ON NONLINEAR DYNAMICS OF A SHEET ELECTRON BEAM

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Abstract

The dynamics of a charged particle beam is studed in the frame of Vlasov theory in the case of non-hollow nonuniform spatial distribution. The nonstationary model based on a beam behaviour description by means of kinetic function dependent on the integrals of the particle motion is presented. The cases of strong and weak nonlinearities are considered.The results of numerical and analitical calculations are discussed.

INTRODUCTION

For a lot of accelerator physics problems the investigation of a charged particle beam dynamics by means of the simple mathematical models is attractive tool because it allows to obtain the knowledge of the beam behaviour with most physical generality. Frequently these models are involved into programm packages which calculate the beam envelopes or particle trajectories. First such a model was proposed by I. M. Kapchinsky and V. V. Vladimirsky (KV-model) in 1959 [1]. This model allows to describe both the charge-dominated and emittance-dominated beams in the case of quasistationary beam propagation. KV-model gives a full kinetic beam description due to the assumption that the kinetic distribution function is a function of particle motion integral and hence automatically satisfies to Vlasov equation. Yarkovoy's model [2] may be considered as a development of KV-model. It allows to describe nonstationary 2D-beams without axial symmetry. Subsequent development of 2D- and 3D-models is presented, for instance, in papers [3-6]. All the models mentioned above describe the beam with linear own forces that corresponds to uniform charge density in a transverse beam cross-section in the case of continuous beam or in a bunch volume in the case of bunched beam. The models that take into account the nonuniform charge density were proposed, as example, in [7-11]. In papers [9-11] only self-similar beam oscillations are under consideration, in contrast to papers [7, 8], where the particle distributions are not stationary.

The present paper studies a charged particle beam with parabolic current density profile. The aim is to predict a behavior of the beam envelope with time. The model is used which doesn't require the particle distribution to be stationary.

MODEL DESCRIPTION AND NUMERICAL CALCULATIONS

Let us consider, for example, a quasistationary relativistic intense electron beam, the own charge of the beam being neutralized. For the mathematical simplicity let us suppose the sheet geometry of the beam. Since as a rule the beam lifetime is significantly more than the time of transition processes in the beam one can describe the beam behaviour by means of a smooth function R(z), where R(z) – the beam tranverse size, z – longitudinal coordinate. In the case of the continuos beam with uniform charge density KV-invariant [1] may be written as:

$$I = (R'x - Rx')^2 + \frac{\varepsilon_0 x^2}{R^2} \tag{1}$$

Where x' is the derivative of x with respect to z, R' – the derivative of R with respect to z, ε_0 - the beam rms emittance squared, x – the transverse coordinate.

Let us consider the beam with the charge density distribution n(x,z), which falls down with x as parabola and reaches zero at the beam boundary. It is a good approximation for the density distribution of the real non-hollow continuous beam:

$$n(\mathrm{x,z}) = a_0(z) - a_2(z)x^2 \tag{2}$$

Hence we obtain that the particle transverse motion is described by the equation

$$x'' = -\alpha_1(z)x + \alpha_3(z)x^3. \tag{3}$$

Here $\alpha_1(z) = \mathrm{k}\,a_0(z), \alpha_3(z) = \mathrm{k}a_2(z)/3$, $k = 4\pi e^2/\mathrm{mc}^2$

For equation (3) the integral I as analogue of KV-invariant (1) may be constructed with the help of the next relation:

$$x'(\mathrm{x,z,I}) = \sum_{k=0}^{\infty} a_k(z,I)x^k \pm \left(\sum_{k=0}^{\infty} b_k(z,I)x^k\right)^{1/2} \tag{4}$$

Let us substitute (4) in (3) and neglect all summands with 5[th] power and higher. Then let us introduce a kinetic distribution function as:

$$f(I) = 2\mathrm{n}_0\sigma(1 - I).$$

Here n_0 – the time-independent normalization constant, σ – Heaviside function. The distribution function differed from Maxwellian may be used in our task because charged particle beam is not thermodynamically equilibrium system. So one can obtain for the beam charge density:

$$n = (n_0/u)(1 - \varepsilon_0^2 x^2 / 2u^2)\sigma(R - |x|), \quad (5)$$

Where

$$R = u\sqrt{2/\varepsilon_0}(1 + \sqrt{1 + (\varepsilon_0' u^2)' u^2 / 3\varepsilon_0^2})^{-1/2} \quad (6)$$

Here function u is the solution of equation

$$u'' = -\alpha_1(z)u + \varepsilon_0(z)/u^3 \quad (7)$$

The whole current conservation should be taken into account, so for dimensionless beam radius and rms emittance the next equation system may be obtained:

$$(\beta'\alpha^2)' = 12(1-\beta)/\alpha^2$$
$$\alpha'' + 1 = \beta/\alpha^3 \quad (8)$$

Here α and β are dimensionless rms radius and emittance respectively,

$$\alpha = u(l_0/l_1)^{2/3}, \quad \beta = \varepsilon_0 l_{02}, \quad l_1 = c/\omega_p,$$
$$l_0 = J/2evn_0 L,$$

J is the whole beam current, L - the width of the beam, ω_p is the plasma frequency, corresponding to the density value n_0, v is the beam velocity.

In (8) time-dependence of rms emittance was obtained in self-consistent manner, because function $f(I)$ automatically satisfies to Vlasov equation, and relation (5) for the density, i.e. for zero moment of the distribution function, has a parabolic dependence from x.

From the system (8) one can find the stationary equilibrium state for the beam. This state corresponds to the values $\alpha = \beta = 1$.

If we consider the case of small deviation of the beam characteristics from equilibrium ones, the beam behavior may be described by means of equations

$$q'' = p - 3q \quad p'' = -12p \quad (9)$$

p, q – small deviations from equilirium values of dimensionless rms emittance and dimensionless envelope radius respectively.

System (9) describes the small transverse oscillations corresponding to the case of the absence of rms emittance growth – the phase curve is finite.

The beam radius that corresonds to equilibrium solution is

$$R = l_0(c/\omega_p l_0)^{2/3}$$

and effective emittance value corresponding to the equilibrium is

$$\varepsilon_0 = \eta / l_0^2 .$$

Here η is the normalization constant.

The results obtained are valid under condition

$$|\alpha'| < |\beta|$$

The systems (8) and (9) are solved numerically by means of Runge-Kutta-Feldberg method of 4[th] order. The results are presented at Figures 1 and 2 that indicate the envelope oscillation build-up possibility.

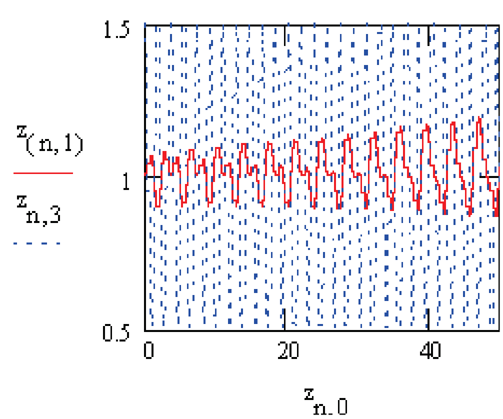

Figure 1: Dimensionless rms radius $z_{n,1}$ vs dimensionless longitudinal coordinate $z_{n,0}$ (case of significant deviation of the beam parameters from equilibrium ones)

From Figures 1 and 2 it is evident that in the case of strong nonlinearity, i.e. in the case of significant deviation of the beam parameters from the equilibrium ones, the essential growth of rms emittance is observed.

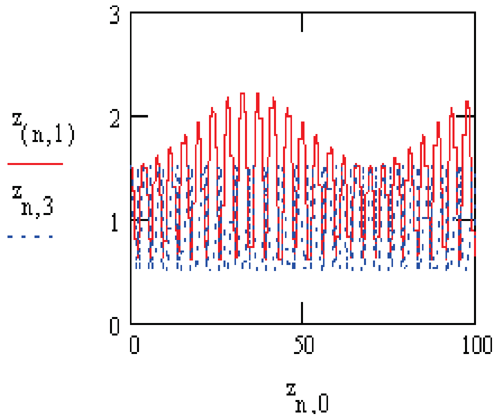

Figure 2: Dimensionless rms radius $z_{n,1}$ vs dimensionless longitudinal coordinate $z_{n,0}$ (case of small deviation of the beam parameters from equilibrium ones).

The self-consistent systems (8) and (9) describing the beam rms parameter oscillations correspond to 4th orde envelope equation. The coefficient near of 1st radius derivative in both cases is not equal to zero unlike the case of envelope equation for the beam with uniform density. Systems (8) and (9) describe the situation when the current distribution is not stationary during the time of one particle flight, and oscillations inside the beam are not self-similar, so the model describes the most general case of the continuos non-hollow beam behaviour with nonuniform charge density.

CONCLUSIONS

Nonlinear dynamics of the charged particle beam is studied. Transverse charge current nonuniformity is shown to lead to essentially nonlinear particle transverse oscillations. Depending on nonlinearity power the growth of effective emittance can be observed at a time corresponding to about a quarter of the plasma wavelength. The exact beam parameters exist corresponding to the case when the effective emittance and the beam transverse size do not grow. In the case of small deviation of the initial beam parameters from equilibrium ones the phase curves are finite, the effective emittance growth is absent. The results obtained are valid under the condition $l_0 > c/\omega_p$, i.e. when minimum system linear size is more than maximum beam plasma wavelength.

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