Recent Trends in Beam Size Measurement Using the Spatial Coherence of Visible Synchrotron Radiation

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Agenda of talk

1. Simple imaging system for beam profile measurement.

2. Profile and size measurement via spatial coherence

3. SR interferometer

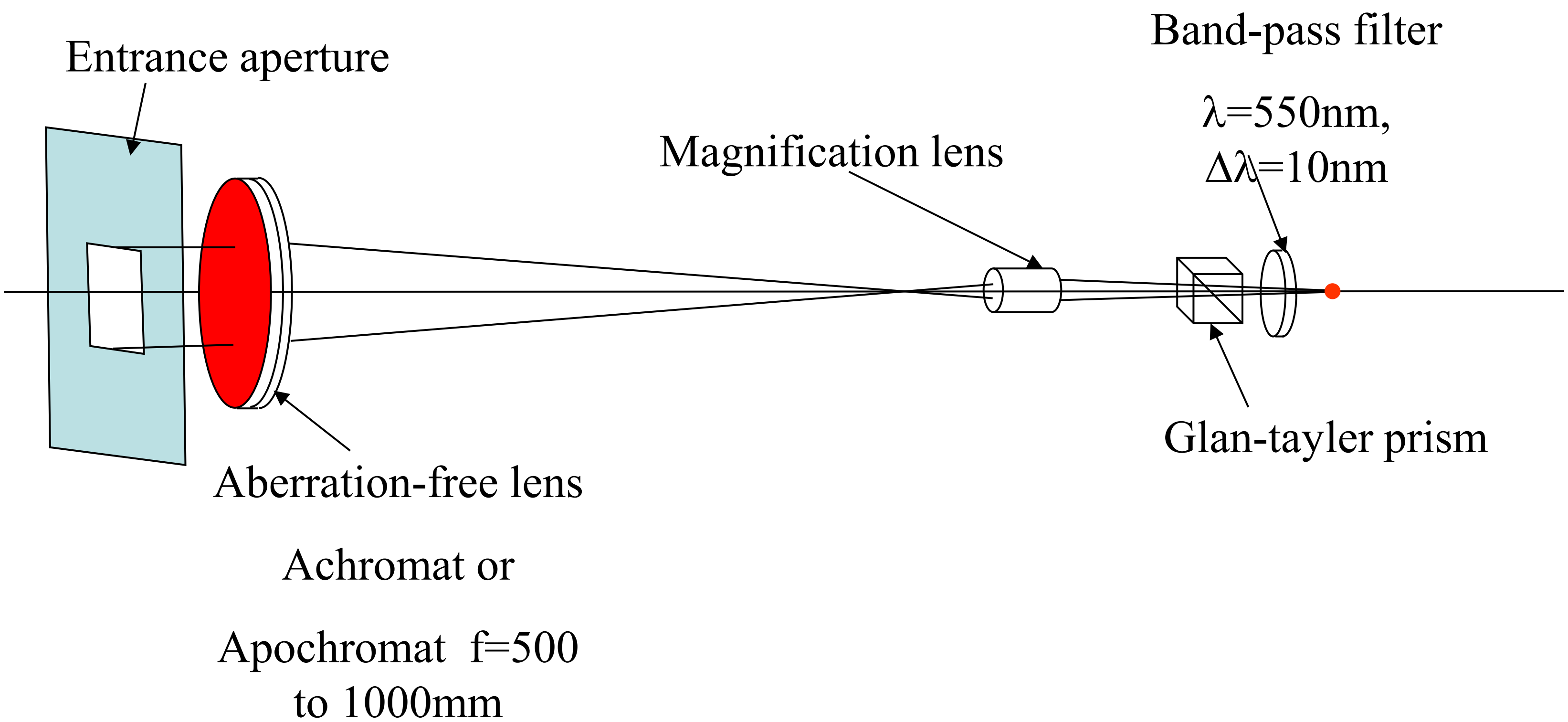
4. Theoretical resolution and error transfer

5. Measurement of vertical small beam size

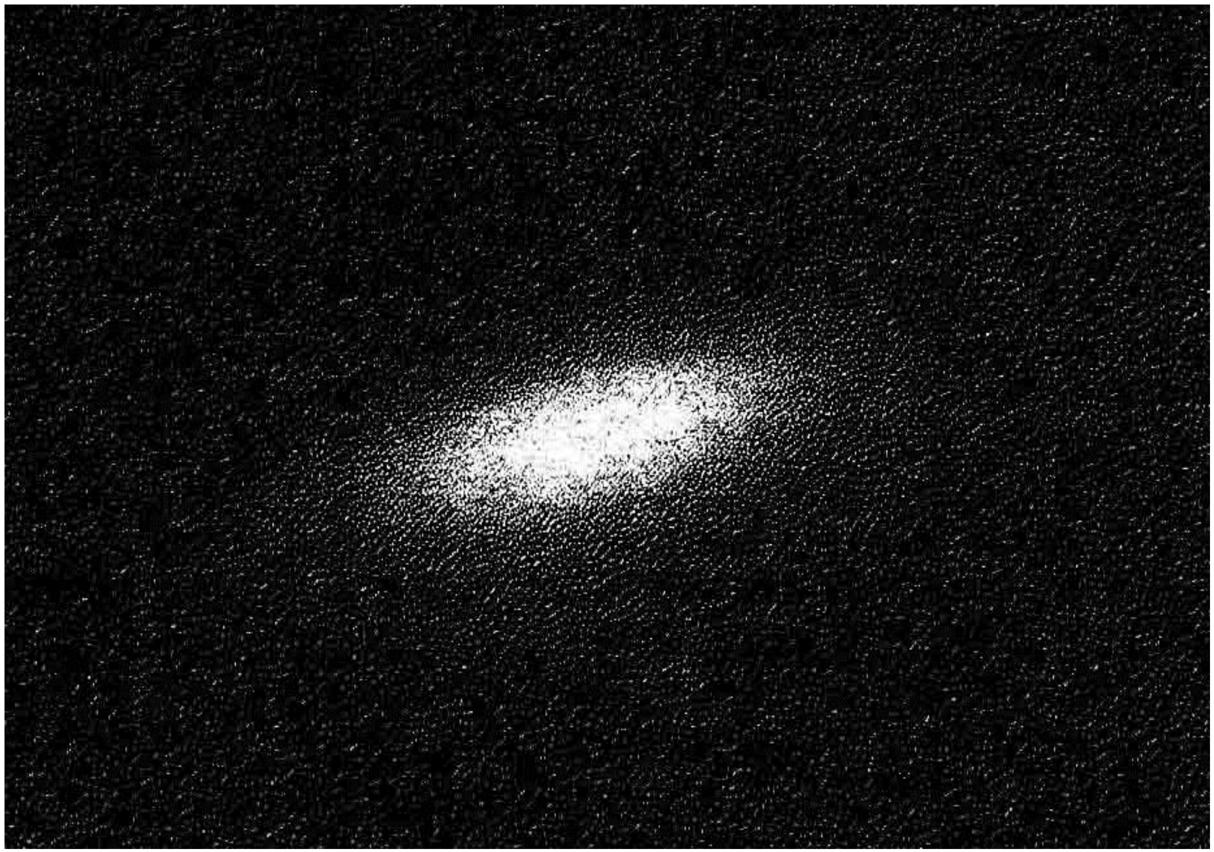
6. **Measurement of horizontal beam size**
   The incoherent depth of field effect (IDOF)

7. **Application of interferometry for the beam size measurement in the LHC**

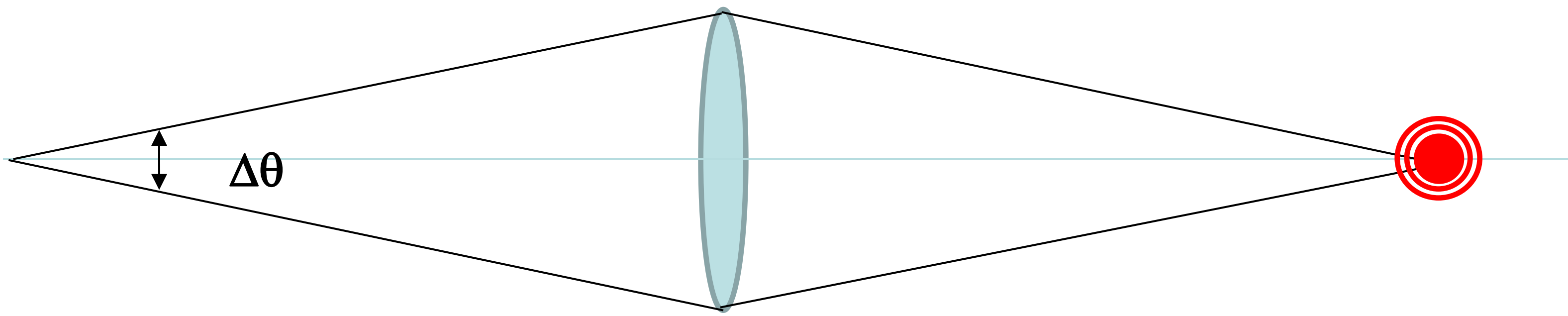
1. Simple imaging system for beam profile measurement.



Example of image of the beam



Uncertainty principal in imaging.



$\Delta\theta/\lambda \cdot \Delta x \geq 1$

So, large opening of light will necessary to obtain a good spatial resolution.

For example, λ=400nm and opening of 7mrad, $\Delta x \approx 57\mu m$ for 1:1 image.

Deconvolution with MEM method by using the Wiener inverse filter

Fourier transform of blurred image G(u,v) in spatial frequency domain (u,v) is given by,

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

where H(u,v) is thought as a inverse filter (Fourier transform of PSF), F(u,v) is a Fourier transform of geometric image, and N(u,v) is a Fourier transform of noise in the image). The Wiener inverse filter H_w is given by,

$$H_w(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + \dfrac{\phi_n(u,v)}{\phi_f(u,v)}}$$

where asterisk indicates the complex conjugate of H, ϕ_n is a power spectra of the noise, and ϕ_f is a power spectra of the signal.

Original image

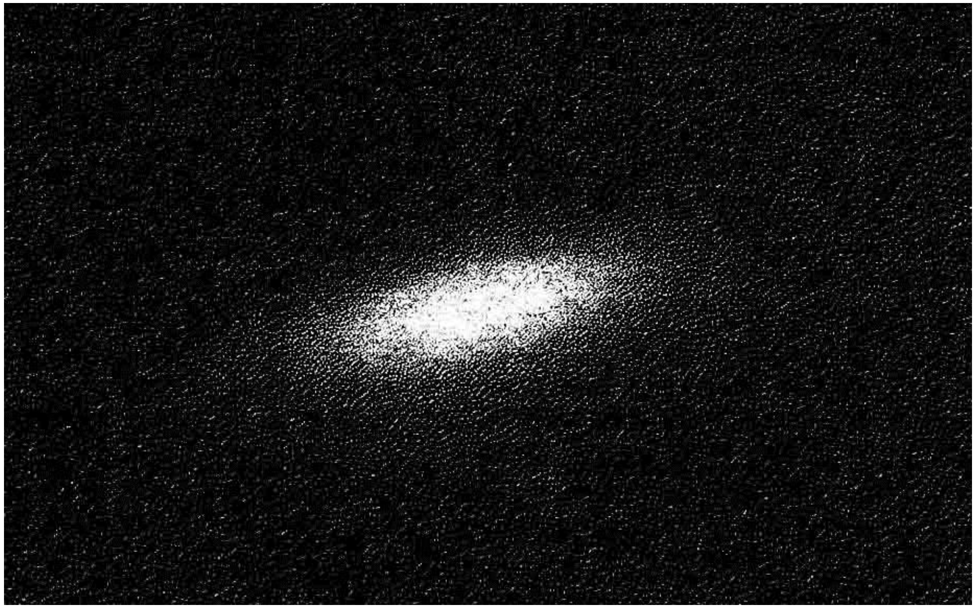
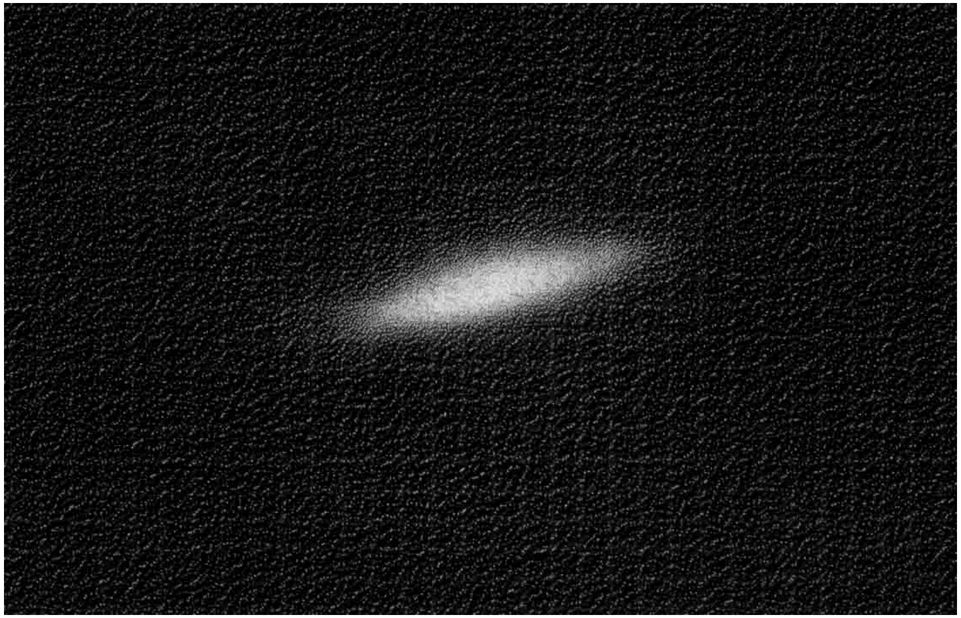


Image after deconvolution



2. Profile and size measurement via spatial coherence

To measure a size of object by means of spatial coherence of light (interferometry) was first proposed by H. Fizeau in 1868!

This method was realized by A.A. Michelson as the measurement of apparent diameter of star with his stellar interferometer in 1921.

This principle was now known as " Van Cittert-Zernike theorem" because of their works;

1934 Van Cittert

1938 Zernike.

Spatial coherence and profile of the object
Van Cittert-Zernike theorem

According to van Cittert-Zernike theorem, with the condition of light is 1st order temporal incoherent (no phase correlation), the complex degree of spatial coherence $\gamma(\upsilon_x,\upsilon_y)$ is given by the Fourier Transform of the spatial profile f(x,y) of the object (beam) at longer wavelengths such as visible light.

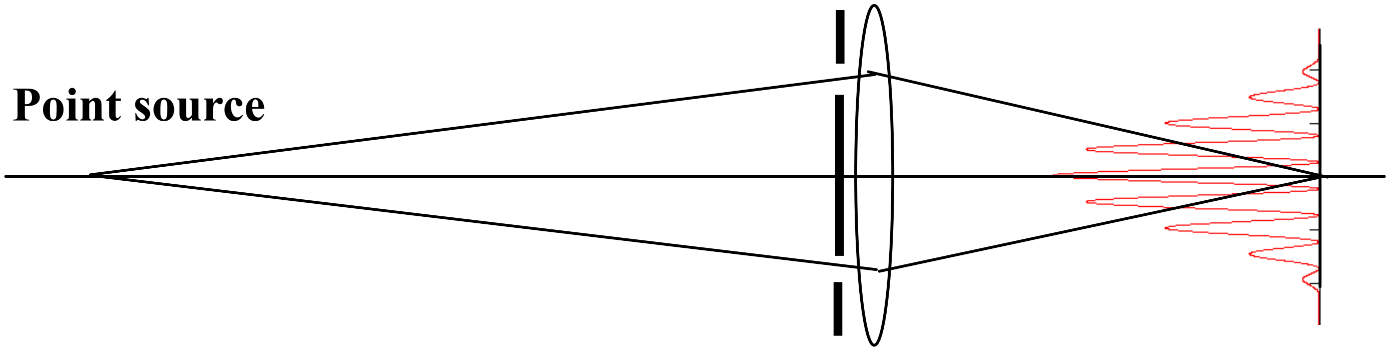
$$\gamma(\upsilon_x,\upsilon_y)=\iint f(x,y)\exp\{-i\cdot 2\cdot\pi(\upsilon_x\cdot x+\upsilon_y\cdot y)\}dxdy$$

where υ_x,υ_y are spatial frequencies given by;

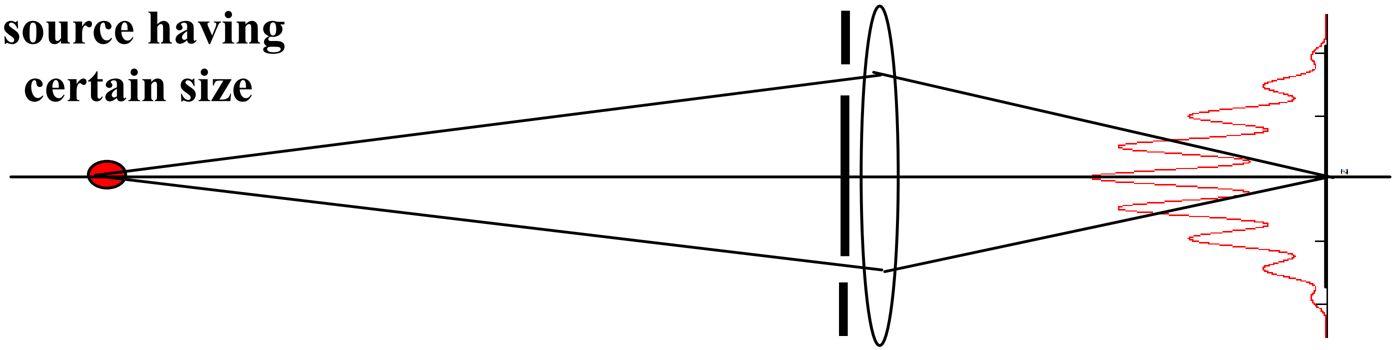
$$\upsilon_x=\frac{D_x}{\lambda\cdot R_0}\ ,\quad \upsilon_y=\frac{D_y}{\lambda\cdot R_0}$$

Simple understanding of van Cittert-Zernike theorem

Point source

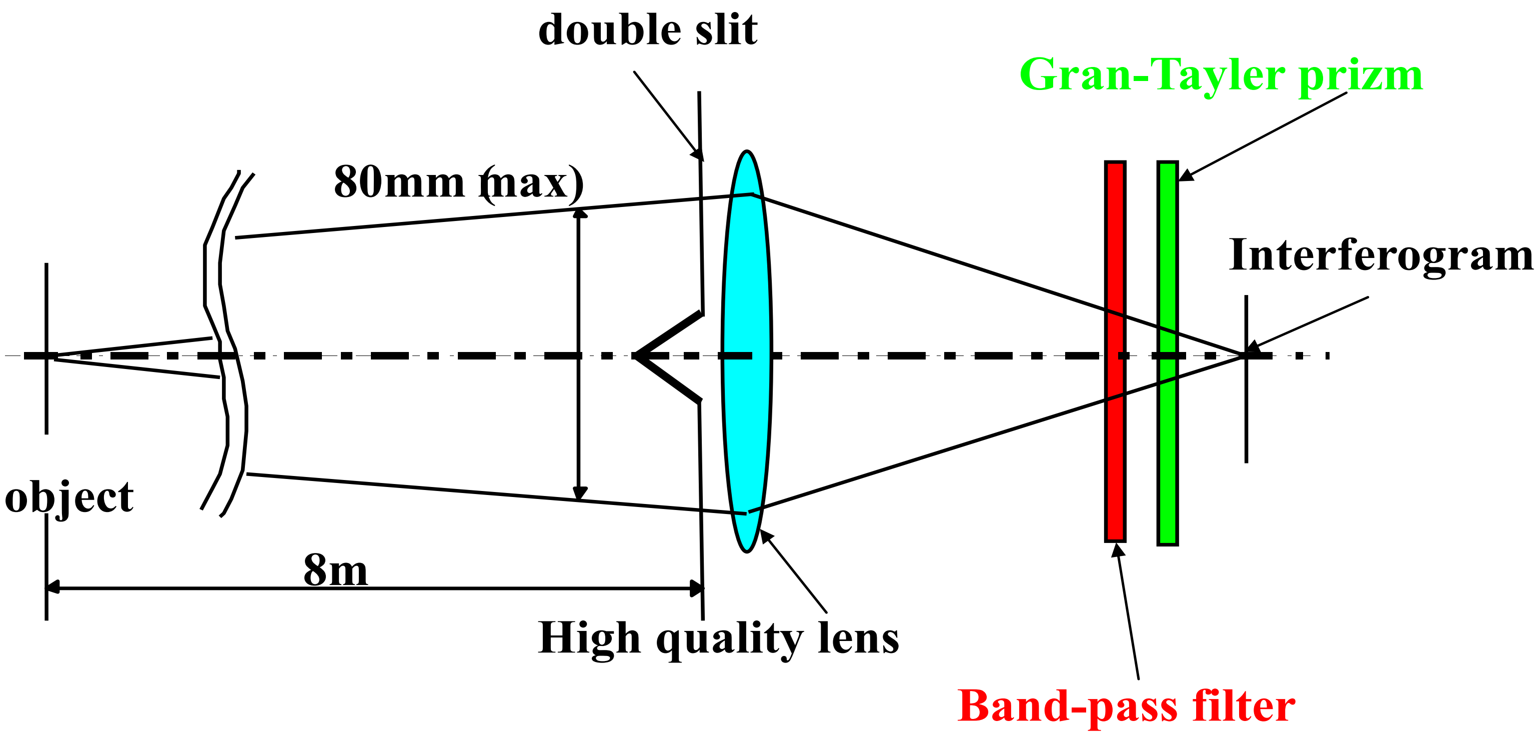


source having certain size



3. SR interferometer

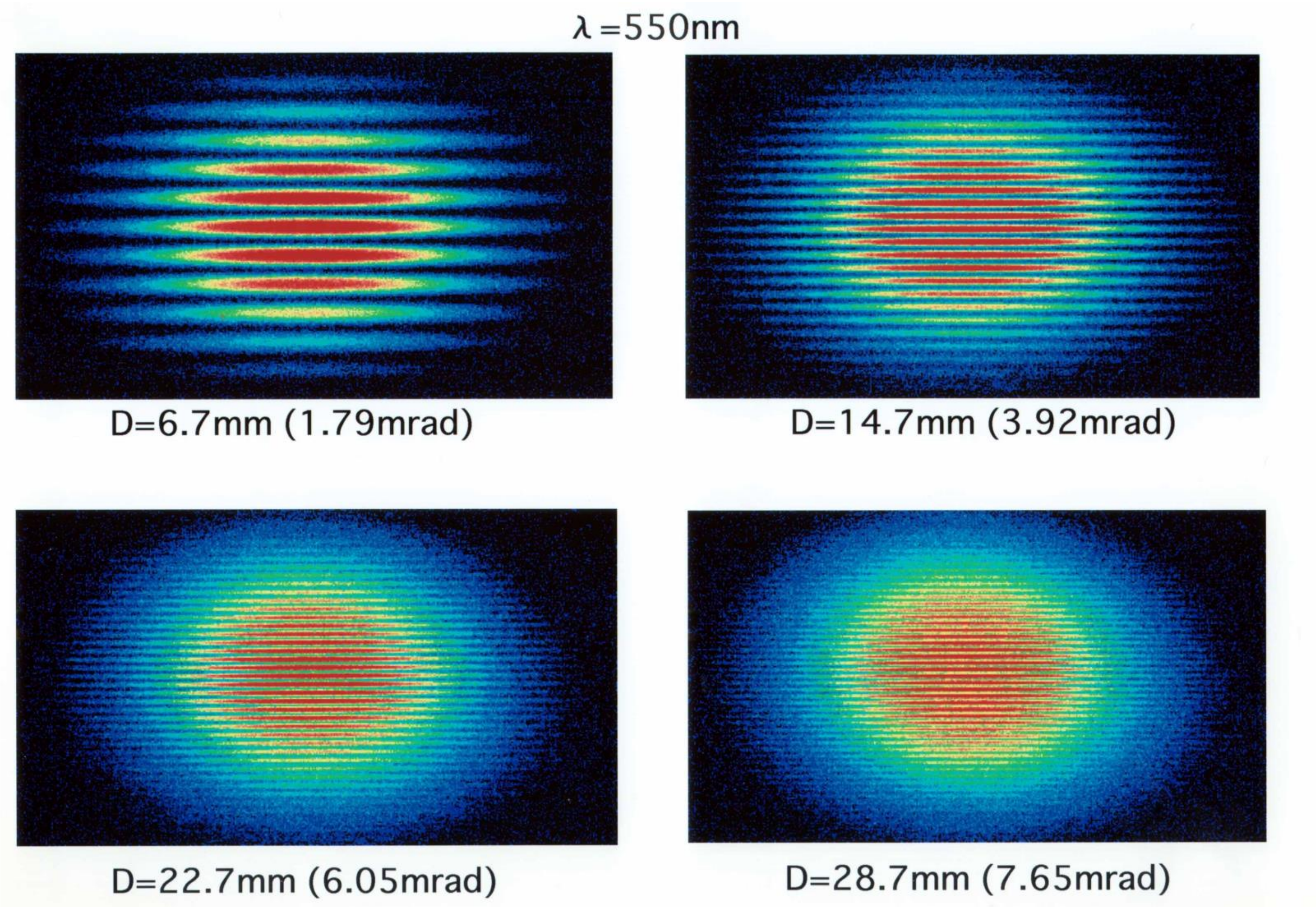
a wavefront-division type interferometer
using polarized, quasi-monochromatic rays



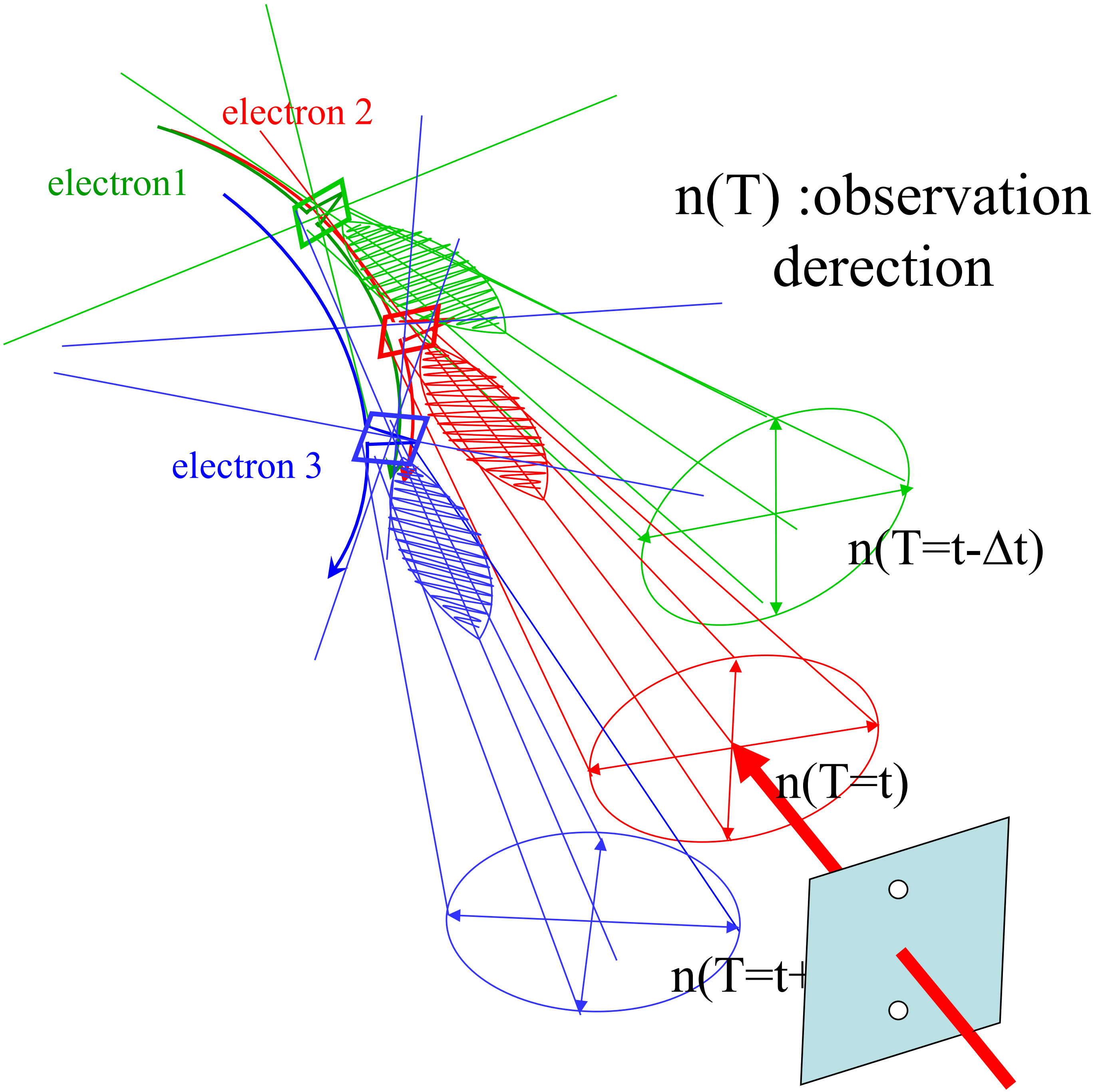
$$I(y,D)=\int (I_1+I_2)\cdot\left\{\sin c\left(\frac{\pi\cdot a\cdot y\cdot \chi(D)}{\lambda\cdot f}\right)\right\}^2\cdot\left\{1+\gamma\cdot\cos\left(k\cdot D\cdot\left(\frac{y}{f}+\psi\right)\right)\right\}d\lambda$$

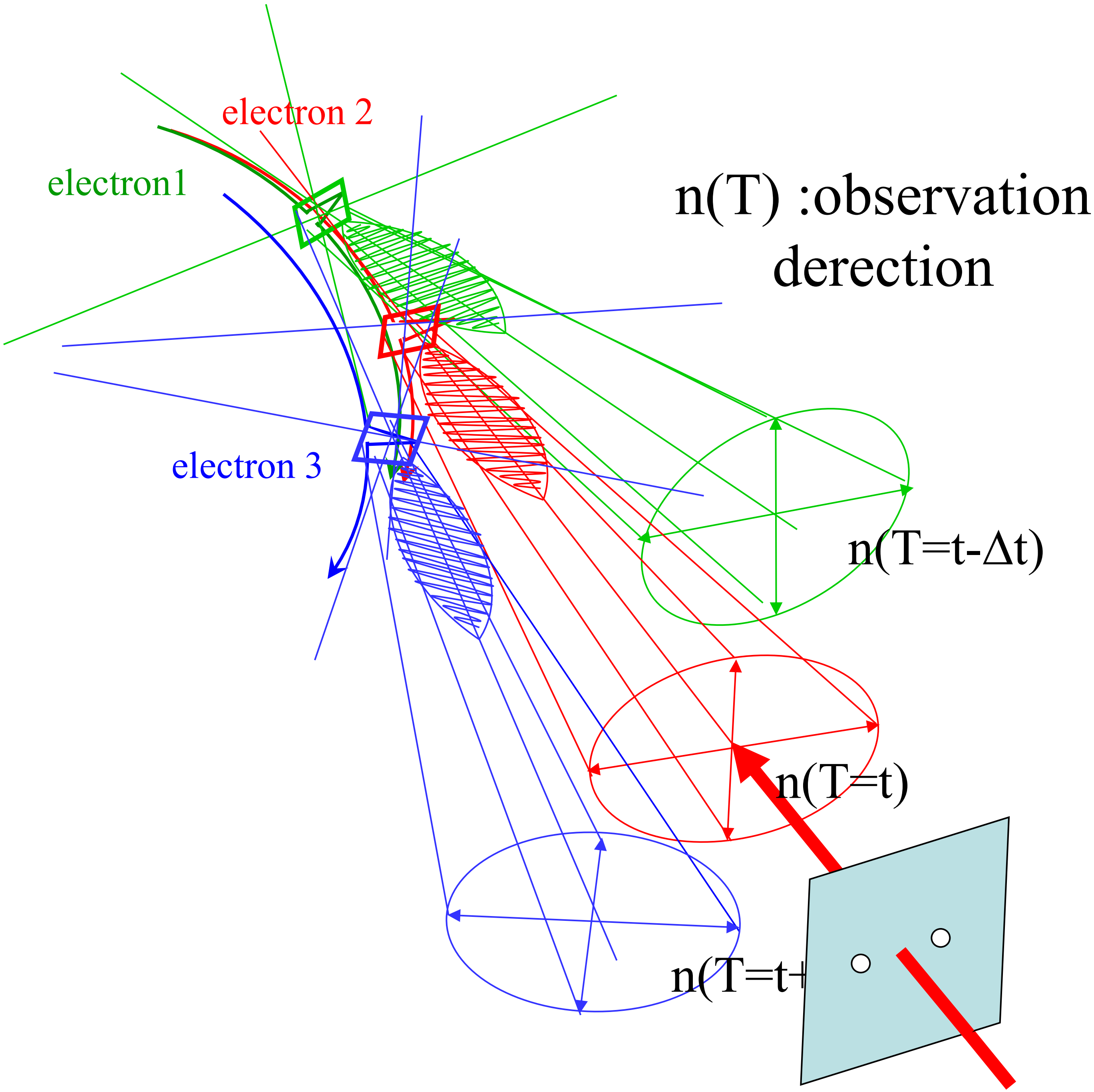
$$\gamma=\left(\frac{2\cdot\sqrt{I_1\cdot I_2}}{I_1+I_2}\right)\cdot\left(\frac{I_{max}-I_{min}}{I_{max}+I_{min}}\right)$$

Typical interferogram in vertical direction



Effect of incoherent depth of field of (IDOF) from independent electrons


electron 2
electron1
n(T) :observation
derection
electron 3
n(T=t-Δt)
n(T=t)
n(T=t+


electron 2
electron1
n(T) :observation
derection
electron 3
n(T=t-Δt)
n(T=t)
n(T=t+

IDOF has no effect for vertical beam size measurement when the beam size is not strongly change along the orbit.

But in the horizontal direction, apparent beam profile is changed with IDOF, we need include this effect to horizontal beam size measurement.

When the incoherent depth of field < Airy disk length of the diffraction

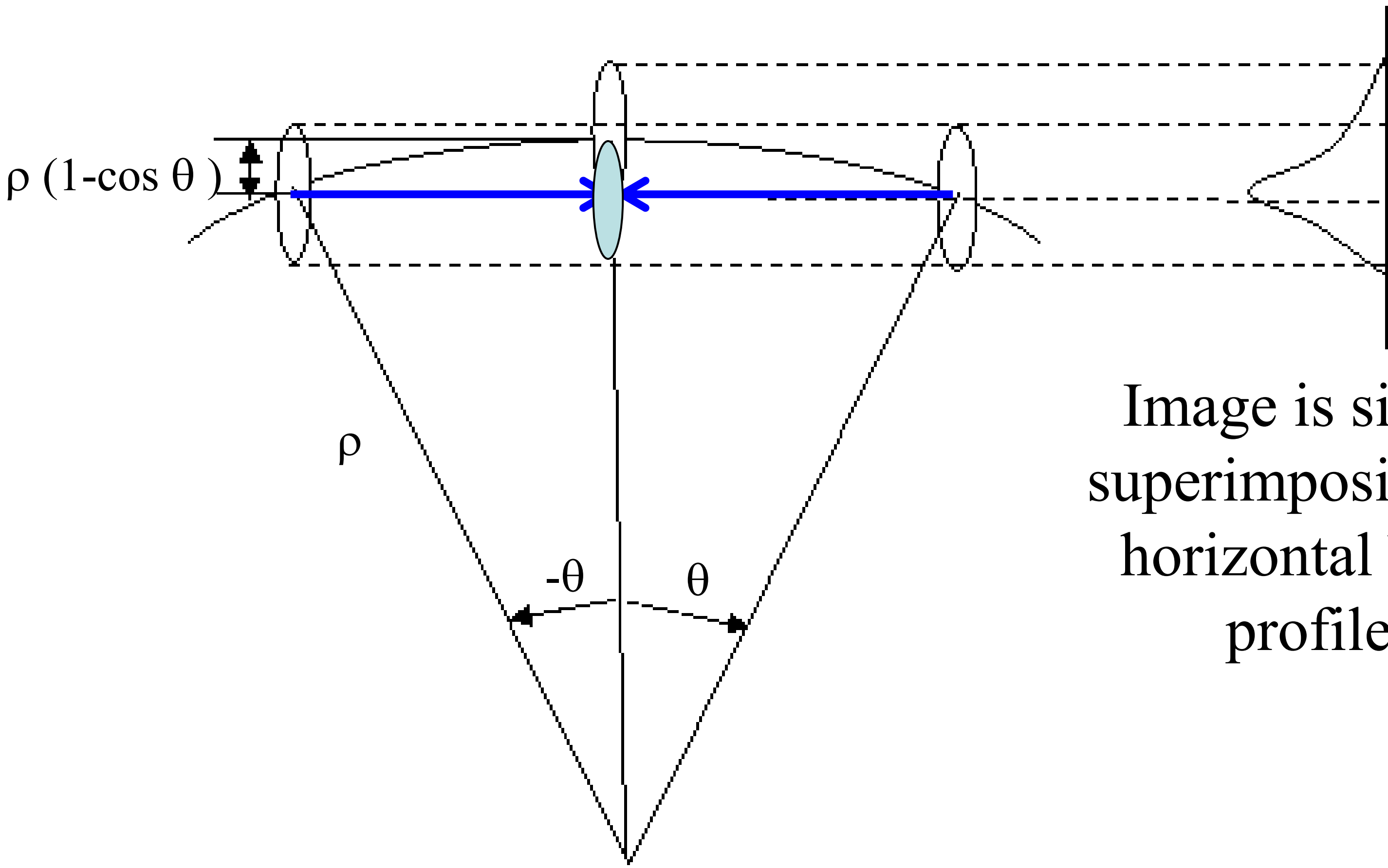


Image is simply superimposition of horizontal beam profiles

For example $\rho \approx 6m,\ \theta \approx 2.4mrad,\quad \rho(1-\cos\theta) \approx 17.3\mu m$

The apparent beam profile is given by ,

$$\int \frac{\exp\left[\dfrac{-\left[x-\rho\left\{1-\cos(\theta)\right\}\right]^2}{2\sigma^2}\right]}{\sigma\cdot\sqrt{2\pi}}\,d\theta$$

Where σ denotes beam size and term means displacement of center of Gaussian distribution represented by ρ and θ. θ is rotation angle of optical axis of pencil of light.

Then the real part of spatial coherence is given by Fourier cosine transform of this apparent distribution is given by ,

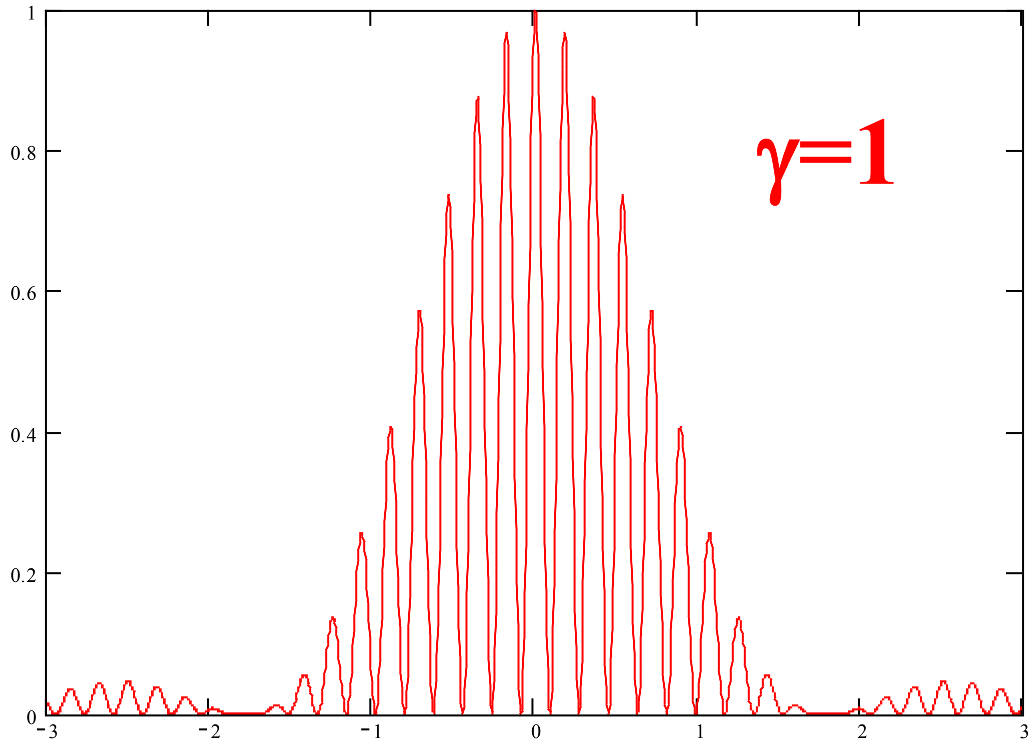
$$\gamma_h(D) = \iint \underbrace{\frac{2\sqrt{I\left(\theta+\frac{D}{2R}\right)I\left(\theta-\frac{D}{2R}\right)}}{I\left(\theta+\frac{D}{2R}\right)+I\left(\theta-\frac{D}{2R}\right)}\cdot I(\theta)}_{\text{Intensity Imbalance factor}} \cdot \frac{\exp\left[\frac{-\left[x-\rho\{1-\cos(\theta)\}\right]^2}{2\sigma^2}\right]}{\sigma\cdot\sqrt{2\pi}} \cdot \underbrace{\cos\left(\frac{2\pi\cdot D\cdot x}{R\cdot\lambda}\right)}_{\text{Fourier cosine transform}} d\theta\, dx$$

4. Theoretical resolution and error transfer

If the phase of light has no uncertainty ($\Delta\phi=0$), the interference fringe by point source is given by;

$$I(y,D)=(I_1+I_2)\left\{\sin c\left(\frac{\pi a y}{\lambda f}\right)\right\}^2\left\{1+\cos(kD\frac{y}{f}+\phi)\right\}$$

Interference fringe with no phase fluctuation



No theoretical uncertainty ⟶ infinite resolution!

According to quantum optics,

In the large number limit, uncertainty principle concerning to phase is given by

$$\Delta\phi \cdot \Delta N \geq 1/2$$

where ΔN is uncertainty of photon number.

Using the wavy aspect of photon in small number of photons, Forcibly ;

From uncertainty principal

$$\Delta\phi \cdot \Delta N \geq 1/2,$$

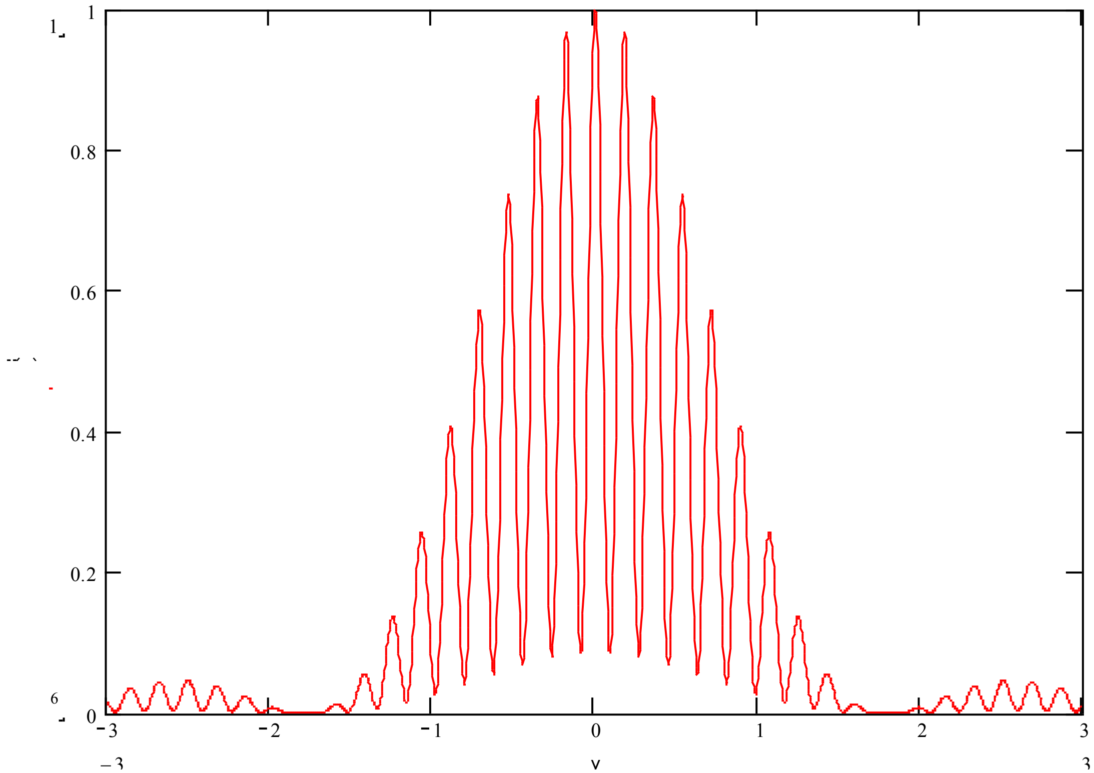
then,

$$\Delta\phi \geq 1/(2 \cdot \Delta N).$$

Even in the case of coherent mode, interference fringe will be smeared by the uncertainty of phase.

$$I(y, D) = (I_1 + I_2)\left\{\sin c\left(\frac{\pi a y}{\lambda f}\right)\right\}^2 \left\{1 + \int_{\Delta\phi} \cos(kD\frac{y}{f} + \phi)\xi(\phi)d\phi\right\}$$

Introducing some uncertainty ($\pi/2$)



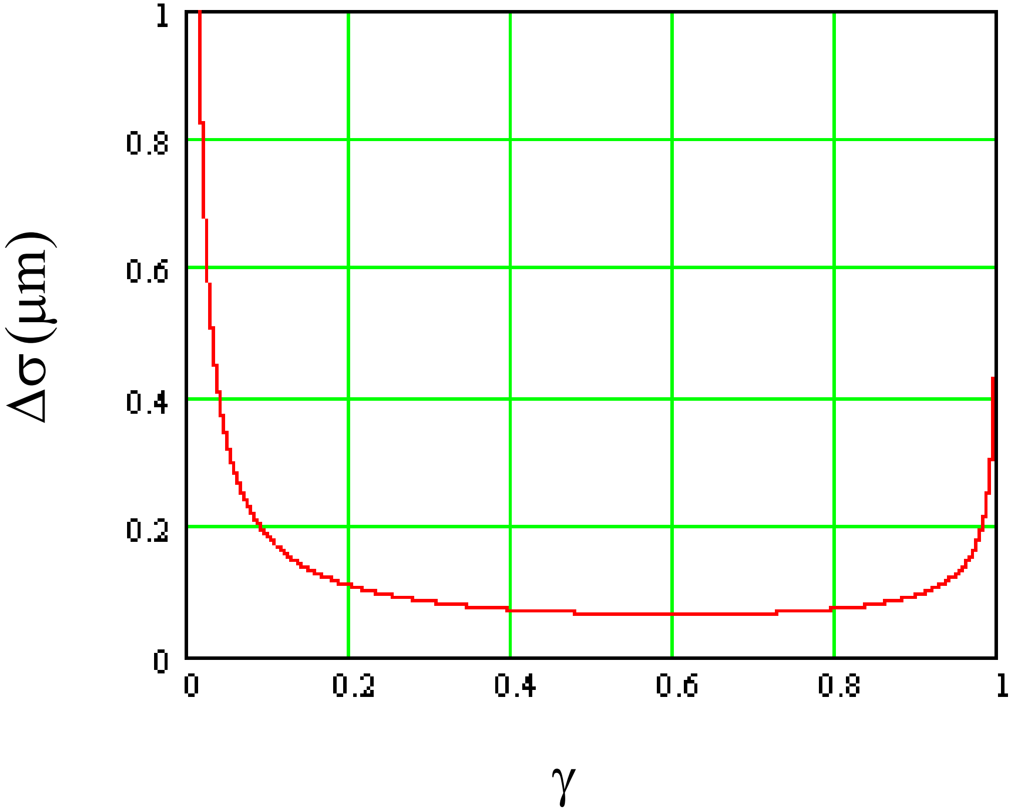
The visibility of interference fringe will reduced by uncertainty of phase under the small number of photons. **But actually, under the small number of photons, photons are more particle like, and difficult to see wave-phenomena.**

Actually, we can have sufficient photons for an interferogram, and ***<u>theoretical limit due to the phase uncertainty is negligible small</u>***.

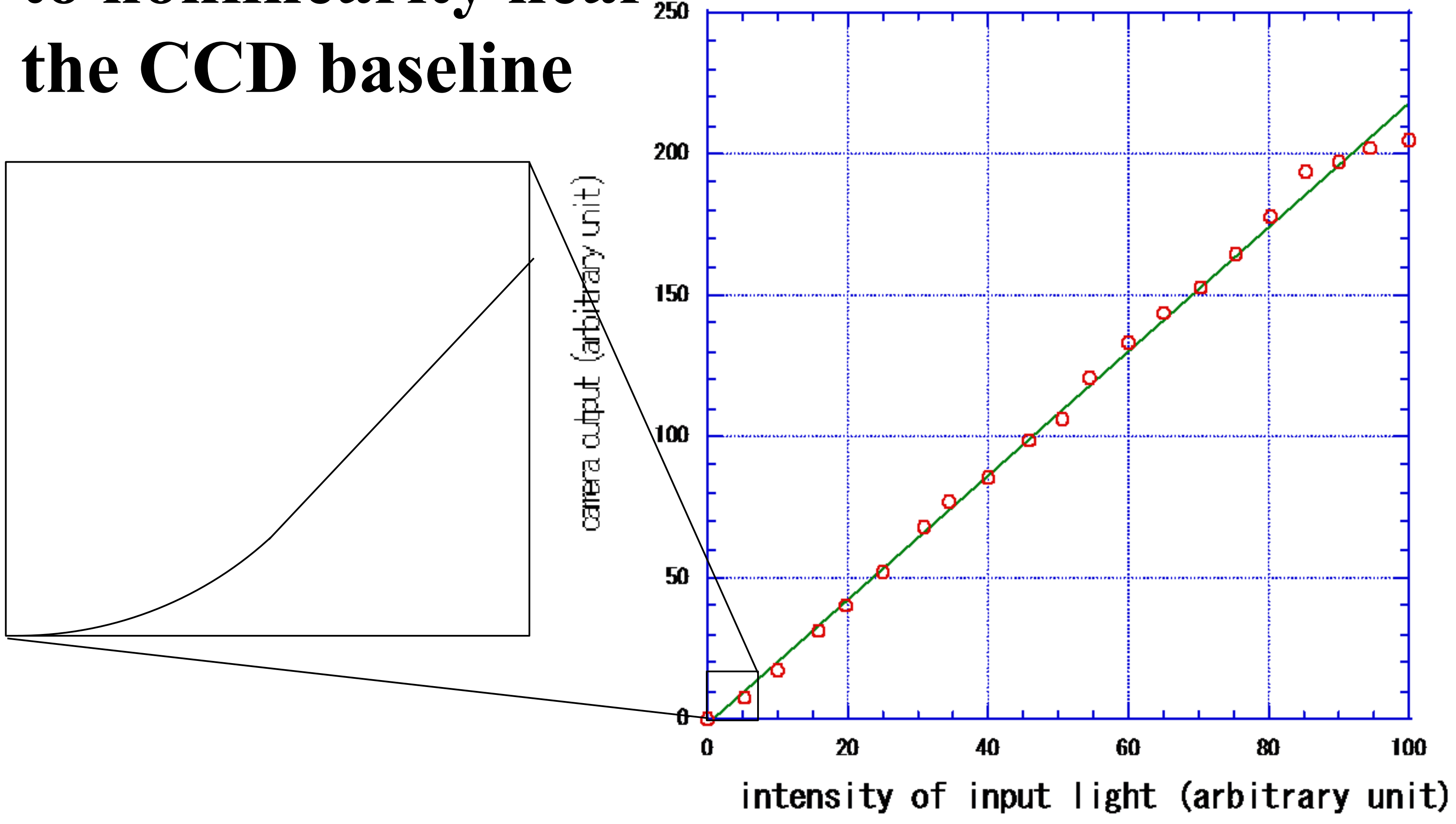
In actual optical component, with $\lambda/10$, this error corresponds to 0.26μm

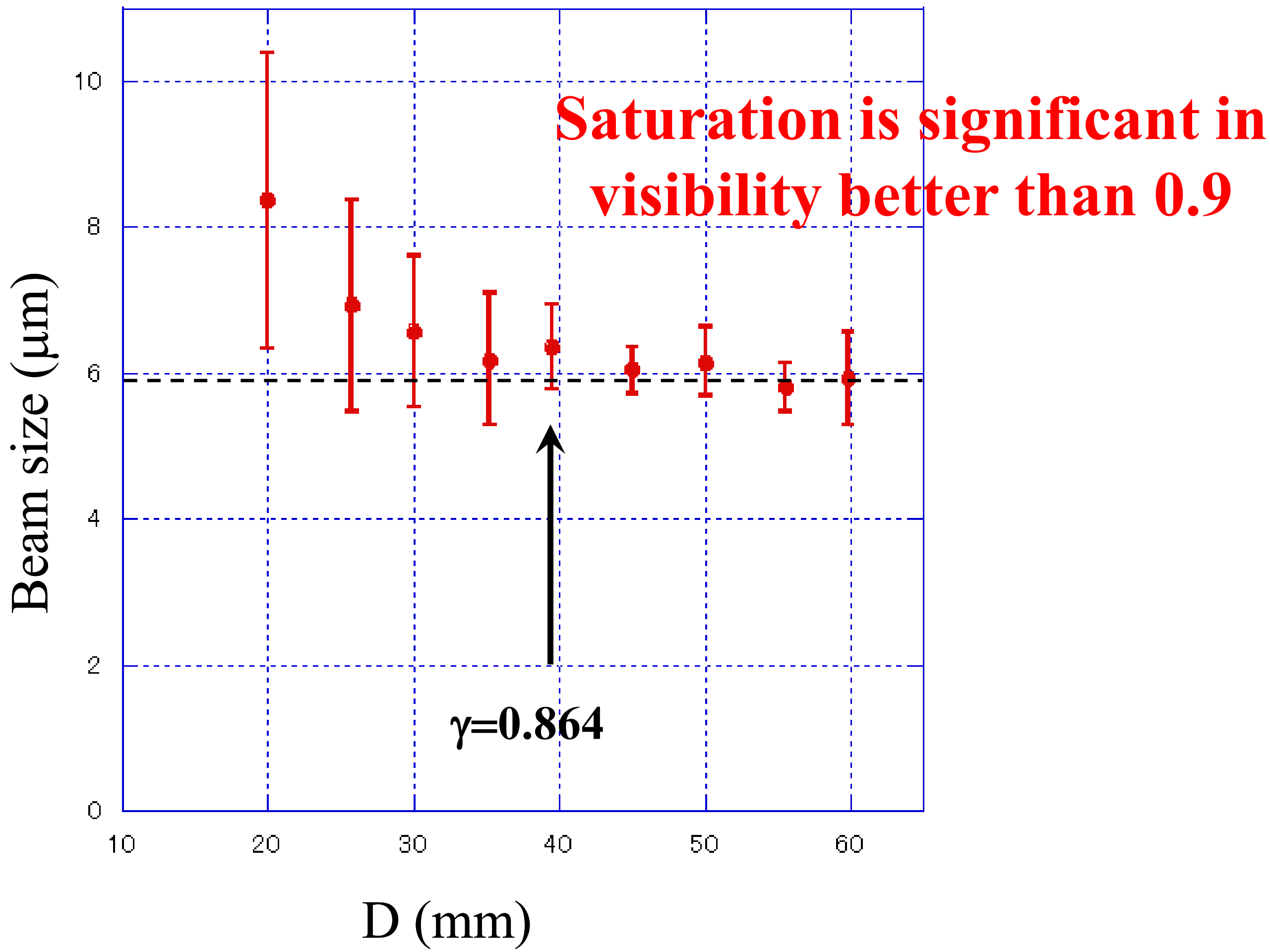
Statistical Error transfer from Δγ to Δσ under assuming Δγ=0.01 as a function of γ.

$$\Delta\sigma = \frac{\lambda \cdot F}{\pi \cdot D} \cdot \frac{1}{\gamma \cdot \sqrt{8 \cdot \ln\left(\frac{1}{\gamma}\right)}} \cdot \Delta\gamma$$

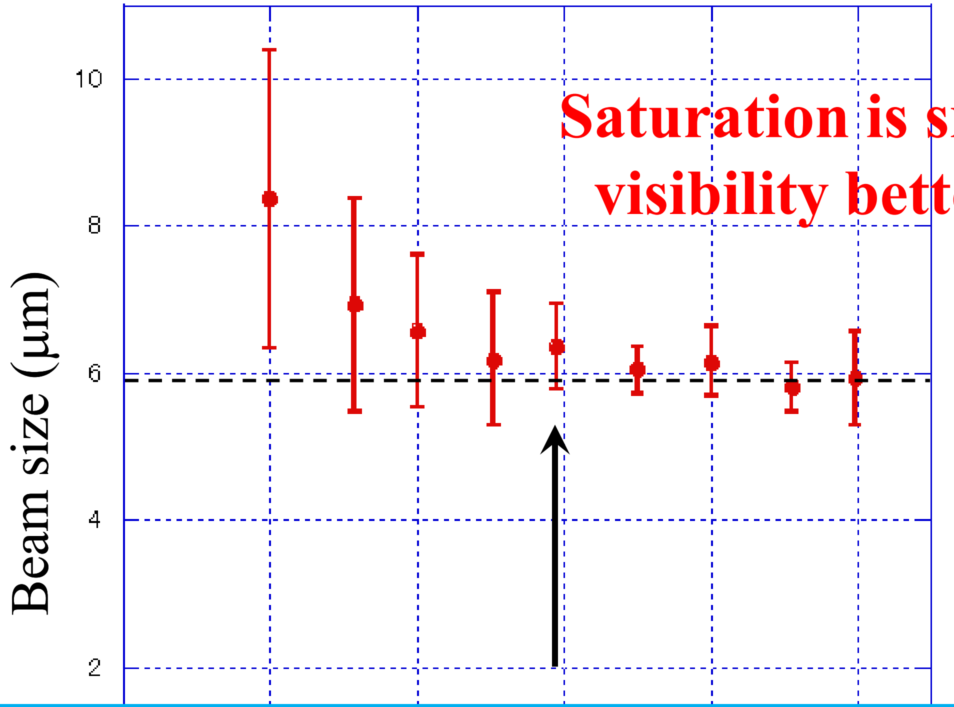


Systematic error due to nonlinearity near the CCD baseline




Saturation is significant in
visibility better than 0.9
γ=0.864
Beam size (µm)
D (mm)
0
2
4
6
8
10
10
20
30
40
50
60

Convert visibility into beam size. We can see clear saturation in smaller double slit range which has visibility near 1.

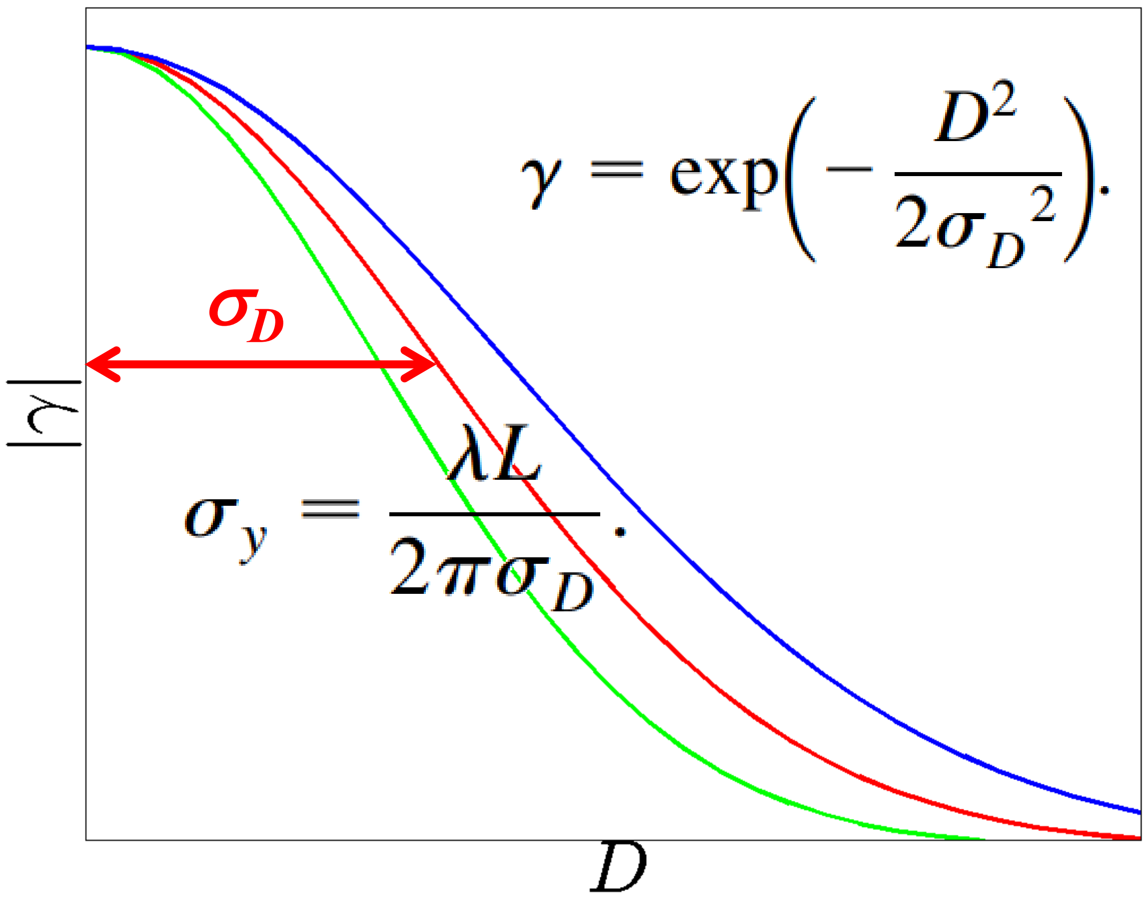


Saturation is significant in visibility better than 0.9

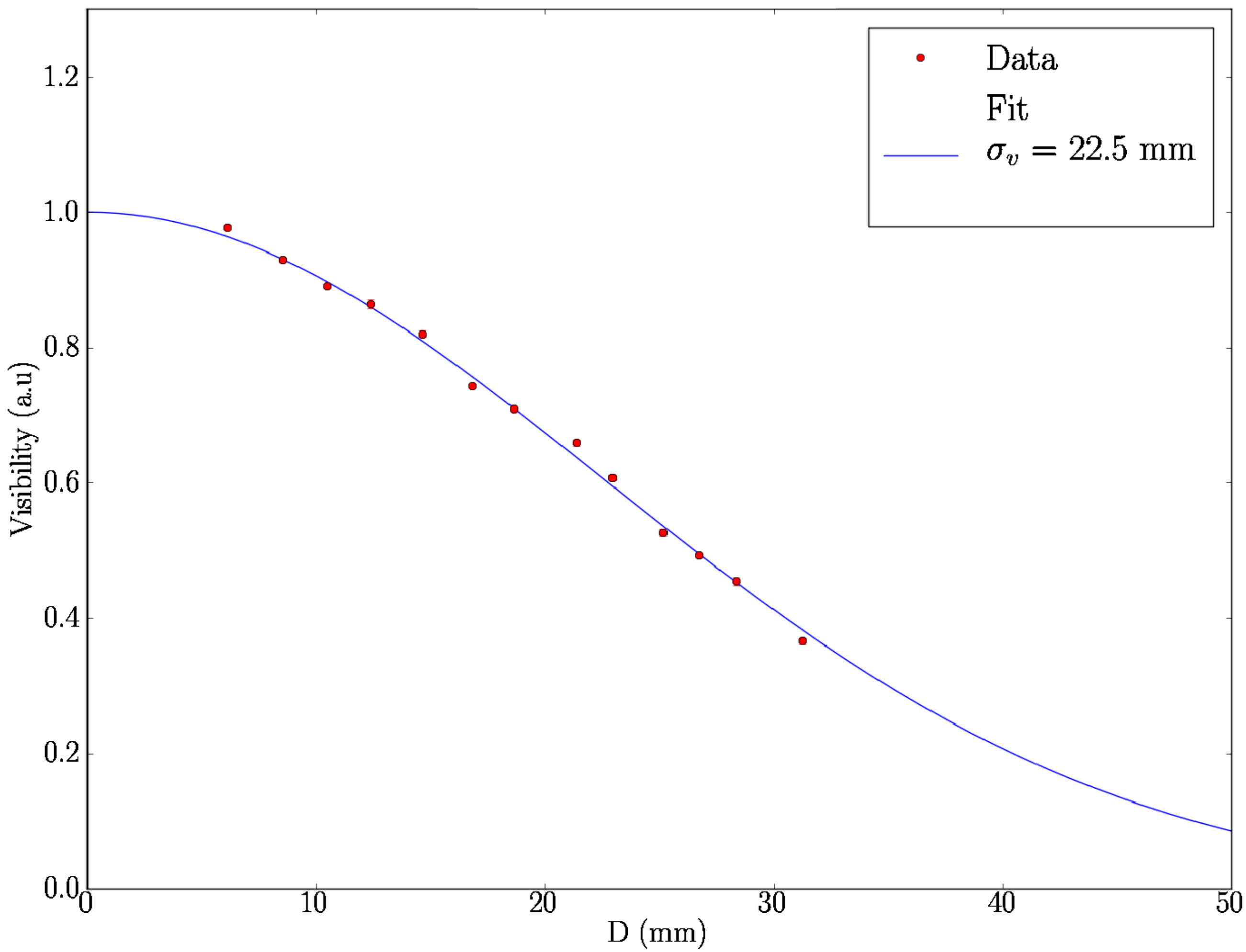
The visibility 0.90 with D=60mm, λ=400nm for the ATF interferometer corresponds to a beam size of 3.5μm. This size seems smallest measurable size with a normal setup of the interferometer.

5. Review of vertical small beam size measurements

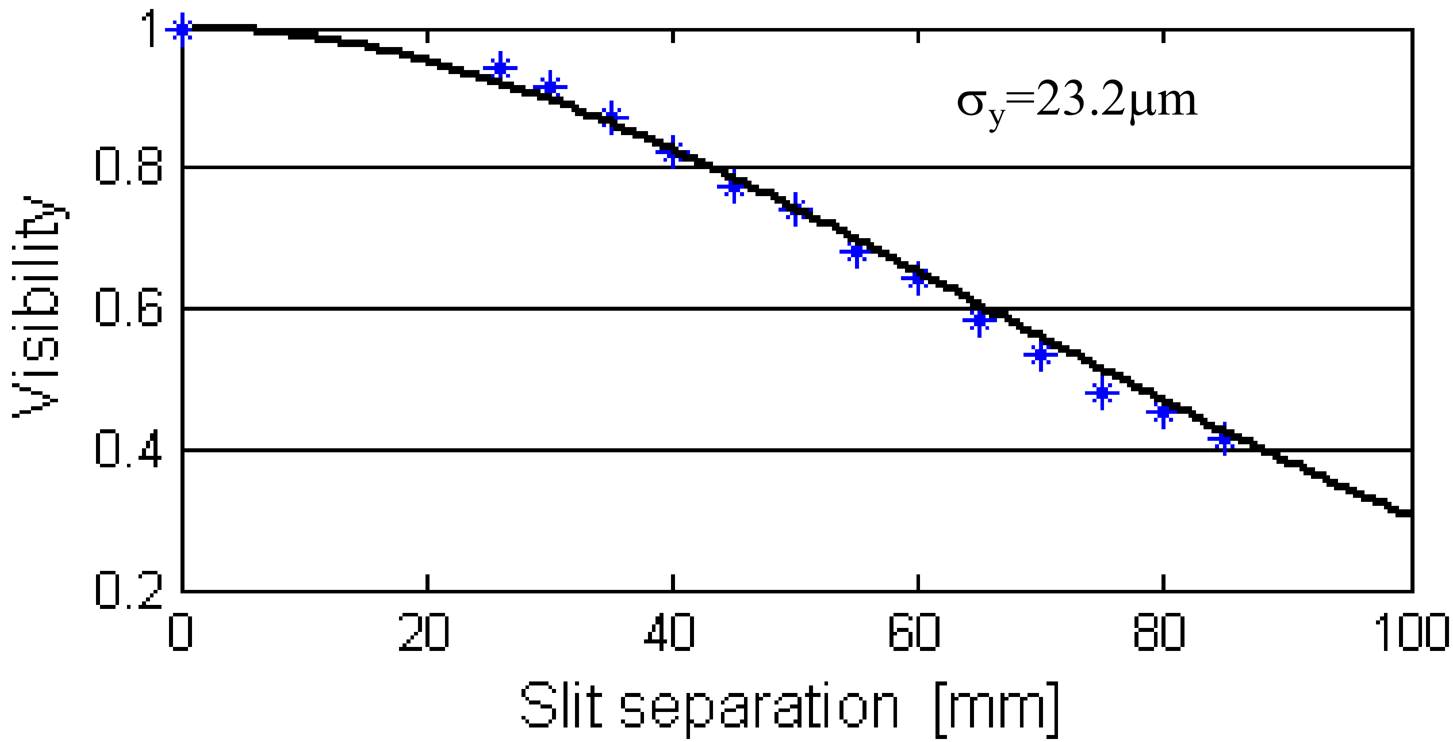
Gaussian profile approximation



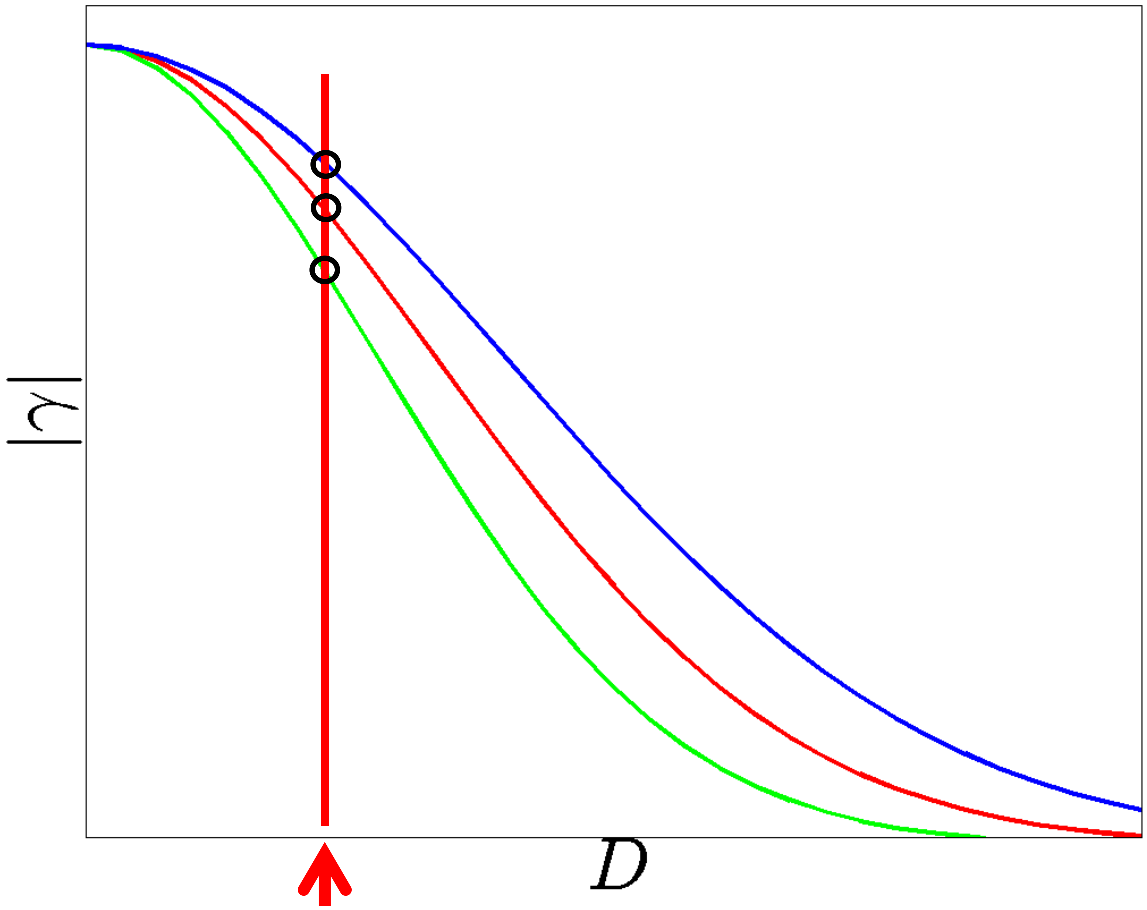
Visibility as a function of double slit separation at ALBA. Solid blue line is the Gaussian fitted result.



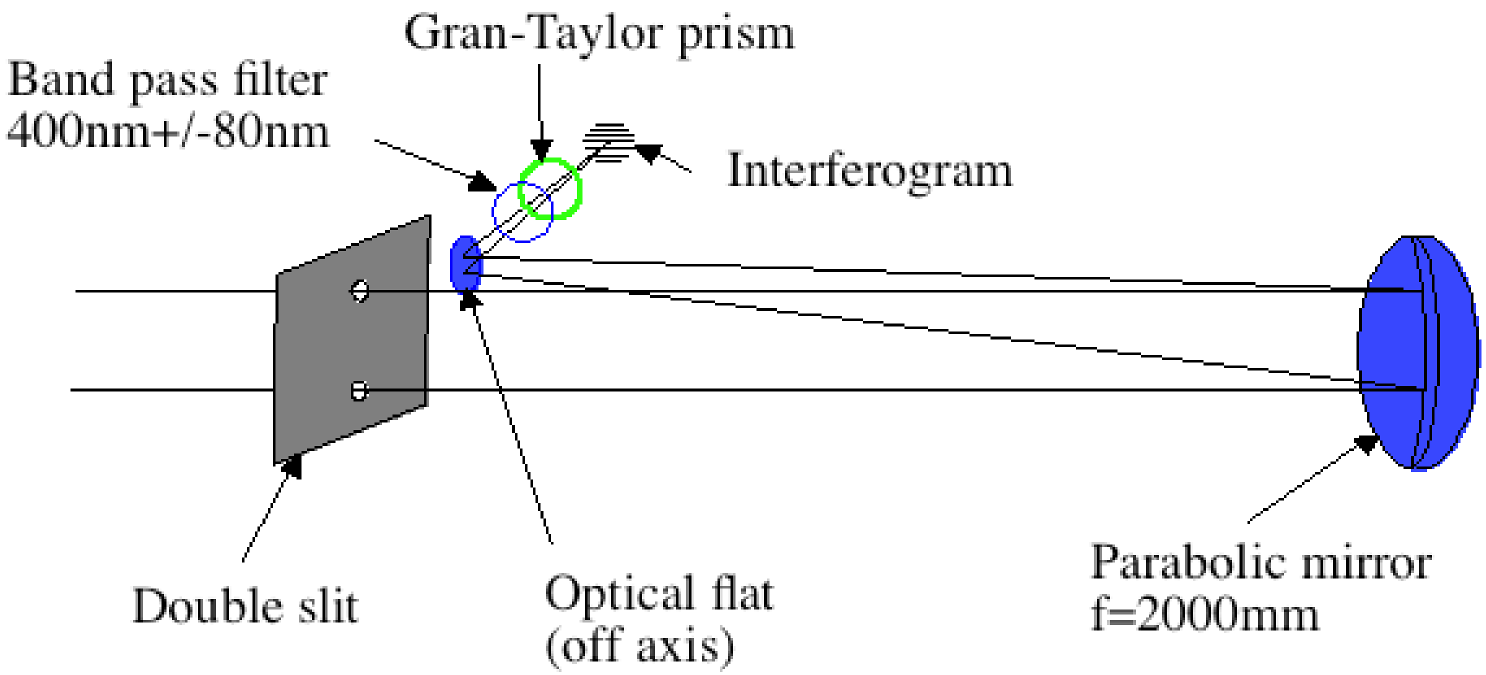
Visibility as a function of double slit separation at SPEAR3. The dotted line is measurement result, and the solid line is a Gaussian fit.



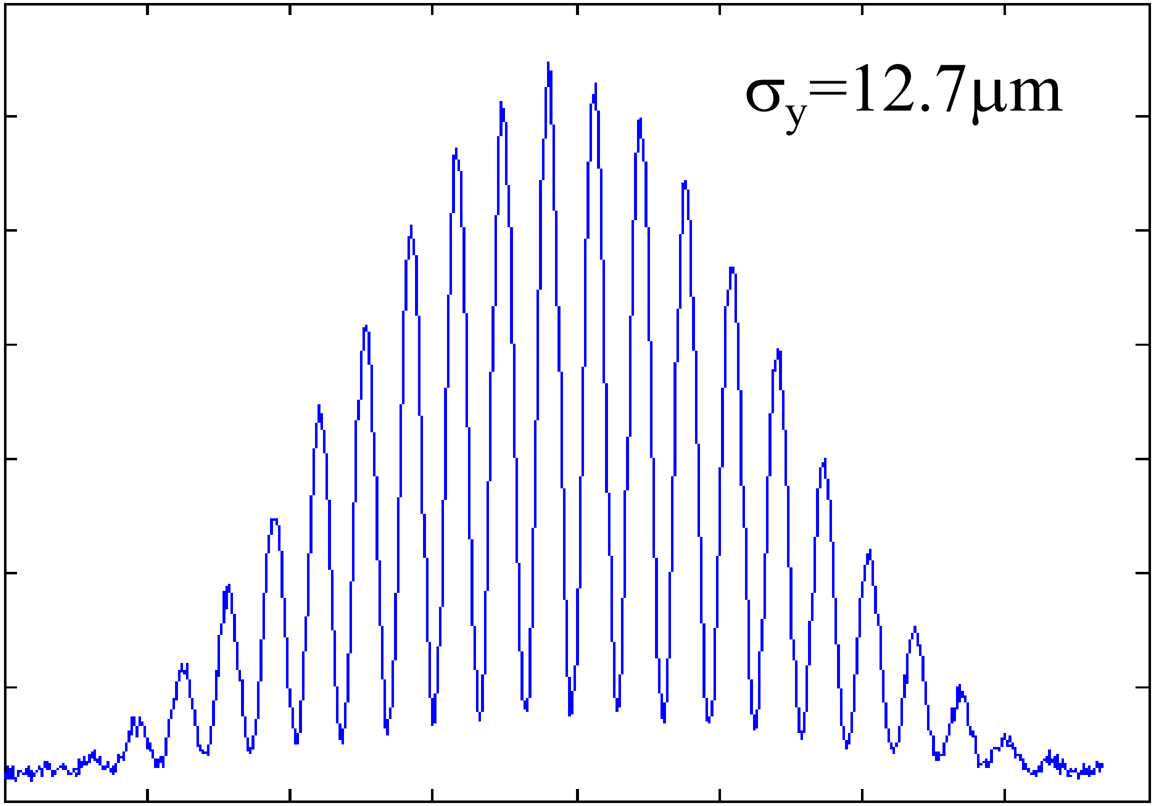
choose a fixed D and determine the beam size from the measurement of the visibility



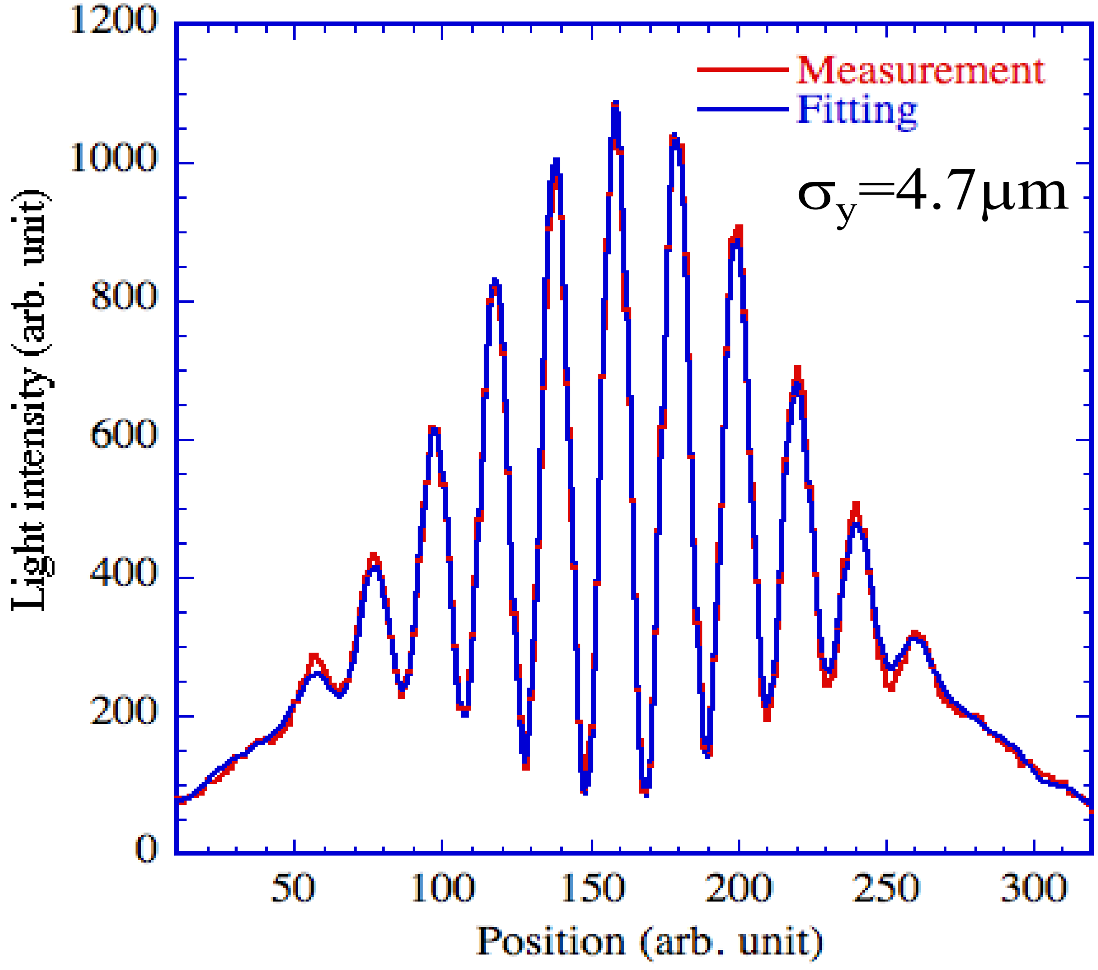
$$\sigma_y = \frac{\pi D}{\lambda R}\sqrt{\frac{1}{2}\ln\left(\frac{1}{|\gamma|}\right)}.$$


Gran-Taylor prism
Band pass filter
400nm+/-80nm
Interferogram
Double slit
Optical flat
(off axis)
Parabolic mirror
f=2000mm

Interferogram measured at D=50mm and l=550nm for low coupling mode at SPEAR3.



Interferogram at the KEK ATF. Red line is measurement, and blue line is fitting. D=60mm and λ=400nm.



7. Review for recent measurement of horizontal beam sizes

The incoherent depth of field effect (IDOF)

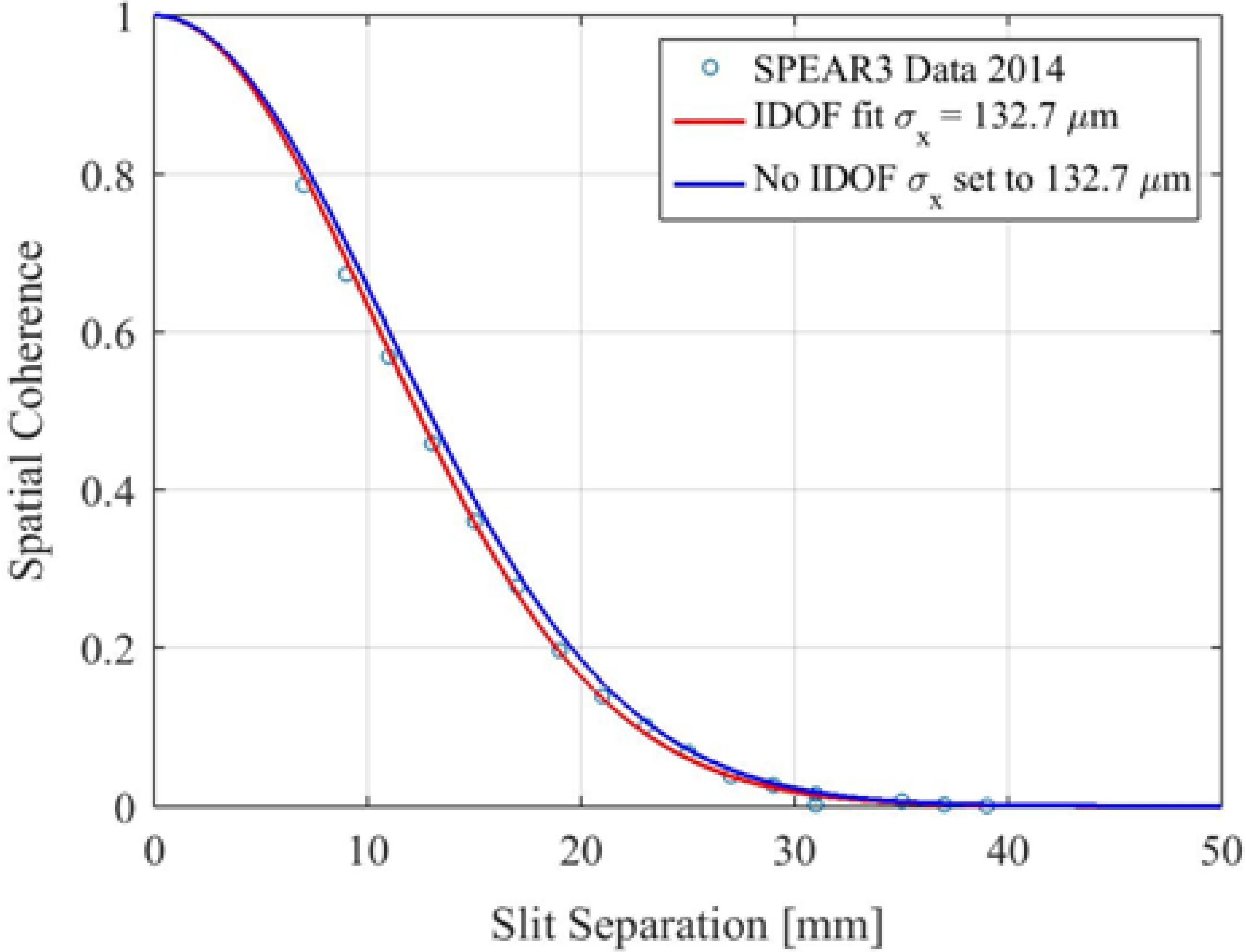
The visibility measurement as a function of slit separation is necessary for the observation of IDOF effect.

Then the real part of spatial coherence is given by Fourier cosine transform of this apparent distribution is given by ,

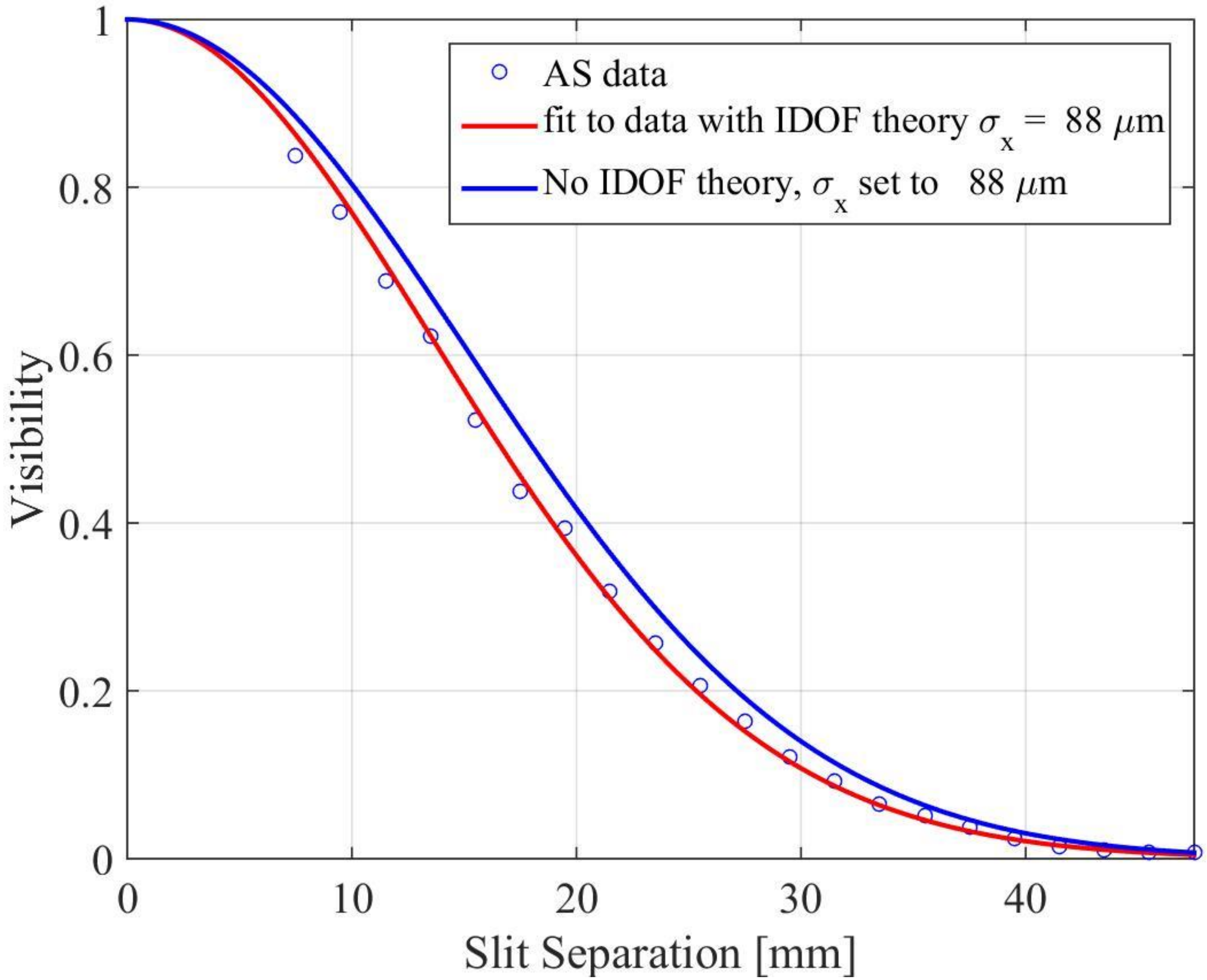
$$\gamma_h(D) = \iint \frac{2\sqrt{I\left(\theta+\frac{D}{2R}\right)I\left(\theta-\frac{D}{2R}\right)}}{I\left(\theta+\frac{D}{2R}\right)+I\left(\theta-\frac{D}{2R}\right)} \cdot I(\theta) \cdot \frac{\exp\left[\frac{-\left[x-\rho\{1-\cos(\theta)\}\right]^2}{2\sigma^2}\right]}{\sigma\cdot\sqrt{2\pi}} \cdot \cos\left(\frac{2\pi\cdot D\cdot x}{R\cdot\lambda}\right) d\theta\, dx$$

Intensity Imbalance factor — Fourier cosine transform

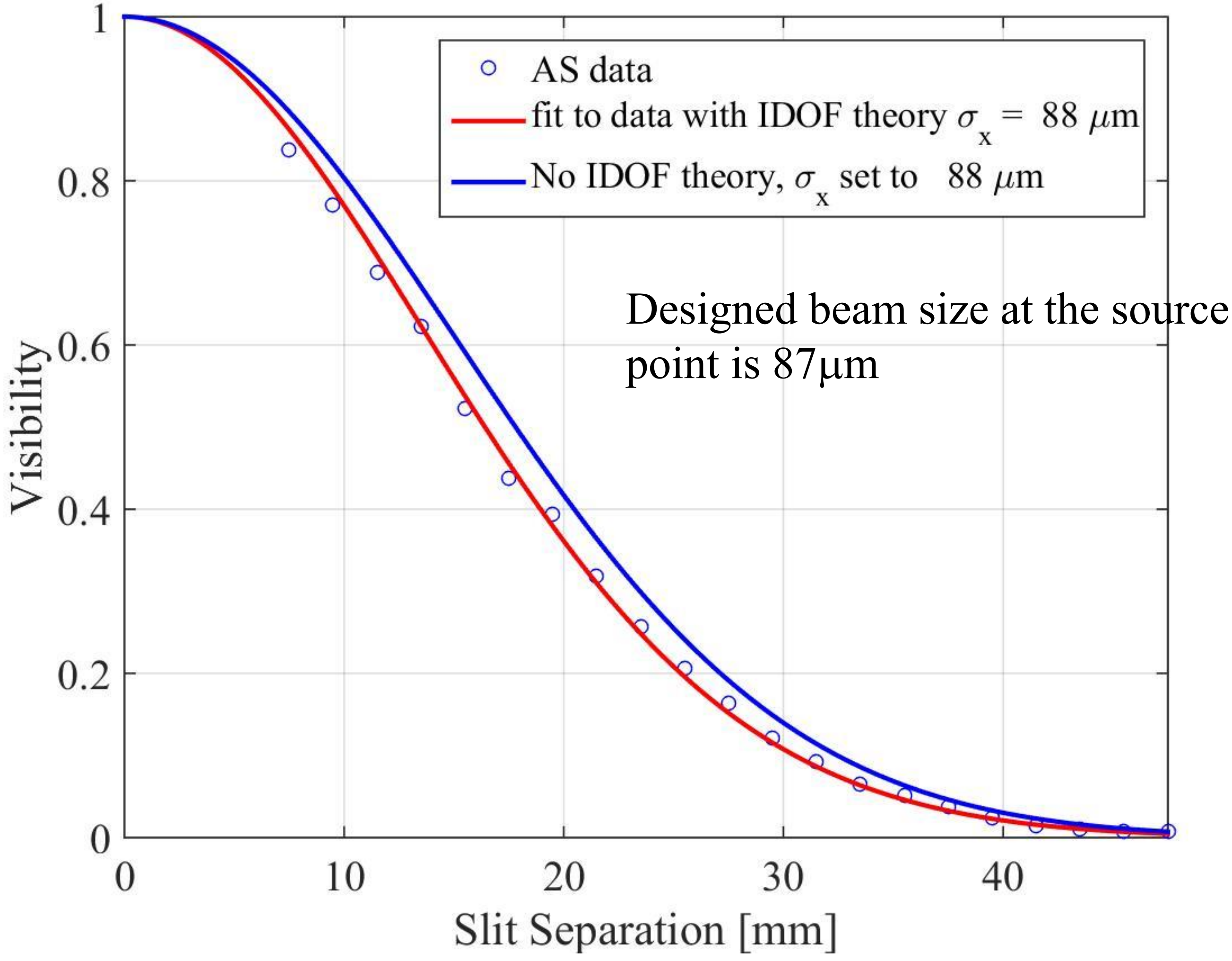
Horizontal beam visibility measurements at SPEAR3. Dots are measurements, red line is fitting with IODF and blue line is calculated visibility curve for a beam size 132.7mm *without* IDOF.



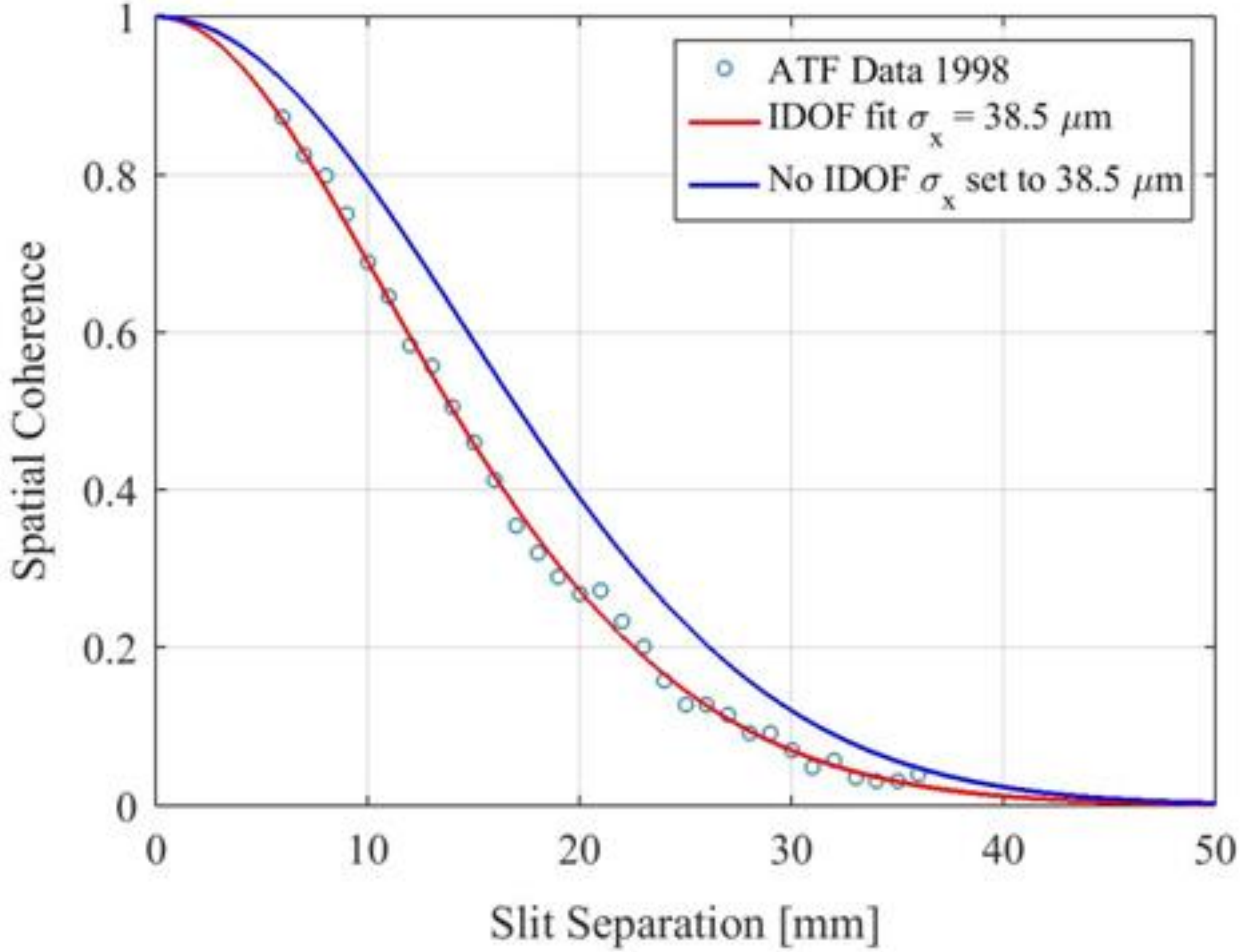
Horizontal visibility measured at ASLS as a function of slit separation. Blue line is calculated visibility curve for a beam size 88μm *without* IDOF.

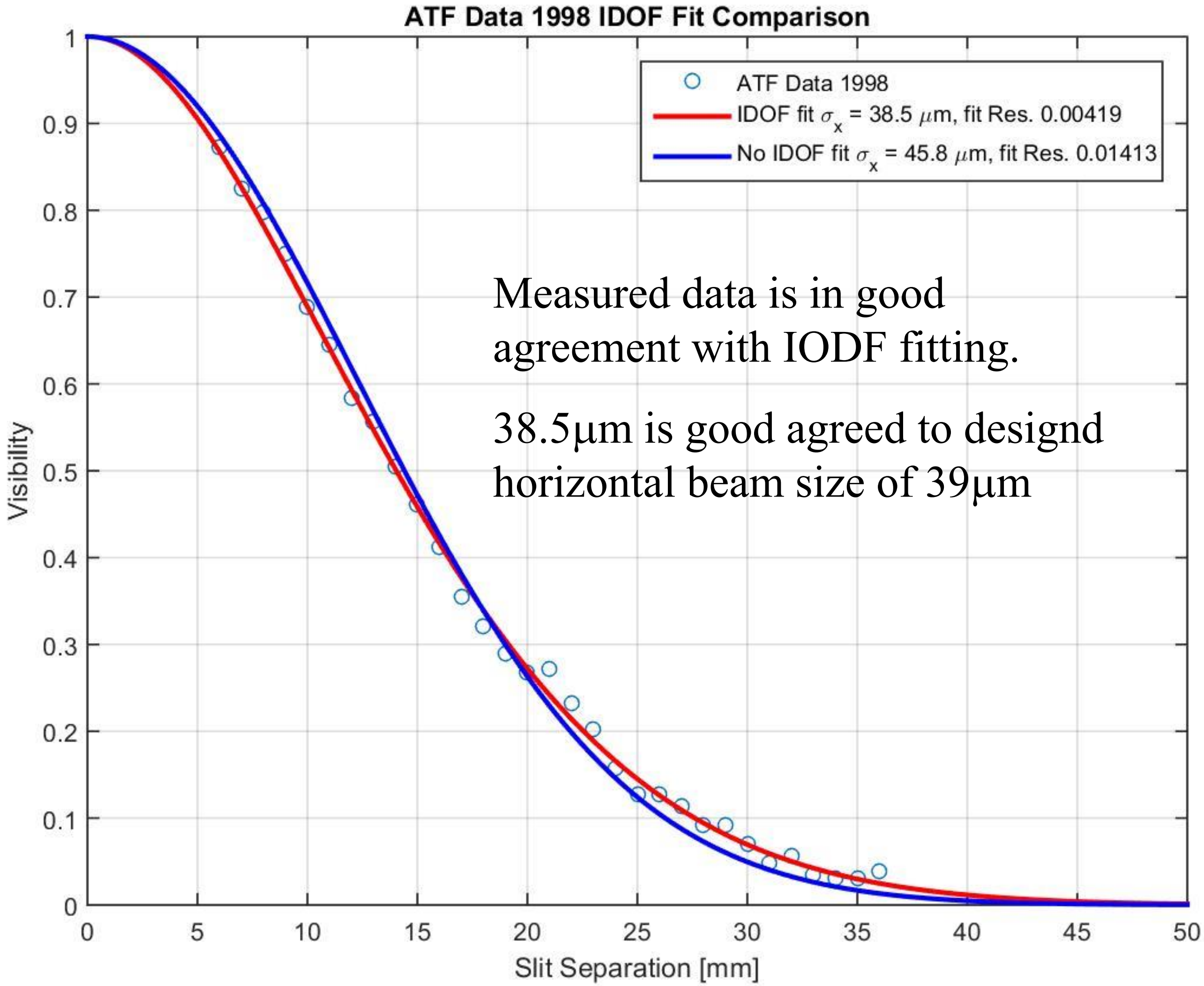


Horizontal visibility measured at ASLS as a function of slit separation. Blue line is calculated visibility curve for a beam size 88μm *without* IDOF.



Horizontal visibility measured at ATF as a function of slit separation. Blue line is calculated visibility curve for a beam size 38.5μm *without* IDOF.



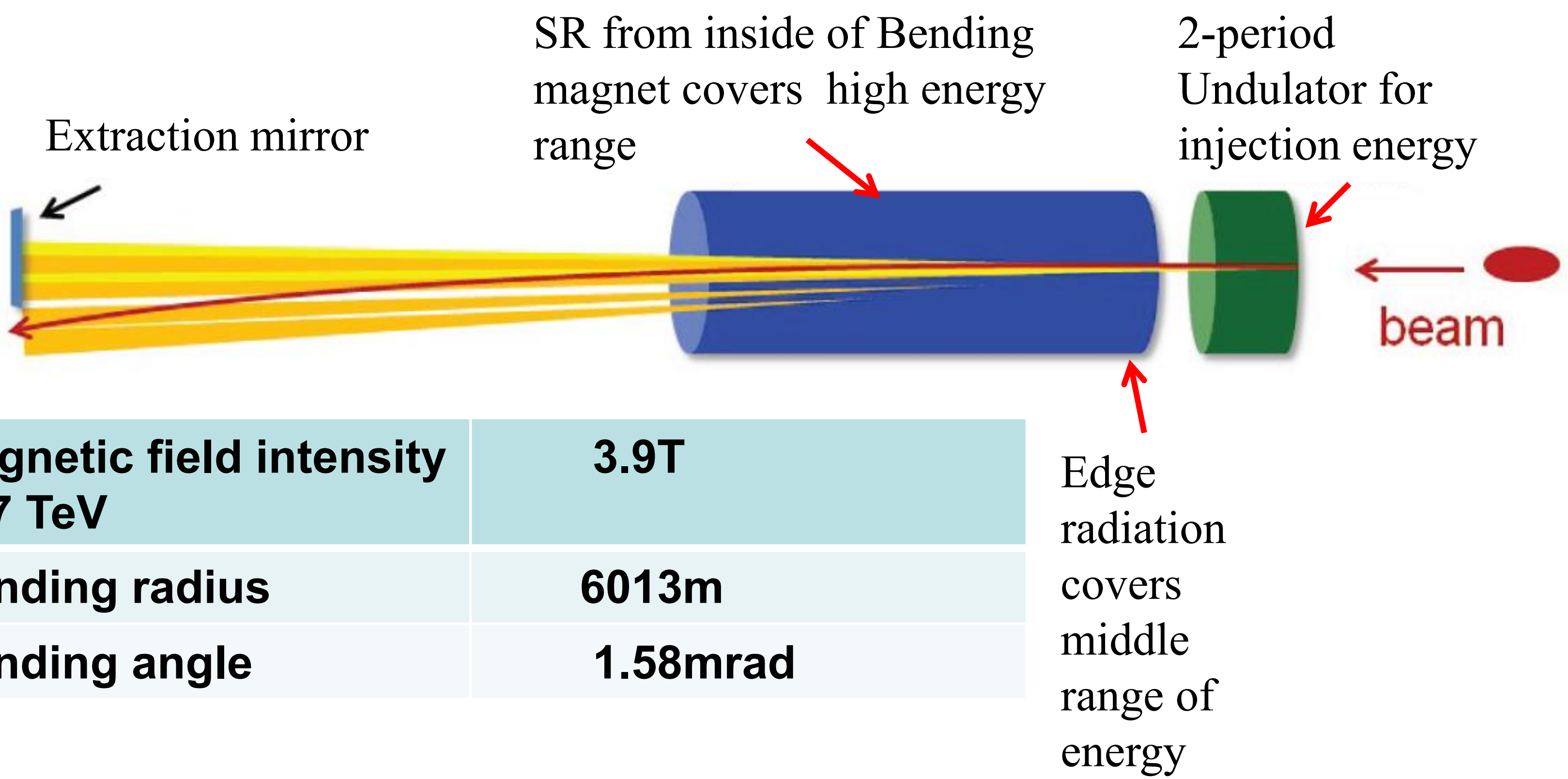


Measured data is in good agreement with IODF fitting.

38.5μm is good agreed to designd horizontal beam size of 39μm

7. Application of interferometry for the beam size measurement in the LHC

Schematic drawing of the BSRT Synchrotron light sources



Magnetic field intensity at 7 TeV	**3.9T**
Bending radius	**6013m**
Bending angle	**1.58mrad**

SR from bending magnet in LHC.

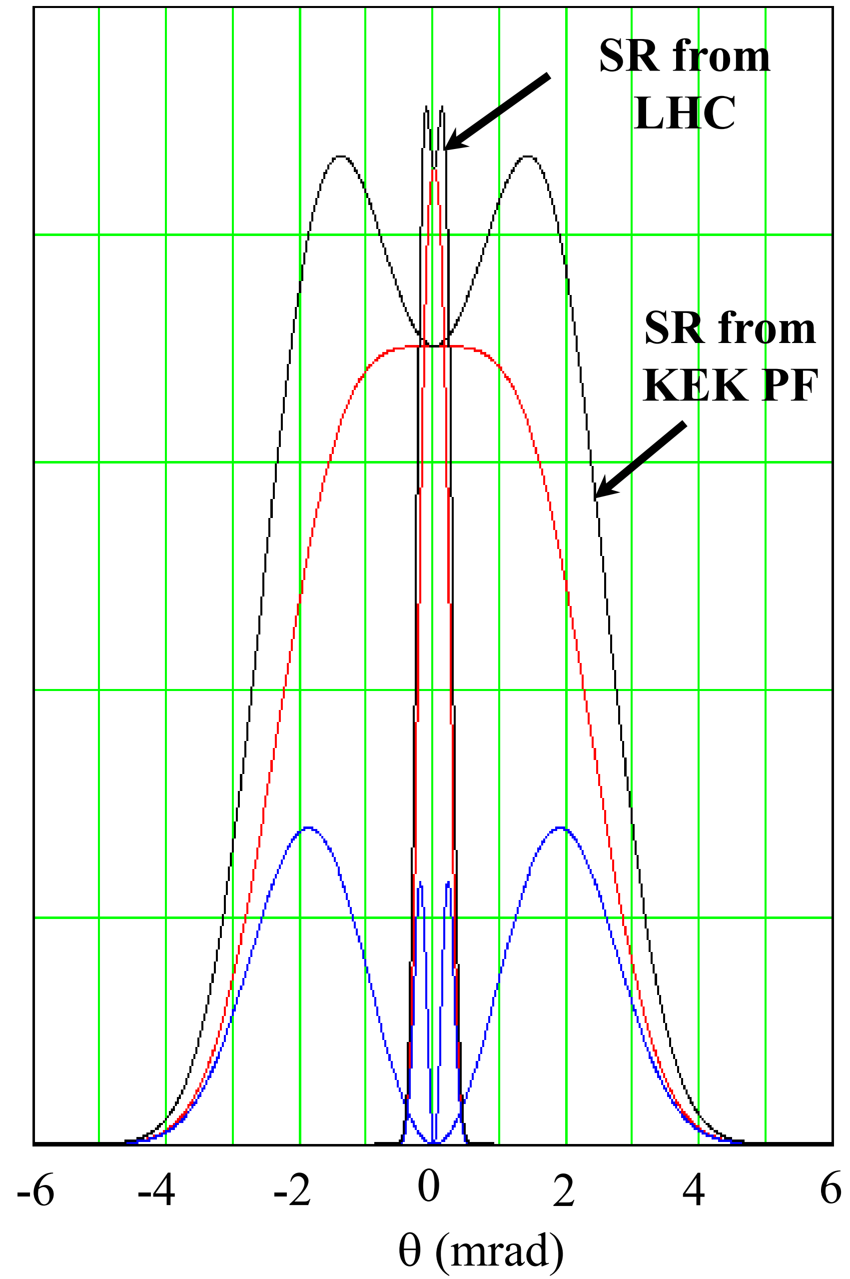
<u>**Opening angle is only about 0.2mrad**</u>

About 10 times narrower than the Electron machine!

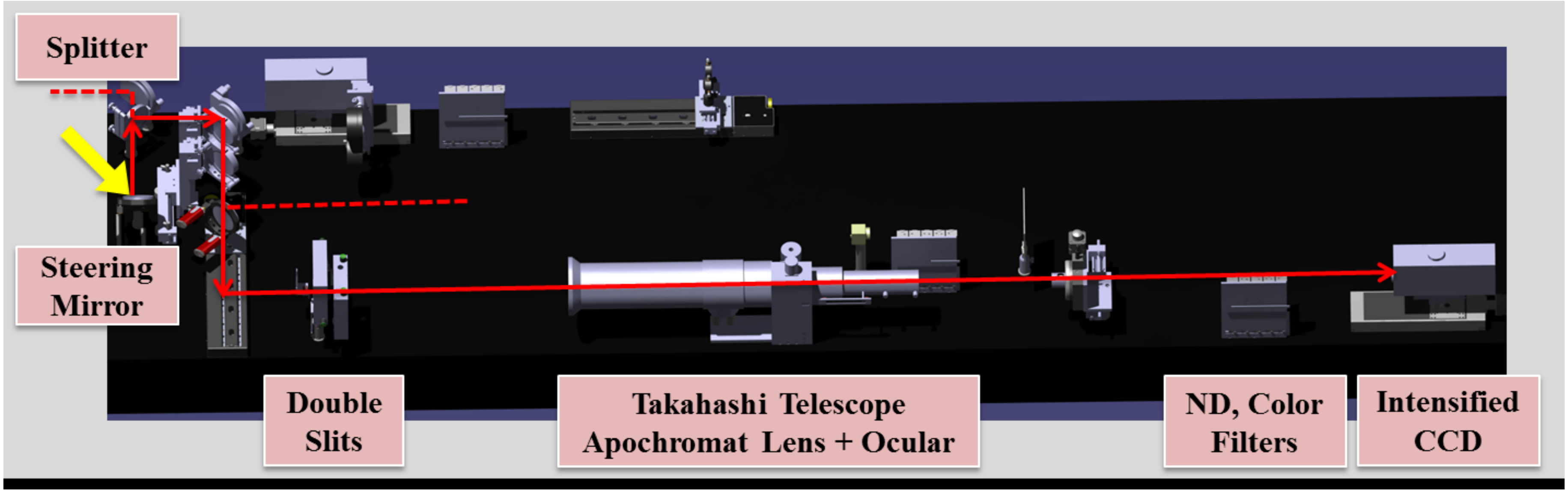
Resolution in imaging is 10 times poor.

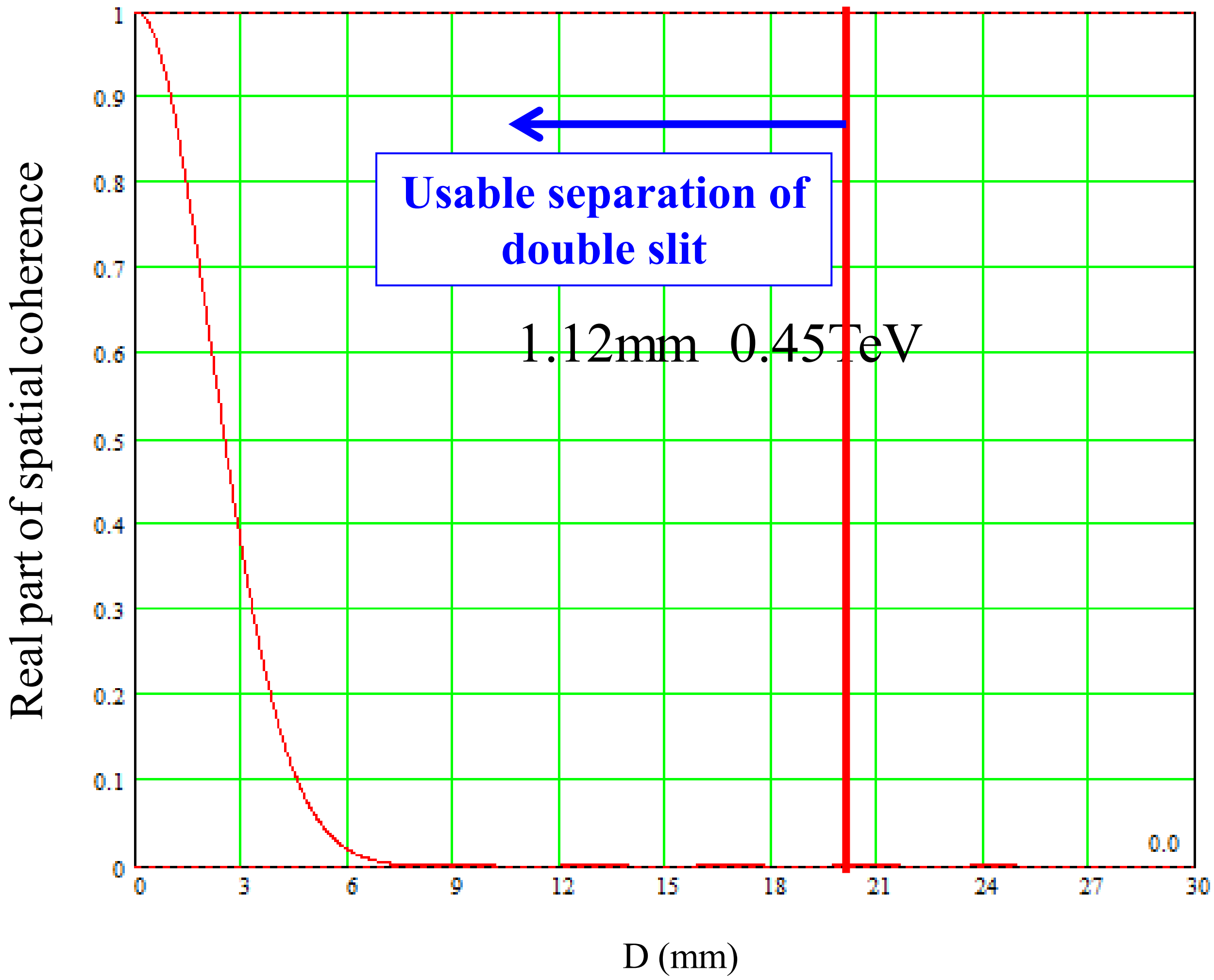
We need a method having 10 times better resolution.

Use the interferomtery!



A Schematic drawing of new interferometer line





Usable separation of double slit
1.12mm 0.45TeV
Real part of spatial coherence
D (mm)
0.0
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
3
6
9
12
15
18
21
24
27
30

IDOF for horizontal beam size measurement is very significant due to very long Bending radius (6013m) and narrow opening (0.2mrad) of SR at 7TeV

When the incoherent depth of field < Airy disk length of the diffraction

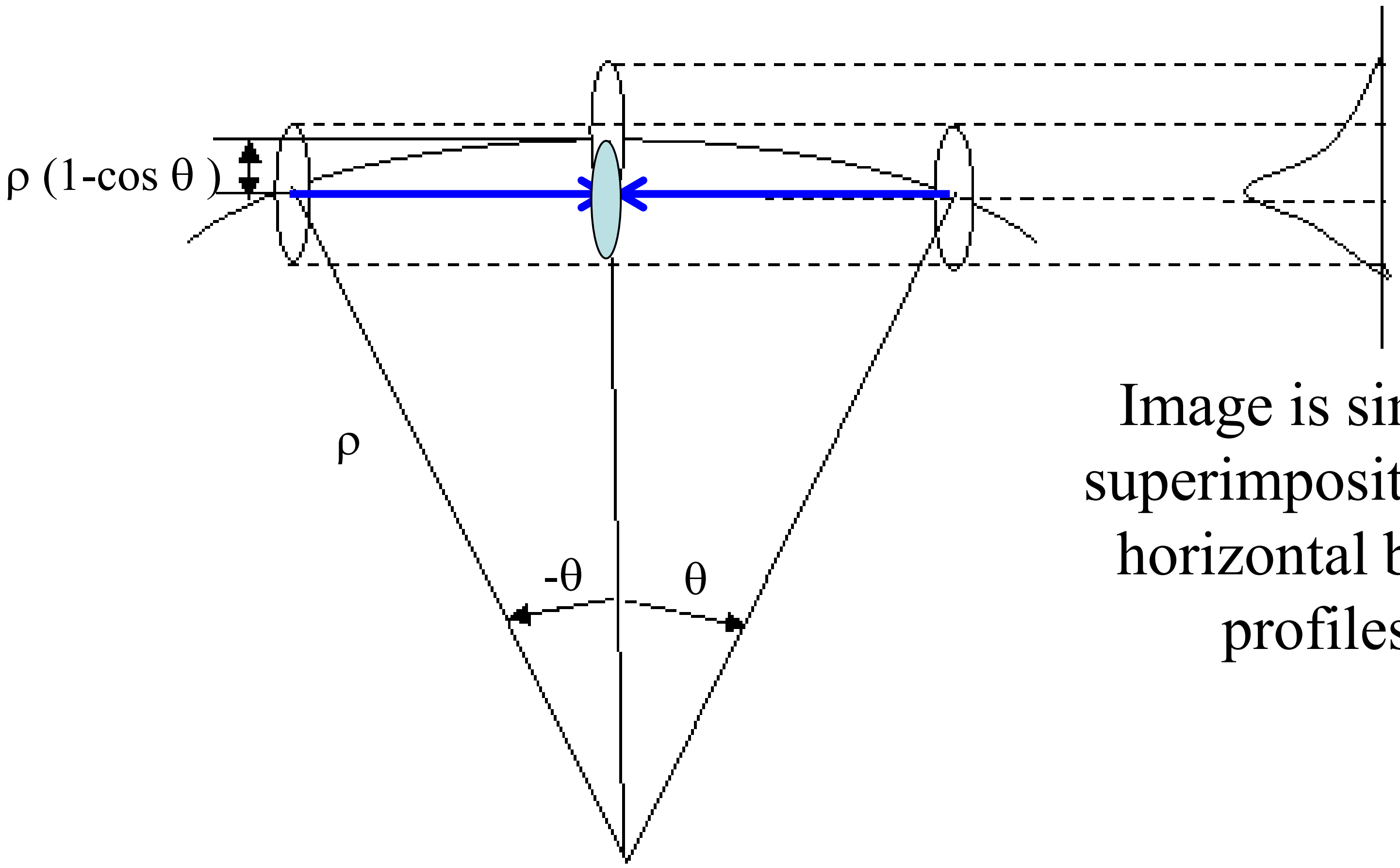
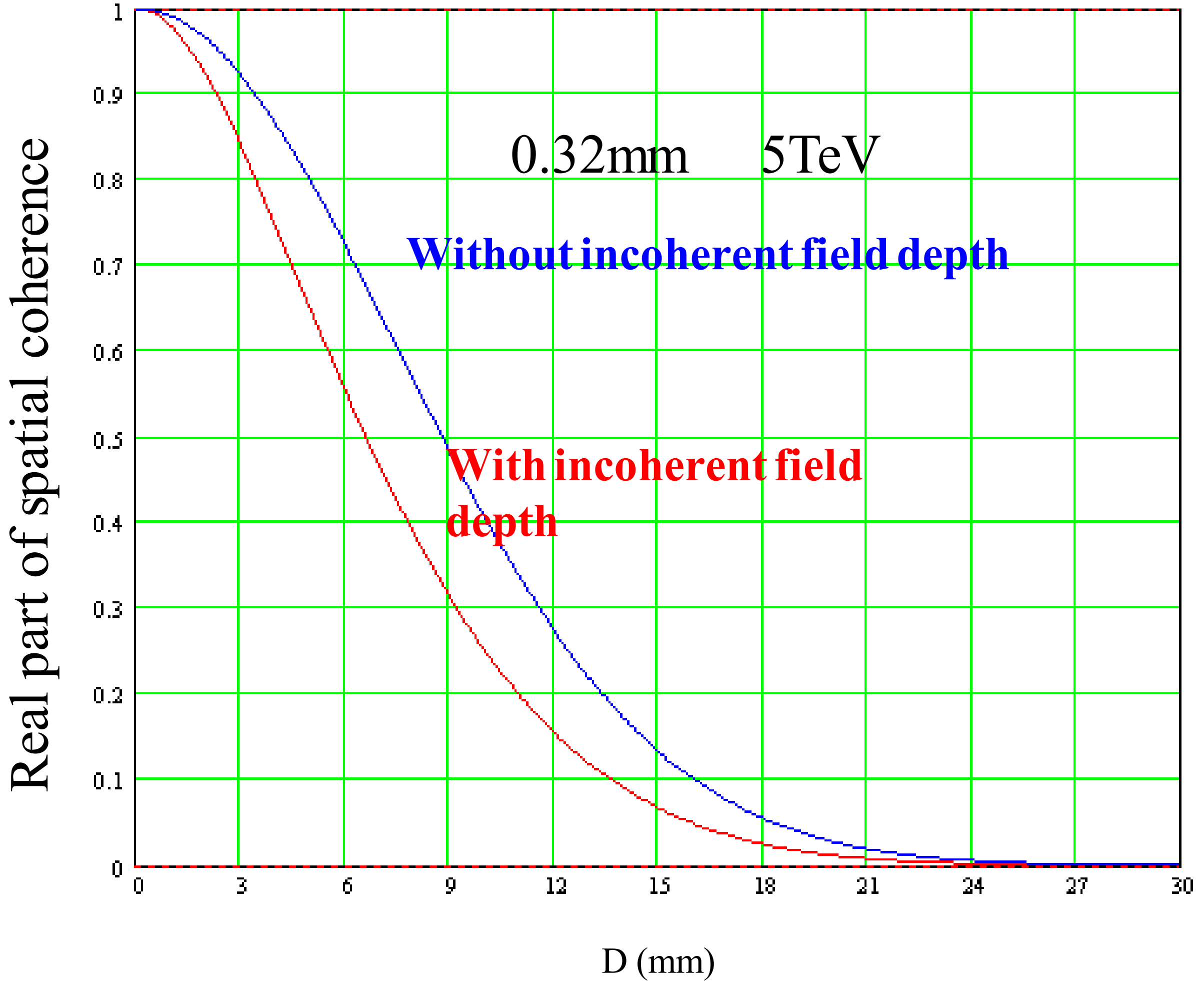
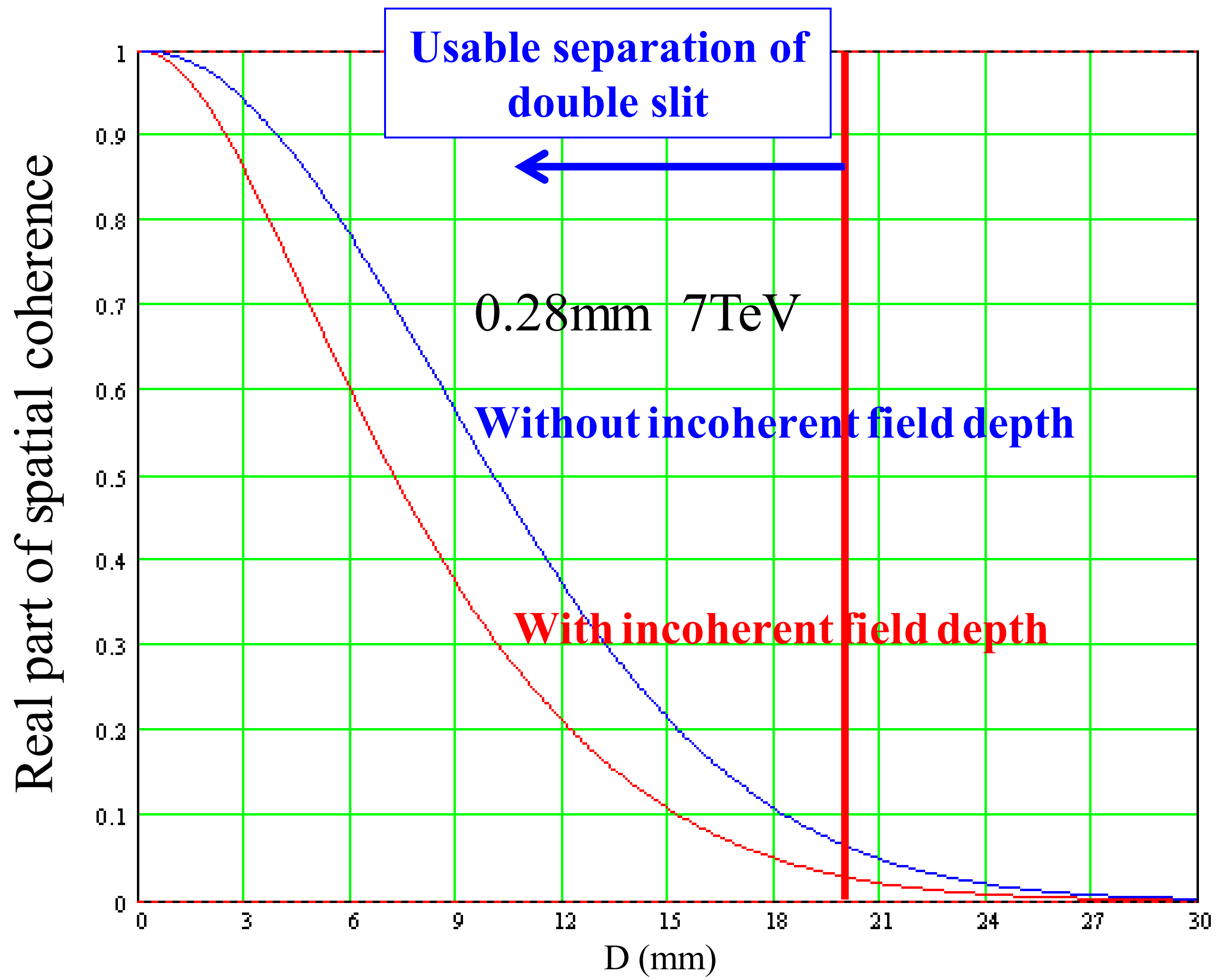


Image is simply superimposition of horizontal beam profiles

For example $\rho \approx 6013m,\ \theta \approx 0.2mrad, \quad \rho(1-\cos\theta) \approx 170\mu m$

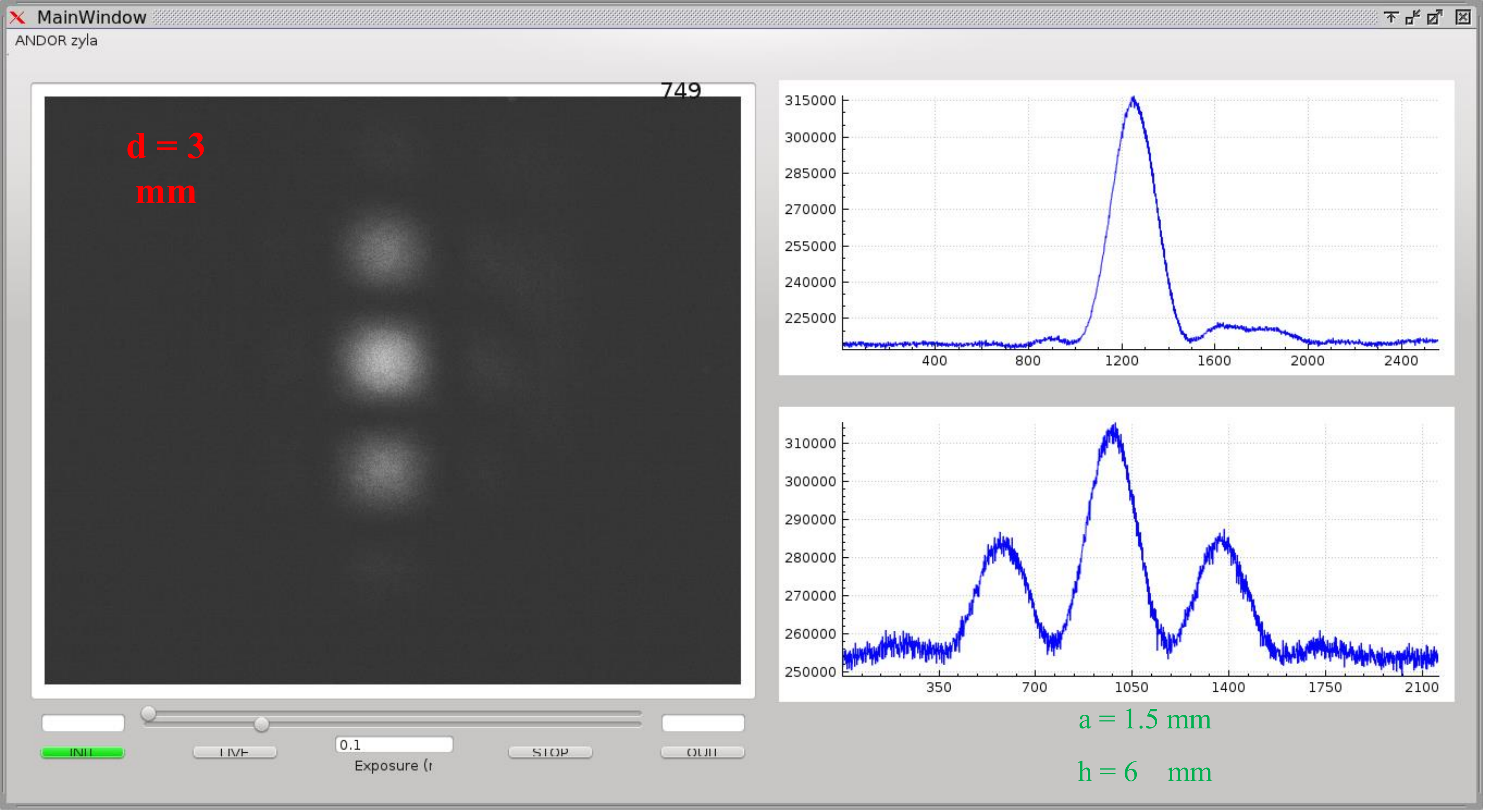


0.32mm 5TeV
Without incoherent field depth
With incoherent field depth
Real part of spatial coherence
D (mm)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
3
6
9
12
15
18
21
24
27
30


Usable separation of double slit
0.28mm 7TeV
Without incoherent field depth
With incoherent field depth
Real part of spatial coherence
D (mm)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
3
6
9
12
15
18
21
24
27
30

First interferograms at LHC

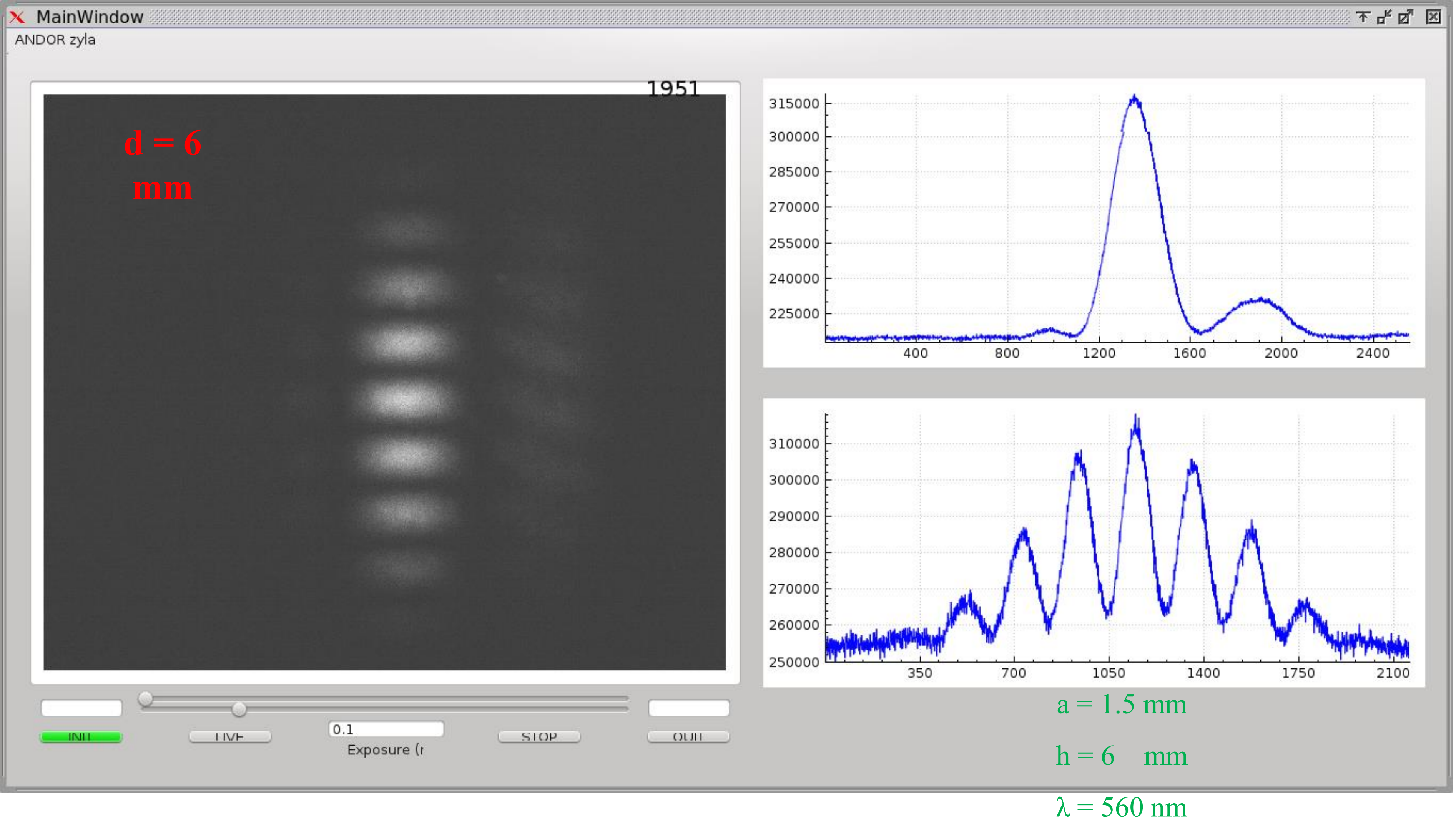
Taken at 6.5TeV with pilot bunch having 5x10^9 protons

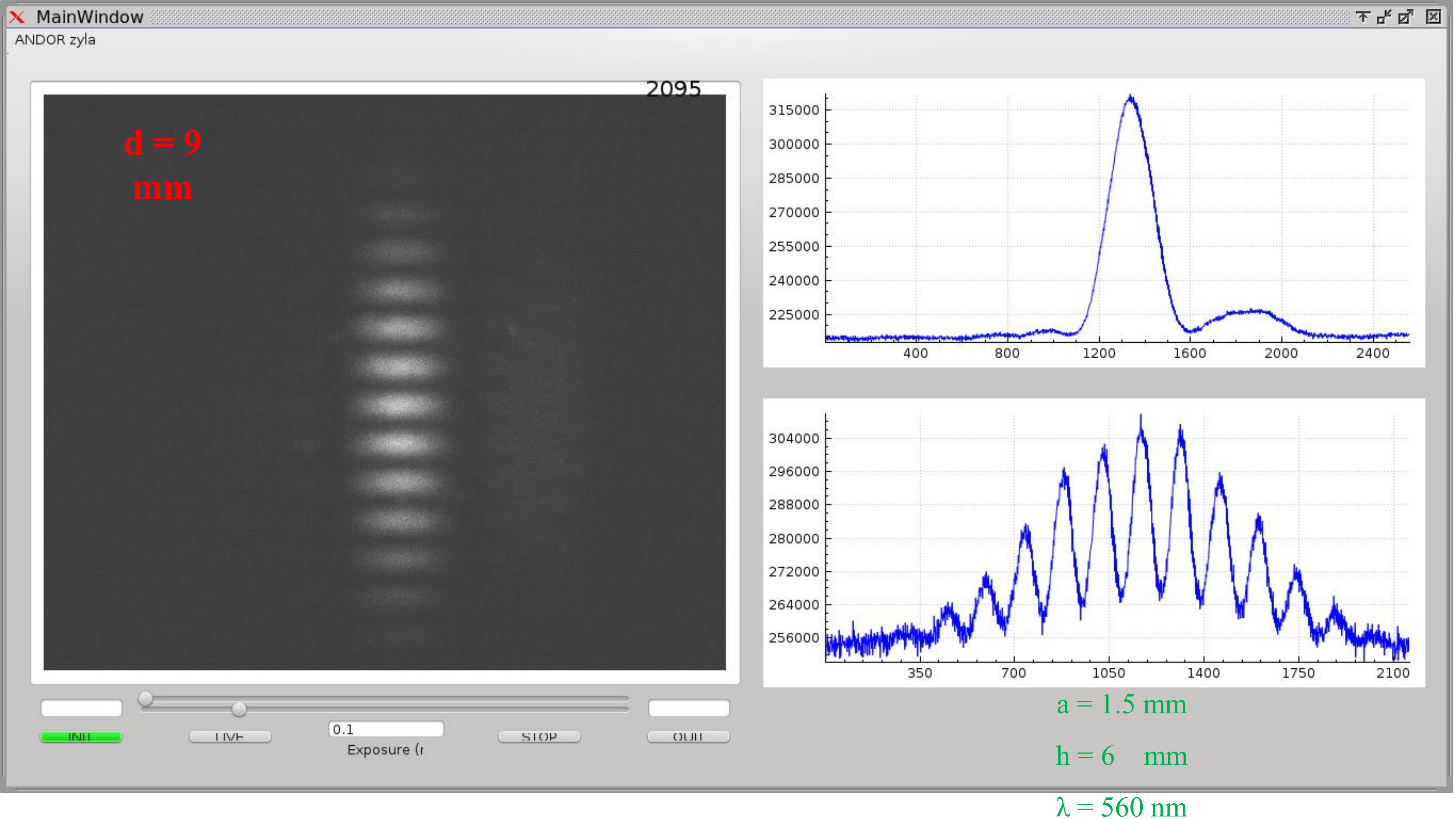


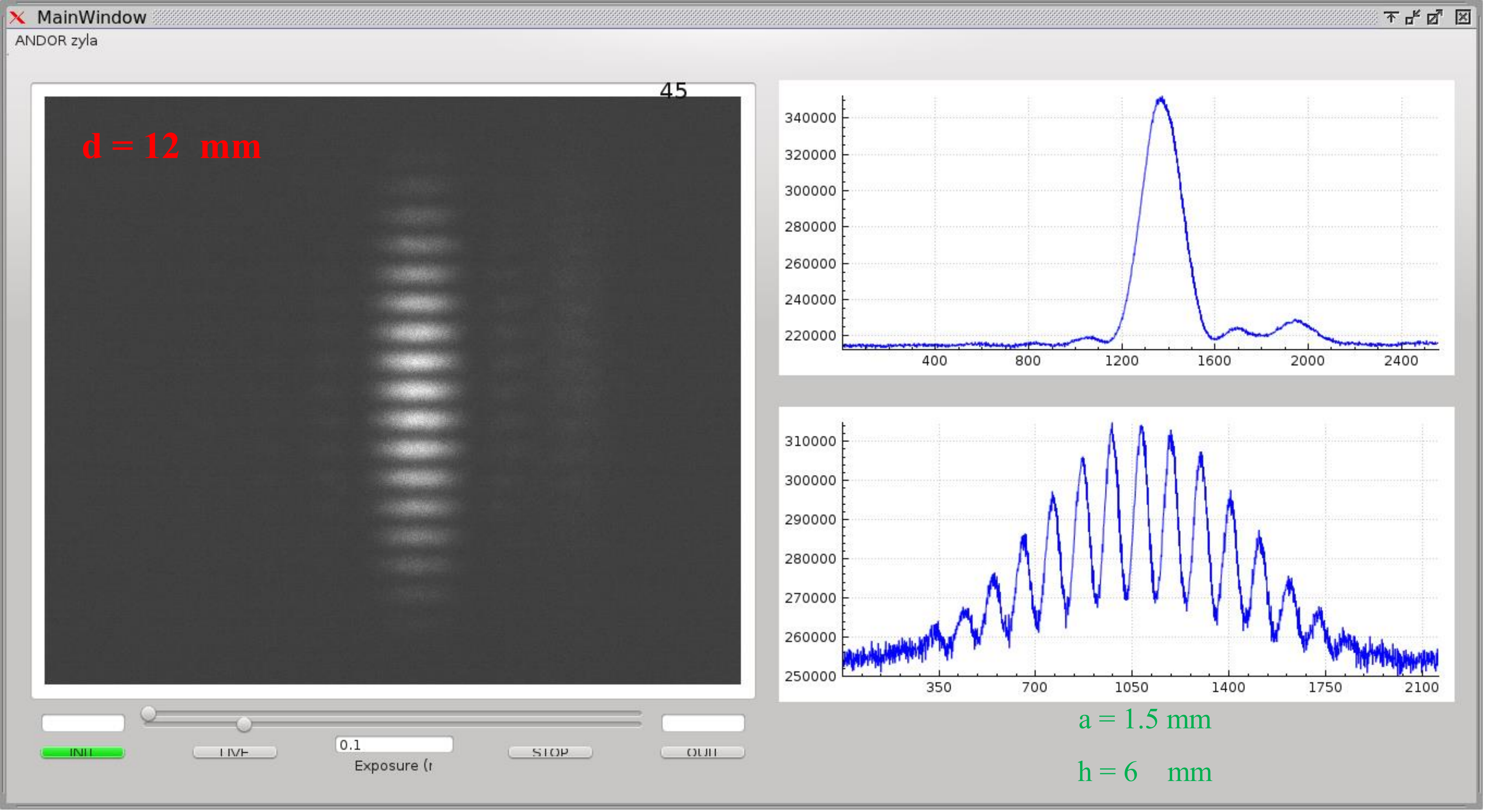
a = 1.5 mm

h = 6 mm

λ = 560 nm


MainWindow
ANDOR zyla
1951
d = 6 mm
315000
300000
285000
270000
255000
240000
225000
400
800
1200
1600
2000
2400
310000
300000
290000
280000
270000
260000
250000
350
700
1050
1400
1750
2100
0.1
Exposure (r
a = 1.5 mm
h = 6 mm
λ = 560 nm


MainWindow
ANDOR zyla
2095
d = 9 mm
315000
300000
285000
270000
255000
240000
225000
400
800
1200
1600
2000
2400
304000
296000
288000
280000
272000
264000
256000
350
700
1050
1400
1750
2100
0.1
Exposure (r
a = 1.5 mm
h = 6 mm
λ = 560 nm


MainWindow
ANDOR zyla
45
d = 12 mm
340000
320000
300000
280000
260000
240000
220000
400
800
1200
1600
2000
2400
310000
300000
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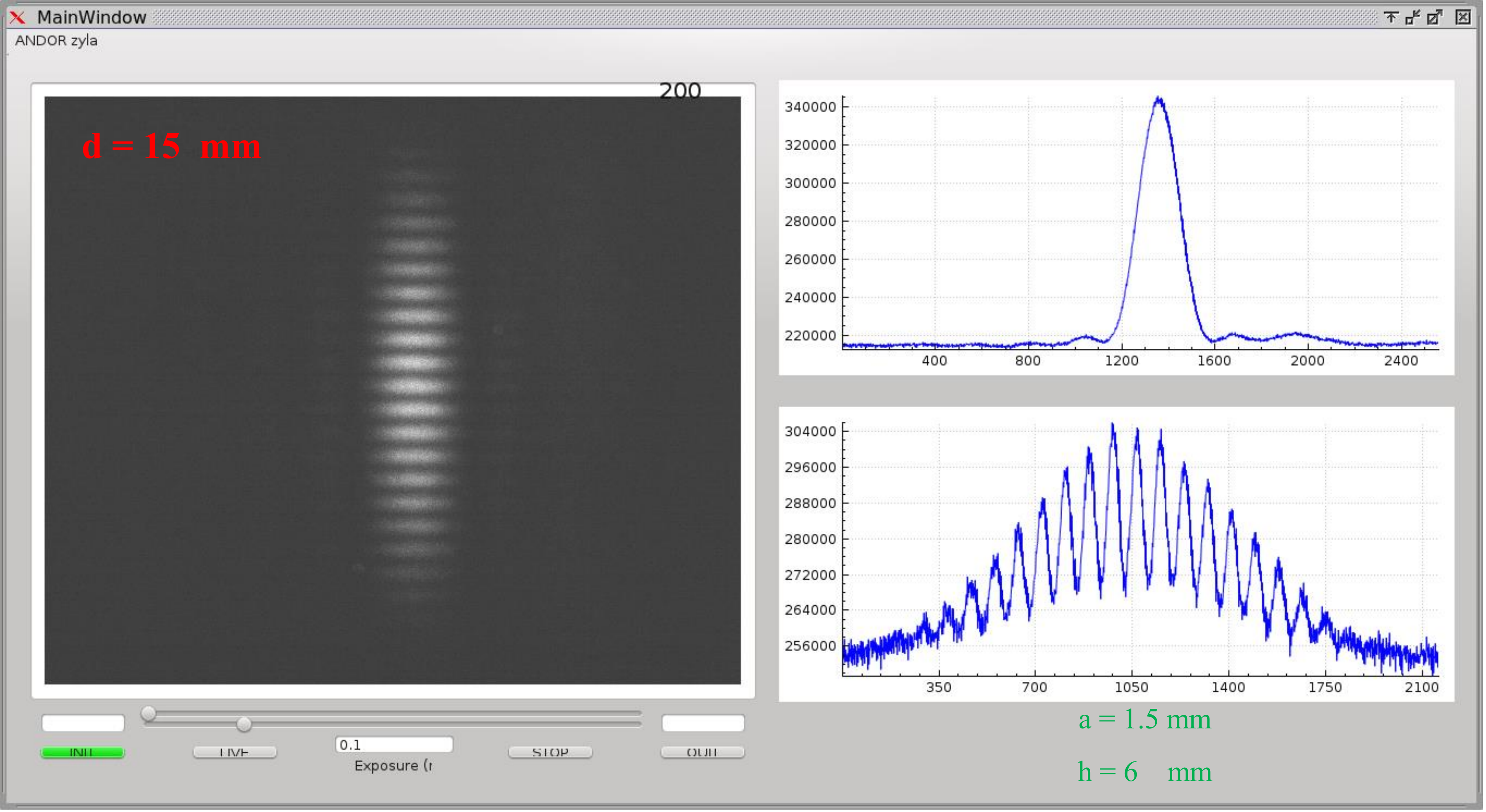
Exposure (r
STOP
a = 1.5 mm
h = 6 mm

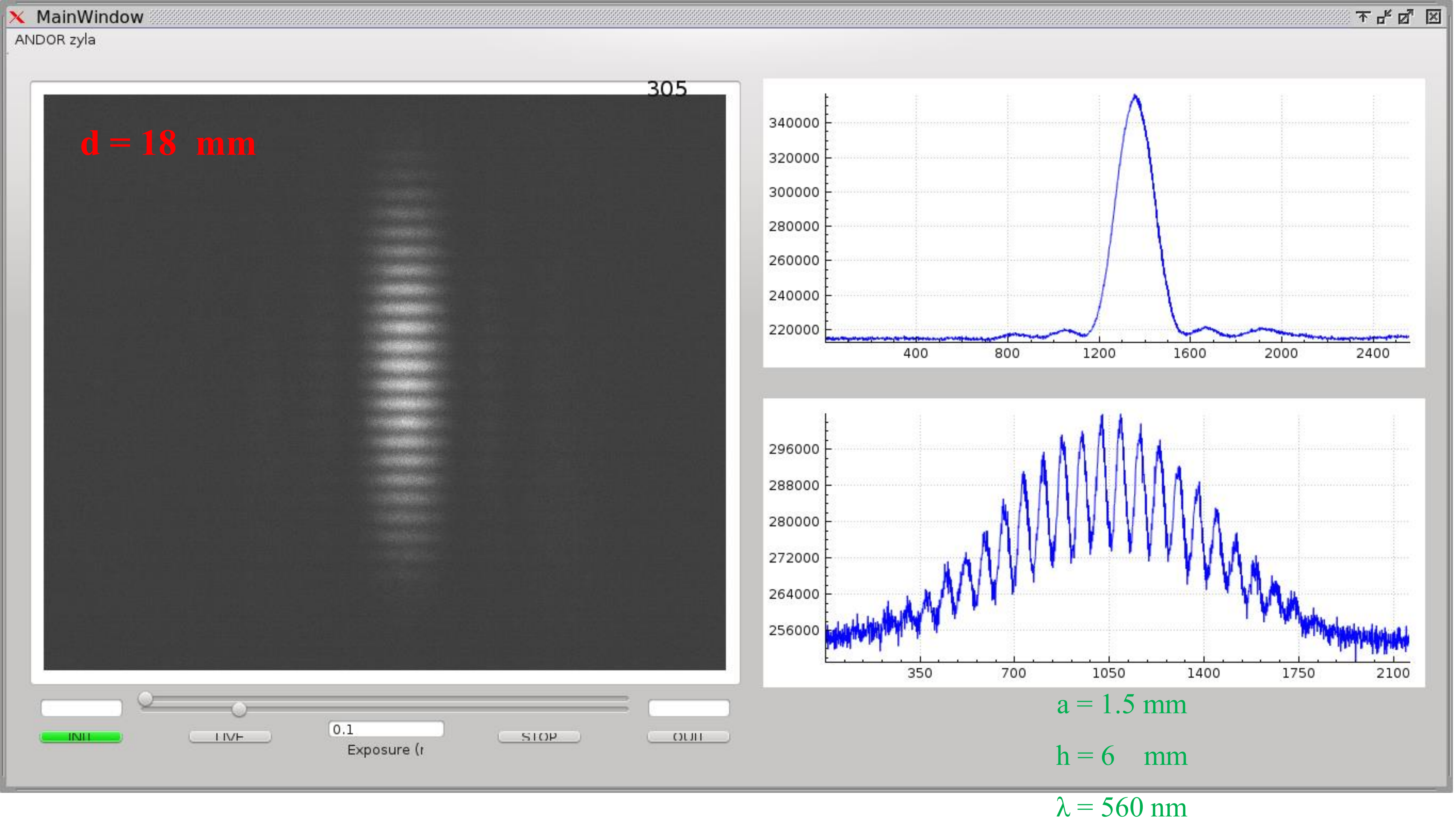
λ = 560 nm



a = 1.5 mm

h = 6 mm

λ = 560 nm


MainWindow
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d = 18 mm
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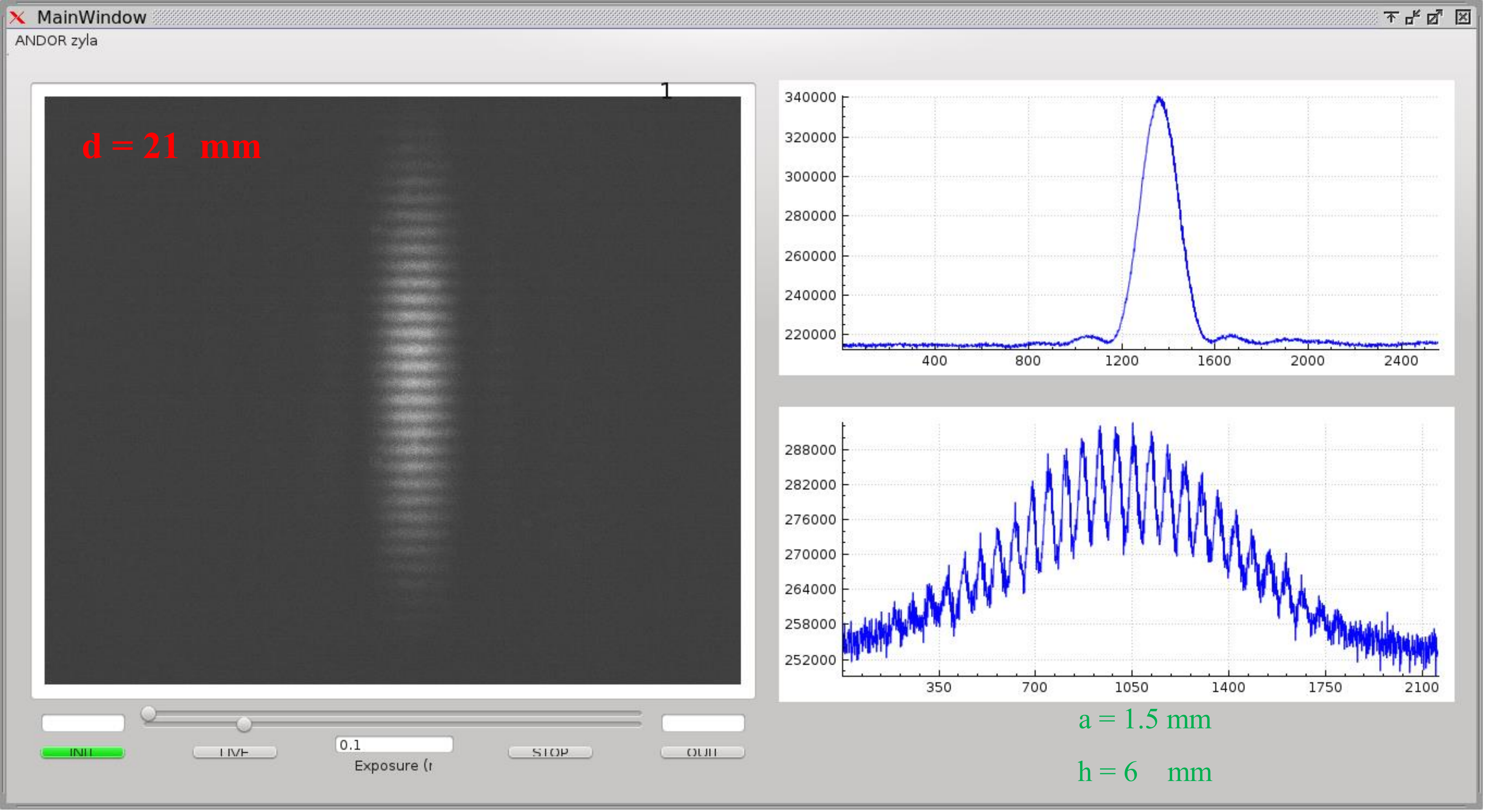
Exposure (r
a = 1.5 mm
h = 6 mm
λ = 560 nm



a = 1.5 mm

h = 6 mm

λ = 560 nm

Very rough estimation of the vertical beam size

At 6.5Tev

Visibility from the interferogram at D=21mm is roughly 0.24

Corresponding vertical beam size is about 200μm

SUMMARY

1.Van Cittert-Zernike's theorem introduced

Design of SR interferometer

Interferograms for vertical and horizontal is discussed.

2. The theoretical resolution and the practical errors are discussed.

We can measure the vertical beam size down to 3-4μm.

3. Recent trends in measurements for vertical and horizontal beam sizes in ALBA, SPEAR3, ASLS and ATF were reviewed.

4. As a fresh topic, SR interferometry for LHC was also introduced.

The beam size measurement with SR interferometry is now popular worldwide, not only for electron machines but also for high-energy the proton machines such as the LHC.

Thank you very much for your attention.