

EXTRACTING SUPERCONDUCTING PARAMETERS FROM SURFACE RESISTIVITY BY USING INSIDE TEMPERATURES OF SRF CAVITIES*

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Abstract

The surface resistance R_s of an RF superconductor depends on the surface temperature T_{in} , the residual resistance R_0 and various superconductor parameters, e.g. the energy gap and the electron mean free path. These parameters can be determined by measuring the quality factor Q_0 of a SRF cavity in helium-baths of different temperatures. The surface resistance can be computed from Q_0 for any cavity geometry, but it is not trivial to determine the temperature T_{in} of the surface when only the temperature of the helium bath is known.

Traditionally, it was approximated that the surface temperature on the inner surface of the cavity was the same as the temperature of the helium bath. This is a good approximation at small RF-fields on the surface, but to determine the field dependence of R_s , one cannot be restricted to small field losses.

Here we show the following: (1) How computer simulations can be used to determine the inside temperature T_{in} so that $R_s(T_{in})$ can then be used to extract the superconducting parameters. The computer code combines the well-known programs, the HEAT code and the SRIMP code. (2) How large an error is created when assuming the surface temperature is the same as the temperature of the helium bath? It turns out that this error can be more than 10% at high RF-fields in typical cases.

INTRODUCTION

The surface resistance R_s of superconductors under an RF-field is a function of the temperature on the RF surface. In the case of a standing-wave resonator, this is the inner surface of an SRF cavity. To determine R_s , the quality factor Q_0 has traditionally been measured in a series of the helium-bath temperatures at low RF-fields, usually around 3-5MV/m. It was typically approximated that the inner temperature was equal to the bath temperature at these low fields. By fitting superconductor parameters in an equation for $R_s(T_{in})$ to the data, quantities like the energy gap, the residual resistance, the electron mean free path *etc.* have been obtained.

Superconductivity theories [1-4] describe the performance of superconductors under low magnetic-field ($H_{RF} \ll H_C$), whether it can be extended to the high RF-fields is unclear [5, 6]. In [7], an effort has been made to empirically establish the relation between the superconducting parameters and the magnetic field by fitting $R_s(T_{in})$ curves to data that was obtained with high RF-fields. This was done up to 100-120mT on the

surface, which corresponds to 25-30MV/m accelerating gradient. For such high fields, it is no longer accurate to approximate the inner temperature by the bath temperature [8-10].

This paper demonstrates a method of computing the inner temperature from the bath temperature T_{bath} and the RF-field (the peak magnetic field H_p or the accelerating gradient E_{acc})¹, so that the correct relation between the surface resistance and the inner temperature can be established at each RF-field.

This paper simulates a 120°C baked case with the energy gap $1.51 \text{ (eV} \times 10^{-3}\text{)}$, corresponding to $\frac{\Delta}{kT_c} = 1.9$,² which error is the minimum case. But it turns out that in typical cases of the high gradient region, the superconductor parameters obtained by fitting $R_s(T_{in})$ can differ by more than 10% from those obtained by the traditional method of approximating T_{in} as the bath temperature. In the high-field region it can therefore often be important to apply the here presented, improved method.

THE FITTING METHODS OF $R_s(T)$ CURVES

The surface resistance of superconductors under RF-fields includes two parts, one is the BCS resistance R_{BCS} and the other part is the residual resistance R_0 shown in Eq. (1). Eq. (2) is the approximate expression of the surface resistance from the BCS theory [5]:

$$R_s = R_{BCS} + R_0, \quad (1)$$

$$R_{BCS} = A \left(\frac{1}{T_{in}} \right) f^2 \exp \left(- \frac{\Delta}{kT_{in}} \right). \quad (2)$$

Here, T_{in} is the temperature on inner surface, the factor A is a constant which is determined by material properties e.g. the electron mean free path l_e *etc.*; Δ is the energy gap; f is the resonant frequency of the cavity. Eq. (1) and (2) have been widely used for fitting $R_s(T_{bath})$ at low fields to extract the residual resistance, A , and Δ .

A better fitting method is based on the SRIMP code. The SRIMP code which incorporates the full BCS theory was written by Jurgen Halbritter [11, 12] for BCS-resistance calculations.

Traditionally, both fitting methods introduced above don't consider the temperature difference between the inside and the bath temperature. The temperature difference relates to the RF-power, hence it can be written as a function of the peak magnetic fields H_p and the surface resistance $R_s(T_{in})$ of the cavity. The ratio of the peak magnetic field H_p and the accelerating gradient E_{acc}

¹ The ratio of E_{acc} and H_p is a constant.

² $\frac{\Delta}{kT_c}$ is used to express the energy gap in this paper.

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is a constant, so we use E_{acc} to replace H_p , which is given in Eq. (3):

$$T_{in} - T_{bath} = f(R_s(T_{in}), E_{acc}). \quad (3)$$

THE HEAT-SRIMP FITTING METHODE

The Temperature Rise on the Interior Surface

To calculate the temperature on the inner surface from the bath temperature, the thermal feedback model has been adopted [5]. The HEAT code [13, 14] and the improved HEAT code which is called HEAT-and-SRIMP program [15] have been developed at Cornell University. Figure 1 compares the inner temperatures with the bath temperature versus E_{acc} of a 1.3GHz cavity at the different energy gaps in 2K helium bath. Cavity-test statistics from Cornell University indicate that $\frac{\Delta}{kT_c}$ of most cavities is around 1.8-1.9. The 120°C bake of cavities reduces the electron-mean-free-path l_e to achieve smaller R_{BCS} ; and it changes the surface of the cavities from the clean limits to the dirty limits [5]. Here we set $l_e=300\text{\AA}$ to represent the baked case shown in Figure 1. The calculation clearly suggests that the temperature increase is at least 0.1K at the accelerating gradient 38MV/m in the baked case with $\frac{\Delta}{kT_c}$ 1.7-1.9, which we define as the typical cases.

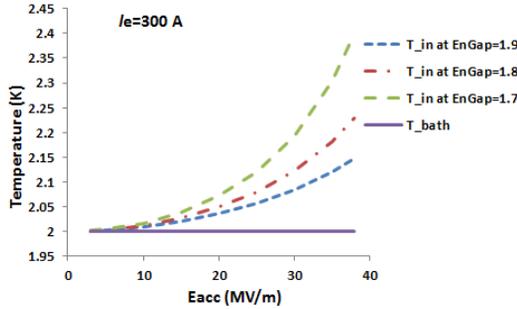


Figure 1: Temperature increases on the inner surface of a 1.3GHz baked-cavity with the energy gap $\frac{\Delta}{kT_c}$ varying from 1.7 to 1.9.

The inner-temperature rise causes the BCS resistance to grow, when E_{acc} increases. This growth of the BCS resistance is depicted in Figure 2,

$$\Delta R_{BCS} = R_{BCS}(T_{in}) - R_{BCS}(T_{bath}). \quad (4)$$

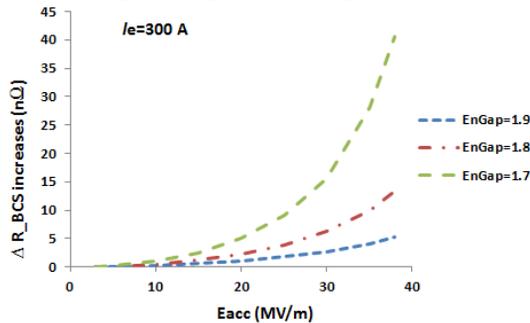


Figure 2: The BCS resistance increase of a 1.3GHz baked-cavity with $\frac{\Delta}{kT_c}$ varying from 1.7 to 1.9.

The HEAT-SRIMP Fitting

We developed the HEAT-SRIMP fitting program which combines the SRIMP code, the HEAT code, and a least square fitting program together.

The HEAT code solves the heat flow equations numerically from the interior wall to the exterior wall of a cavity at an accelerating gradient; and outputs the temperature distribution through the wall [13]. The HEAT code adopts Koechlin and Bonin expression [16] to calculate niobium thermal conductivity. The Kapitza thermal conductivity is calculated from experimental data fitting [17].

In an $R_s(T)$ fitting, the energy gap $\frac{\Delta}{kT_c}$, the electron-mean-free-path l_e , and the residual resistance R_0 are selected to be fitted in most cases. Therefore the form of the surface resistance is possible to be written as Eq. (5):

$$R_{s_calc.} = f(T_{in}, P_{fix}, P_{fit}). \quad (5)$$

Here the constant P_{fix} represents frequency f_0 , and the non-fitting BCS parameters; P_{fit} is the fitting parameters. From Eq. (3), the bath temperature can be expressed as a function of the inner temperature, the surface resistance, and the accelerating gradient in Eq. (6):

$$T_{bath} = g(T_{in}, R_s(T_{in}), E_{acc}, P_T). \quad (6)$$

Here the constant P_T describes the thermal-related parameters. Here T_{in} cannot be expressed as a single formula of T_{bath} , because R_s is T_{in} dependent as well. So we use an iterative method to solve Eq. (6) to obtain T_{in} , when cavity reaches thermal equilibrium. Then we can put T_{in} into Eq. (5) to obtain $R_{s_calc.}$ by Eq. (7):

$$R_{s_calc.}(T_{bath}, E_{acc}, P_{fit}) = f(T_{bath}, E_{acc}, P_T, P_{fix}, P_{fit}). \quad (7)$$

The surface resistance from measurements is an array at a series of the bath temperatures T_0-T_n as well as the accelerating gradients E_0-E_m , which is described in Eq. (8):

$$R_{s_data}(T_{bath}, E_{acc}) = \begin{bmatrix} R_{00} & \cdots & R_{0m} \\ \vdots & \ddots & \vdots \\ R_{n0} & \cdots & R_{nm} \end{bmatrix} \begin{matrix} @T_0 \\ \vdots \\ @T_n \\ @E_0 \\ \cdots \\ @E_m \end{matrix} \quad (8)$$

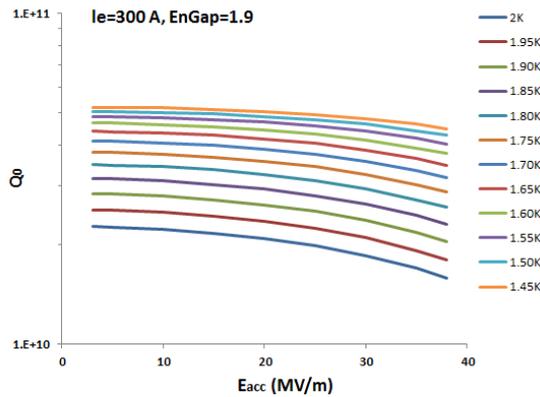
The fitting program takes every $R_s(T_{bath})$ curves at different E_{acc} ; compares Eq. (7) and Eq. (8); and tunes the parameter P_{fit} to achieve the minimum fitting error by the least squares method. The fitting error RSS is given by Eq. (9):

$$RSS = \sum_{i=1}^n \left(R_{s_calc.}^i(T_{bath}^i, E_{acc}, P_{fit}) - R_{s_data}^i(T_{bath}^i, E_{acc}) \right)^2. \quad (9)$$

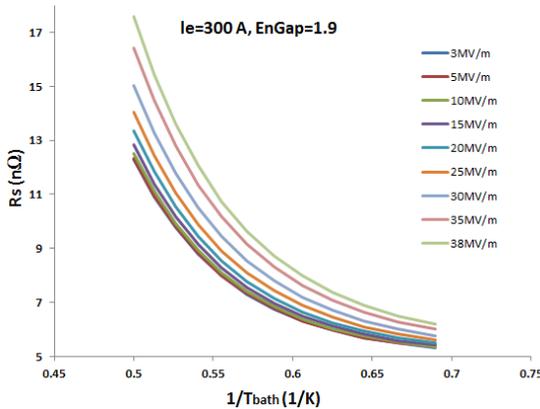
The HEAT code and the SRIMP code are written in C++ and the least squares fitting program is written in Matlab. The Matlab program calls the C++ program as a function.

COMPARISON BETWEEN HEAT-SRIMP FITTING AND SRIMP FITTING

This section will give the comparison between the HEAT-SRIMP fitting and the traditional fitting. Here we use the HEAT-and-SRIMP program to generate the $Q_0(E_{acc})$ curves (Figure 3(a)) of a 1.3GHz 120°C baked-cavity ($l_e=300\text{\AA}$) from temperature 1.45-2K; the energy gap was set 1.9, the residual resistance was set $5\text{n}\Omega$. The correct fitting-method is ought to retrieve the energy gap and the residual resistance back by fitting the $R_s(T_{bath})$ curves (Figure 3(b)) which are converted from The $Q_0(E_{acc})$ curves.

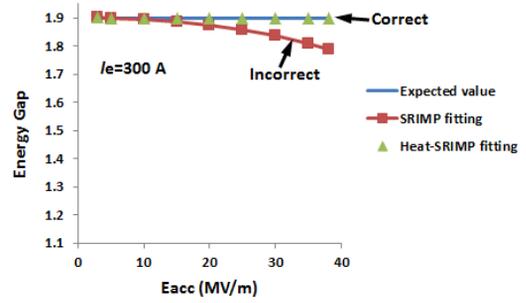


(a) The $Q_0(E_{acc})$ curves

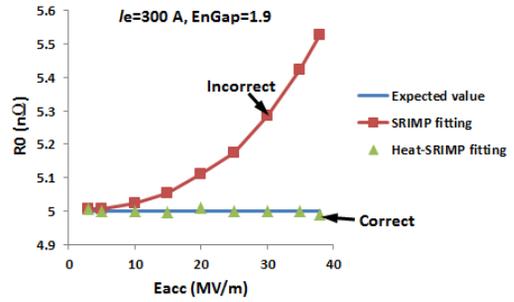


(b) The $R_s(T_{bath})$ curves

Figure 3: The $Q_0(E_{acc})$ and the $R_s(T_{bath})$ curves of a 1.3GHz baked-cavity with the energy gap 1.9 and the residual resistance $5\text{n}\Omega$.



(a) The fitting results of the energy gap,



(b) The fitting results of the residual resistance.

Figure 4: The HEAT-SRIMP fitting and the traditional fitting method. exhibits the fitting results of the residual resistance.

It's very interesting to compare the two fitting methods. The results are shown in Figure 4(a), and (b). The results indicate the SRIMP fitting tends to give the energy gap value smaller than the correct value. The fitting error of the energy gap $\frac{\Delta}{kT_c} = 1.9$ is 0.11 at 38 MV/m. Figure 4(b): The SRIMP fitting results gave a larger residual resistance value than the expected value; the maximum error of the cases is around $0.5\text{n}\Omega$ shown in the figure. It is very clear that the HEAT-SRIMP fitting extracts the correct values of the energy gap and the residual resistance.

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