

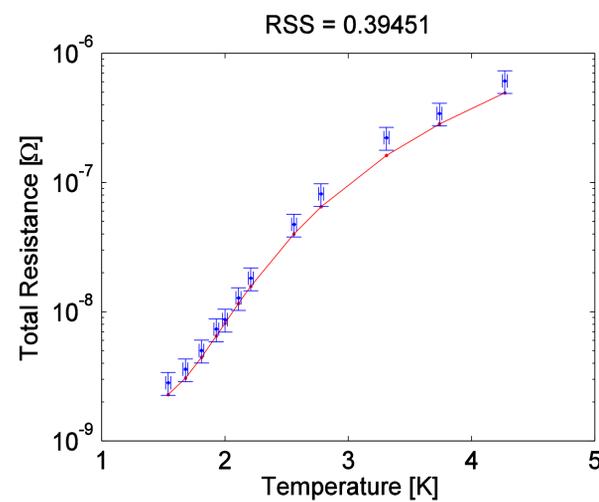
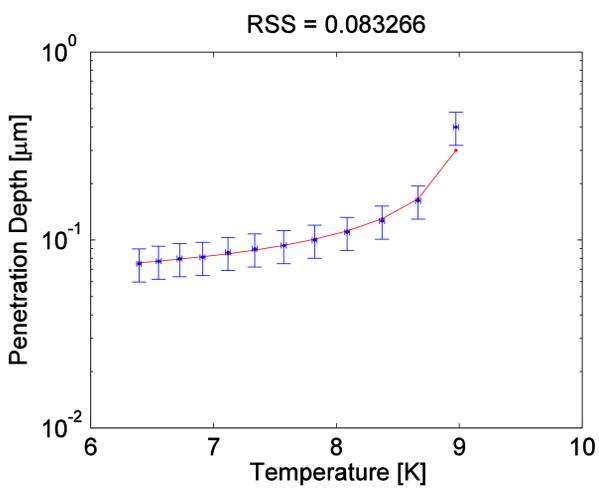
# Error Analysis in SRF Material Parameter Calculations

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## Introduction

A reliable method of material parameter extraction is required to study both new materials on the surface of SRF cavities, such as Nb<sub>3</sub>Sn, and new methods of cavity preparation, such as nitrogen-doping. A computer program based on BCS theory was used to compute surface resistance or penetration depth, given an input array of temperatures for given values of the parameters of interest, including energy gap and mean free path. To study random error of the fit, Monte Carlo simulations were performed of input data with added noise; for systematic error, contour plots of normalized residual sum of squares (RSS) of the polymorphic fit on inputted data were generated. This study aims to show how given methods of obtaining parameters such as energy gap and mean free path minimize their associated uncertainty, and to present a comparison of how the different parameters of interest affect the uncertainty of the fit.

## SRIMP Fitting



Top: Sample fit of nitrogen-doped Nb using  $f$  vs.  $T$  data. Bottom: Sample fit of nitrogen-doped Nb using  $Q$  vs.  $T$  data.

## Discussion

- $\lambda$  sensitive to  $l$  but not  $\frac{\Delta}{k_B T_c}$ , so optimal RSS range in  $f$  vs.  $T$  contour plots will span horizontal axis
- $R_s$  sensitive to  $\frac{\Delta}{k_B T_c}$ , but not  $l$ , so optimal RSS range in  $Q$  vs.  $T$  contour plots will span vertical axis
- Combining results yields tighter range of optimal RSS for both parameters

## Conclusion

- Fitted values and systematic error extracted from contour plots of RSS
- Random error found using Monte Carlo method
- Promising new method for parameter extraction

## Results

Parameter	Fitted Value	Systematic Error	Random Error
$\frac{\Delta}{k_B T_c}$	1.97	$\pm 0.05$	$\pm 0.01$
$l$ [nm]	32	$\pm 30$	$\pm 2$
$R_0$ [nΩ]	1.75	$\pm 0.75$	$\pm 0.18$

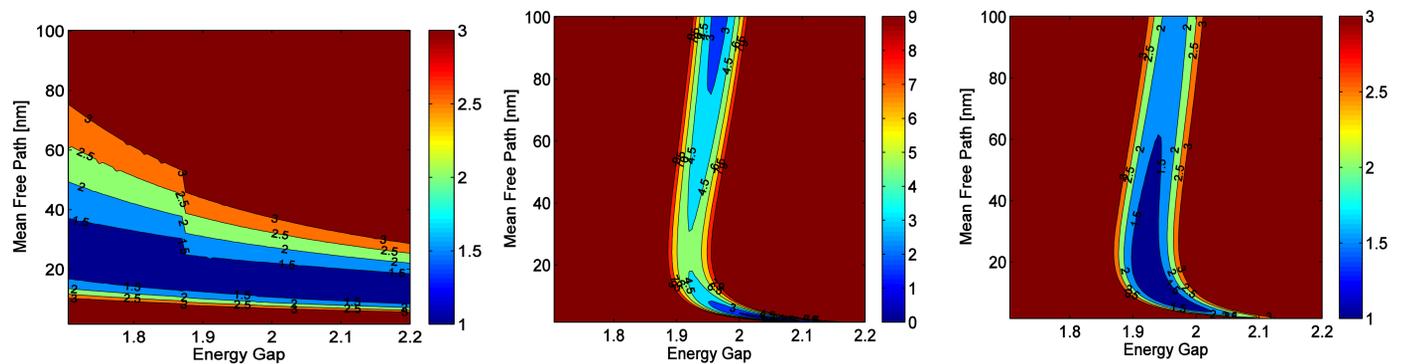
Parameter	Fitted Value	Systematic Error	Random Error
$\frac{\Delta}{k_B T_c}$	2.47	$\pm 0.12$	$\pm 0.02$
$l$ [nm]	3.25	$\pm 0.25$	$\pm 0.12$
$R_0$ [nΩ]	9.25	$\pm 1.25$	$\pm 0.06$

Top: Extracted parameters and associated uncertainty for nitrogen-doped Nb. Bottom: Extracted parameters and associated uncertainty for Nb<sub>3</sub>Sn.

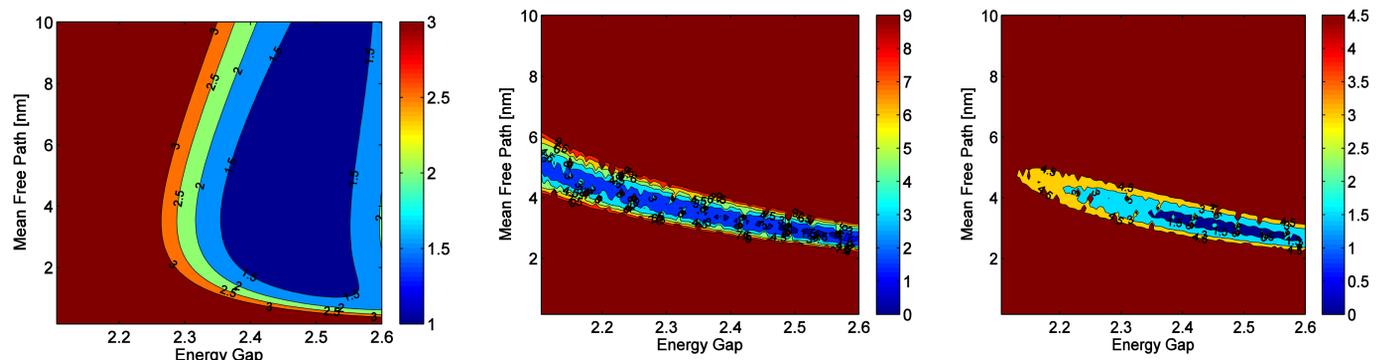
## Parameters of Interest

- Start out with 7 parameters to fit:
  - Critical Temperature ( $T_c$ ) → measured
  - Energy Gap ( $\frac{\Delta}{k_B T_c}$ )
  - Intrinsic Coherence Length ( $\xi_0$ ) → extract value from literature
  - London Penetration Depth ( $\lambda_L$ ) → extract value from literature
  - Mean Free Path ( $l$ )
  - Residual Resistance ( $R_0$ ) → extract from  $Q$  vs.  $T$  data
  - Pen. Depth at  $f_0$  ( $\lambda_0$ ) → from  $f$  vs.  $T$  data
- Further improve the fit by synthesizing the  $f$  vs.  $T$  and  $Q$  vs.  $T$  data sets (combining two 3-parameter fits rather than one 4-parameter fit)
- Reducing 7-parameter fit to a combination of 3-parameter fits significantly reduces associated uncertainty

## Systematic Error



Left: Contour plot of normalized RSS for  $f$  vs.  $T$  data for nitrogen-doped Nb. Middle: Contour plot of normalized RSS for  $Q$  vs.  $T$  data for nitrogen-doped Nb. Right: Combined contour plot of normalized RSS for nitrogen-doped Nb.



Left: Contour plot of normalized RSS for  $f$  vs.  $T$  data for Nb<sub>3</sub>Sn. Middle: contour plot of normalized RSS for  $Q$  vs.  $T$  data for Nb<sub>3</sub>Sn. Right: Combined contour plot of normalized RSS for Nb<sub>3</sub>Sn.