus in G4Beamline and COSY, with varying total charge. G4Beamline is a one of the standard codes for simulating muon beams and provides a particle in cell (PIC) space charge algorithm [8]. The quadrupoles, placed at 2, 3, and 4 m were tuned such that a focus occurs at approximately 6.5 m. The beam was initially parallel with a gaussian spatial distribution in x, y and z, with an initial momentum of 200 MeV/c. Figure 1 shows the standard deviation of the beam in the x direction as a function of position along the channel.

As the total charge in the beam increases, the effect of space charge repulsion becomes drastically more obvious, such that a total charge of 10^{12} results in a very wide focus. The minute variations between G4Beamline and COSY are the result of differences in the space charge algorithms as well as the methods for calculating the beam statistics. In G4Beamline, the beam sigmas at z are calculated from the positions when each particle is at that specific z, while in COSY the calculation is performed as a function of the average z position of the bunch. Despite this difference, the codes show good agreement over a range of total charge.

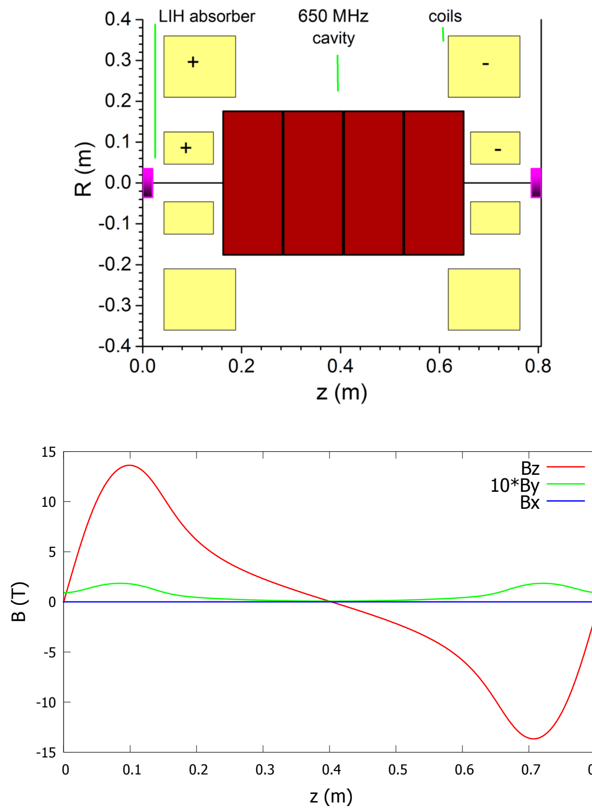

Figure 2: Diagram (top) and magnetic field in the x, y, and z directions (bottom) of a late-stage cooling cell.

COOLING CHANNEL SIMULATIONS

In COSY, we use an 8th order Runge-Kutta integrator with automatic step size control. This integrator automatically selects a time step size based on a prespecified error bound. This allows selection of large initial timesteps, as the integrator reduces the stepping at difficult areas in the cooling cell which typically occur at the boundaries of elements. However, tracking particles through elements with the boundaries behaving as step functions is unnecessarily difficult for the integrator, resulting in very small timestepping at boundaries in order to reach the desired error bound. To improve the efficiency of the integrator, it is advantageous to continuously vary the density of the elements. This is done by applying Enge functions, which have the property of being zero for positions outside and unity for positions inside the element while varying smoothly at the boundary [9].

Simulation Comparison

We have performed simulations of the late stages of a proposed cooling channel design in both COSY and G4Beamline, with and without space charge effects. Stochastic effects and particle decays have yet to be implemented in COSY, so this functionality has been disabled to ensure a clear comparison between the codes.

One cell of this late-stage cooling channel consists of four 26 MV/m, 650 MHz RF pillboxes, two 120 degree Lithium Hydride (LiH) wedge absorbers, and a solenoid arrangement

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producing a maximum on-axis field of 12.9 T. The solenoids have opposing currents and a small tilt to create dispersion in the x direction at the absorbers. Figure 2 contains graphics of the cell geometry and magnetic field. The simulations were run using a bunch of 10^4 muons and Fig. 3 depicts the transverse, longitudinal, 6D emittance, and transmission over 60 cells. For the purposes of space charge, the bunch was given a total charge of 10^{12}. The emittance is calculated at the end of each cell, and as in the case of the triple quadrupole focus, there are differences in calculation between the two codes. G4Beamline records the positions of the particles at the set z positions at the end of each cell, while in COSY this is performed when the average z position of the beam is at the end of each cell. In addition, COSY tends to lose the more extreme particles longitudinally, leading to a lower overall longitudinal emittance.

The 6D emittance is reduced by an order of magnitude over this stage of cooling, showing promising emittance reduction if combined with previous stages. The behavior of the emittance and transmission shows good agreement between the two codes, and there is a small, but noticeable space charge effect causing greater particle loss further down the channel.

In summary, we explained the specifics of muon beam cooling, discussed the methods of cooling channel simulations, and presented results of the simulation. Our future plans involve incorporating an entire cooling channel in COSY while correcting the differences in early transverse emittance behavior, and later longitunal behavior, and further optimizing the efficiency of COSY simulations for time domain particle tracking.

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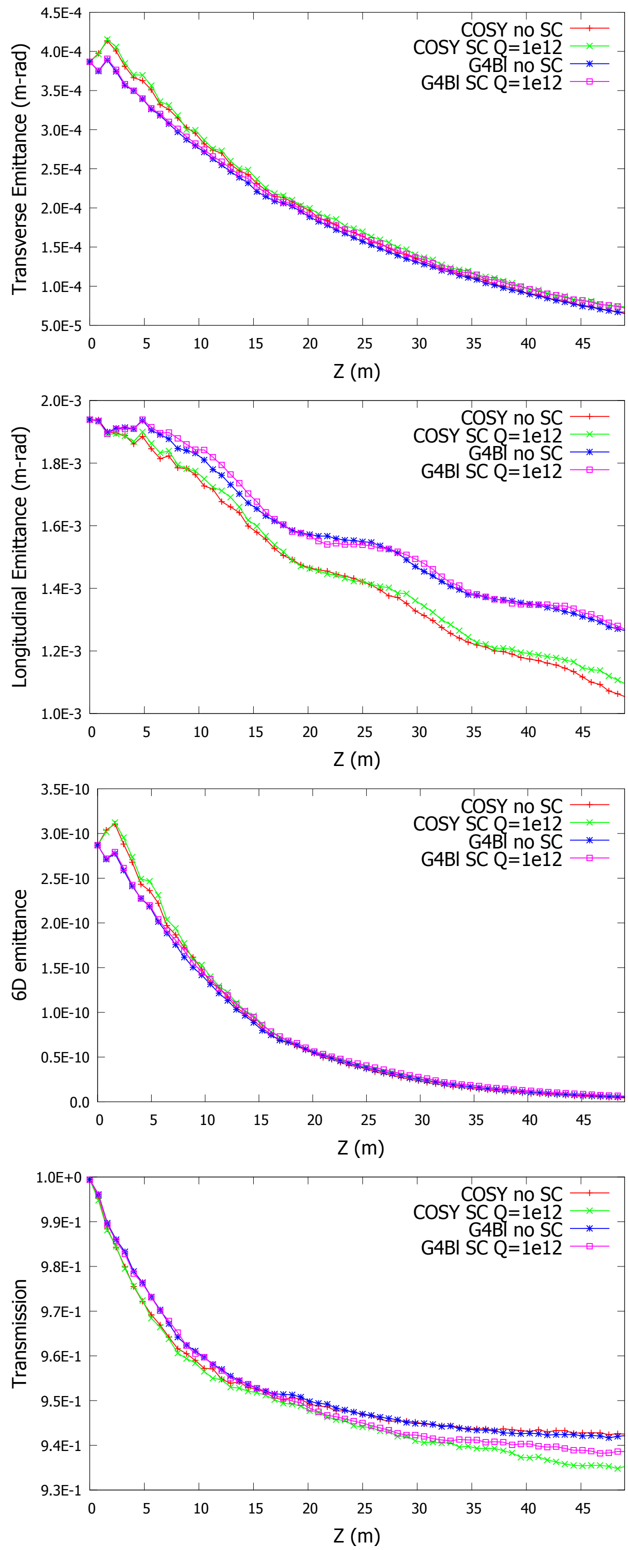

Figure 3: In order from the top, the transverse, longitudinal, 6D emittance, and transmission behavior with and without space charge through 60 cells as simulated in both codes.