

Kenichi YANAGIDA, Shisuke SUZUKI, and Hirofumi HANAKI
Japan Synchrotron Radiation Research Institute / SPring-8

Purpose

- To measure emittances and Twiss parameters easily like as daily (routine) operation.
 - Using profile monitors is difficult to gather all signal elements for calculating **r.m.s. beam size**, because of signal saturation or reduction to noise level.
- => Use six-electrode BPMS and measure second-order relative moment.

Note:

Second-order relative moment P_{g2} is difference of square of **r.m.s. beam size**, $P_{g2} = \sigma_H^2 - \sigma_V^2$, and easily derived from signal difference.

- Higher-order moment correction for signal difference.
- Effective aperture radius.
- Entire Calibration.
- Emittance measurement by Q-scan method.

- Correction : To minimize moment calculation error.

Roughly $-1 < \Delta P_{g2} < 1$ [mm²] ($-\mathbf{X} < P_1, Q_1 < \mathbf{X}$ [mm])

- Definition of mth-order signal difference : $\mathbf{C}_m$, $\mathbf{S}_m$.

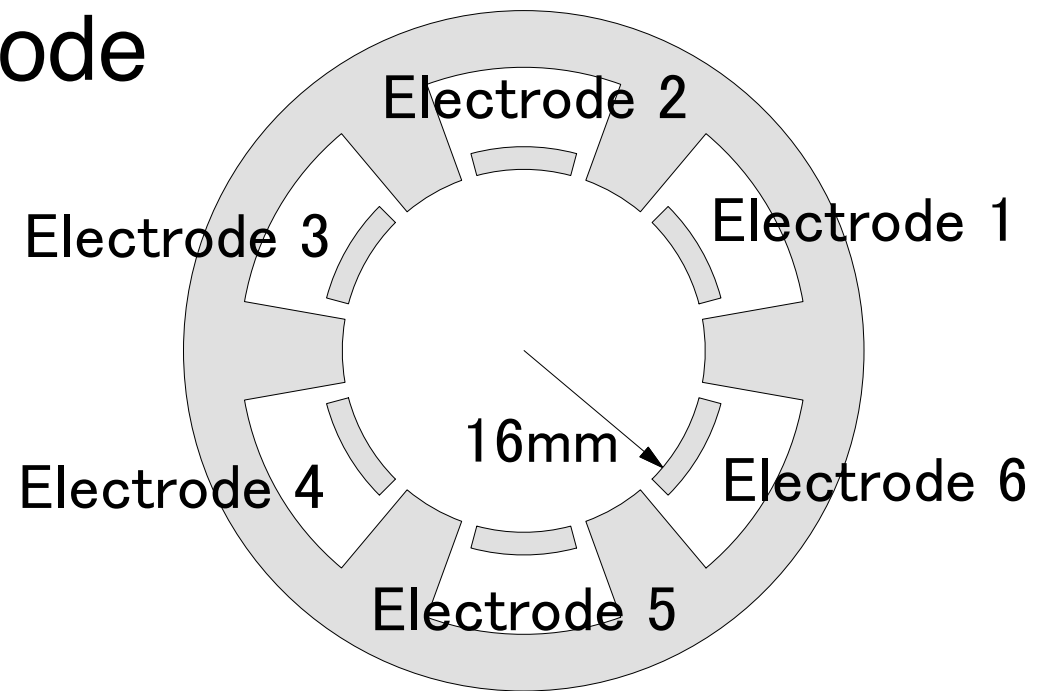
V_d : Voltage from dth-electrode

$$\mathbf{C}_1 = \frac{V_1 - V_3 - V_4 + V_6}{V_1 + V_3 + V_4 + V_6}$$

For horizontal position $\mathbf{P}_1$

$$\mathbf{S}_1 = \frac{V_1 + V_3 - V_4 - V_6}{V_1 + V_3 + V_4 + V_6}$$

For vertical position $\mathbf{Q}_1$



Higer-order moment correction for signal difference

- Without correction (2012)

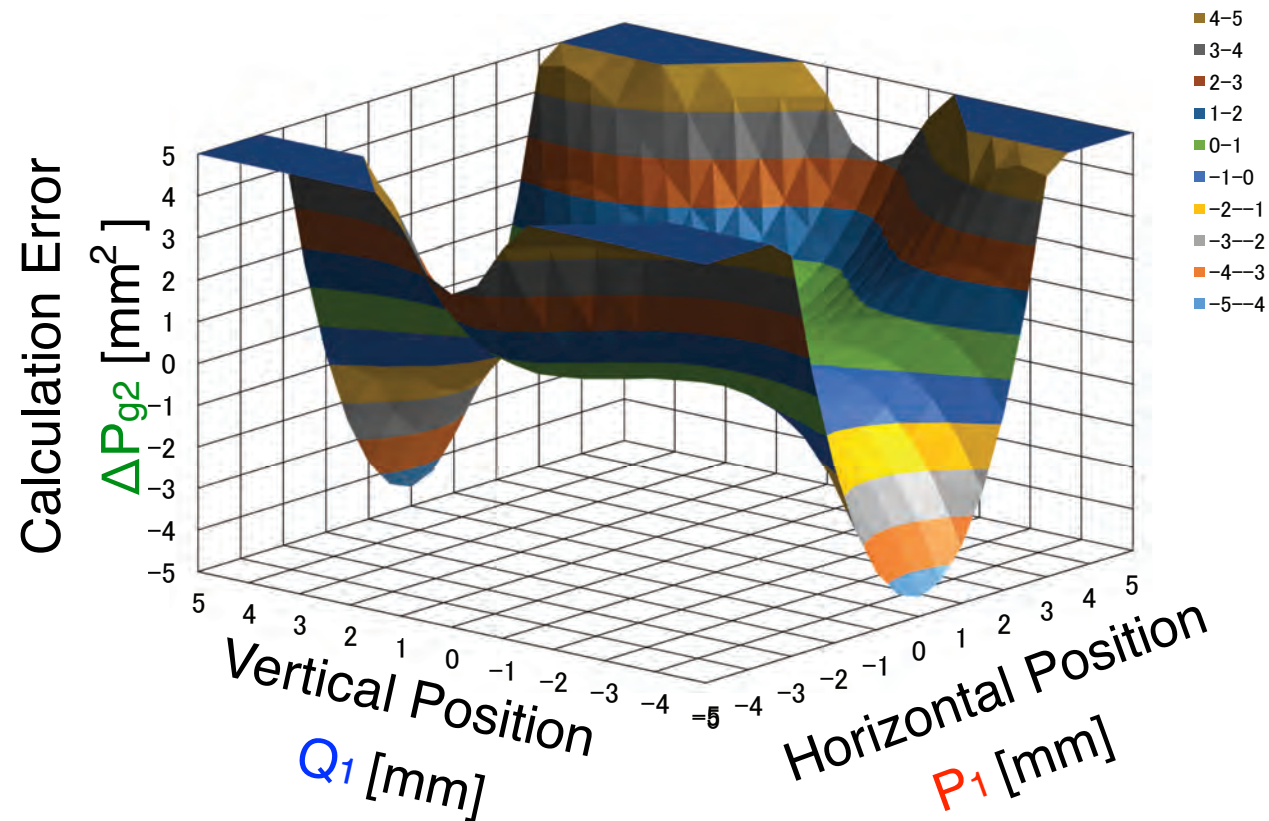
$$C_1 \approx \frac{2}{R_{C1P1}} P_1$$

$$S_1 \approx \frac{2}{R_{S1Q1}} Q_1$$

Effective aperture radius

R_{C1P1} : 18.688 [mm]

R_{S1Q1} : 32.368 [mm]



Roughly $-1 < \Delta P_{g2} < 1$ [mm²] ($-1 < P_1, Q_1 < 1$ [mm])

Higer-order moment correction for signal difference

- With up to third-order moments (2013)

nth-order absolute moments : P_n, Q_n .

$$C_1 \approx \frac{2P_1}{R_{C1P1}} \left(1 - \frac{2P_2}{R_{C1P2}^2} \right) + \frac{2P_3}{R_{C1P3}^3}$$

$$S_1 \approx \frac{2Q_1}{R_{S1Q1}} \left(1 - \frac{2P_2}{R_{S1P2}^2} \right) + \frac{2Q_3}{R_{S1Q3}^3}$$

Effective aperture radius

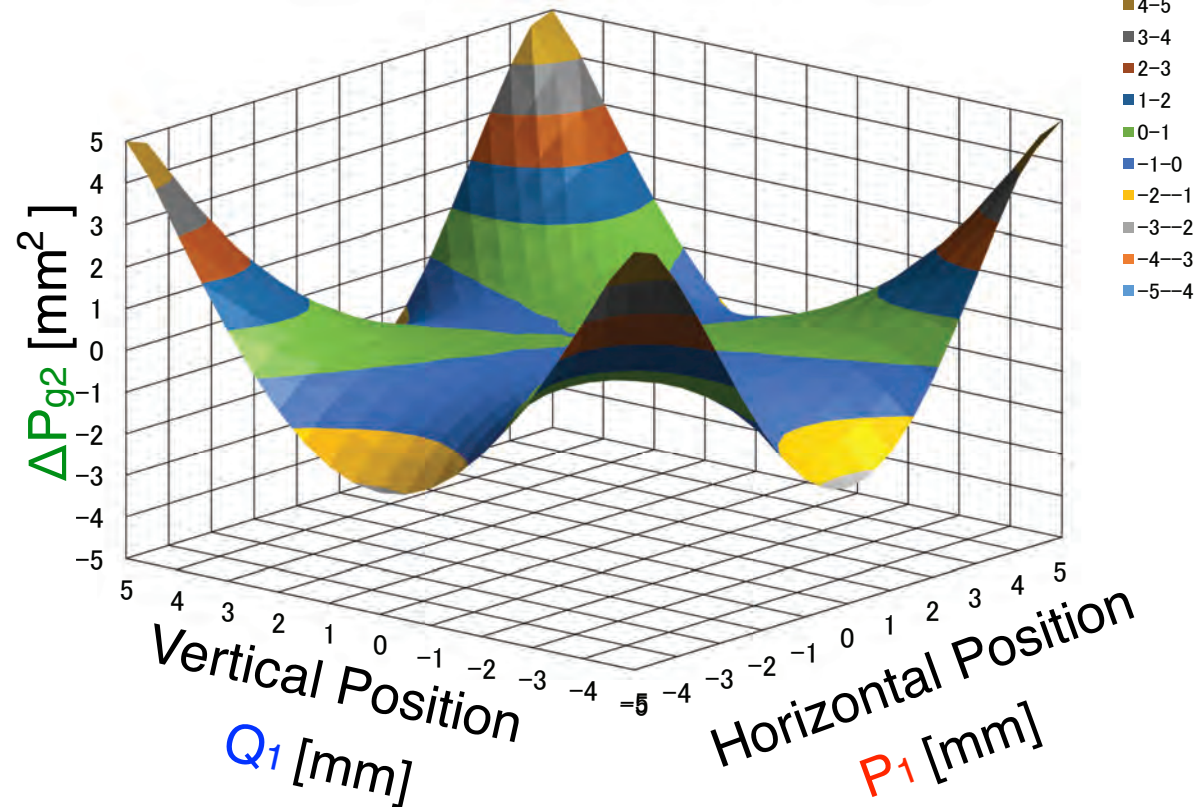
R_{C1P2} : 23.155 [mm]

R_{C1P3} : infinity [mm]

R_{S1P2} : 23.155 [mm]

R_{S1Q3} : 16.570 [mm]

Calculation Error



Roughly $-1 < \Delta P_{g2} < 1$ [mm²] ($-3 < P_1, Q_1 < 3$ [mm])

Higer-order moment correction for signal difference

- With up to fifth-order moments (2014)

$$C_1 \approx \frac{2P_1}{R_{C1P1}} \left(1 - \frac{2P_2}{R_{C1P2}^2} + \frac{4P_2^2}{R_{C1P2}^4} + \frac{2P_4}{R_{C1P4}^4} \right) + \frac{2P_3}{R_{C1P3}^3} - \frac{2P_5}{R_{C1P5}^5}$$

$$S_1 \approx \frac{2Q_1}{R_{S1Q1}} \left(1 - \frac{2P_2}{R_{S1P2}^2} + \frac{4P_2^2}{R_{S1P2}^4} + \frac{2P_4}{R_{S1P4}^4} \right) + \frac{2Q_3}{R_{S1Q3}^3} + \frac{2Q_5}{R_{S1Q5}^5}$$

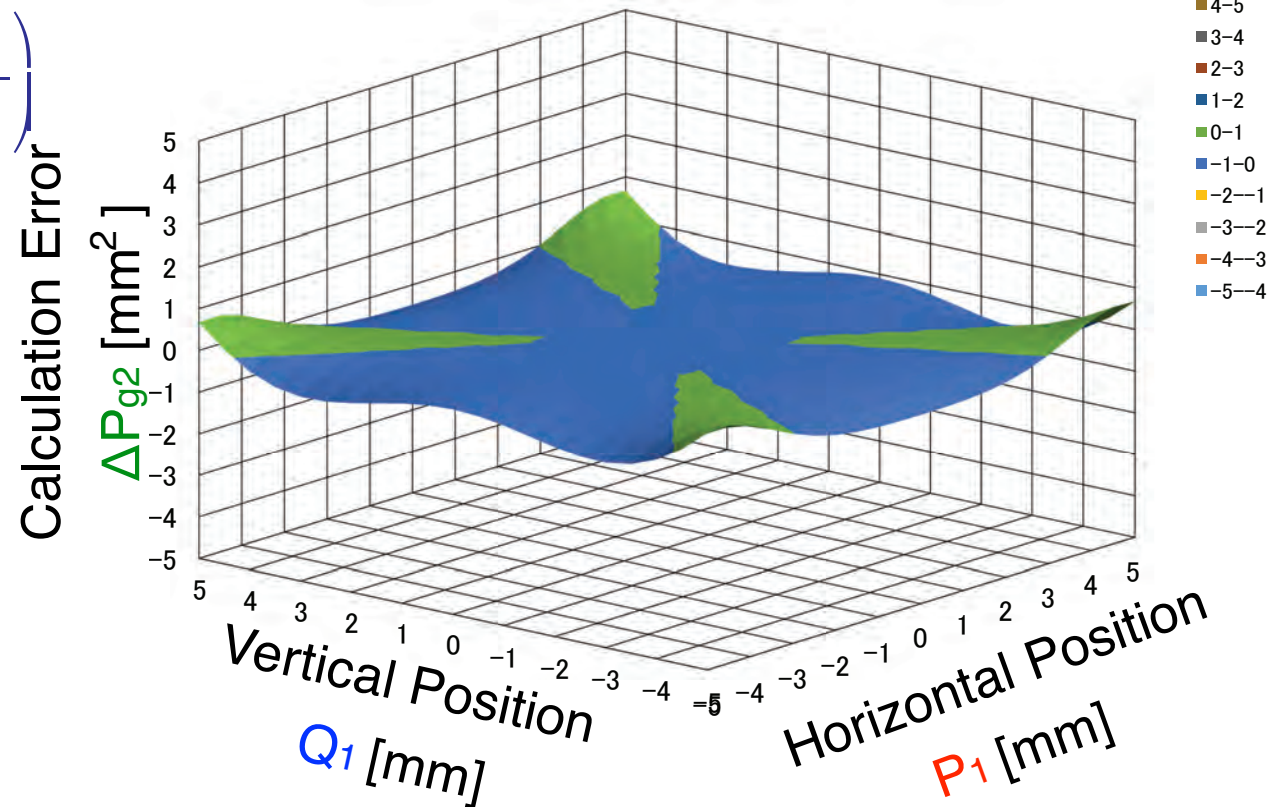
Effective aperture radius

R_{C1P4} : 19.953 [mm]

R_{C1P5} : 17.499 [mm]

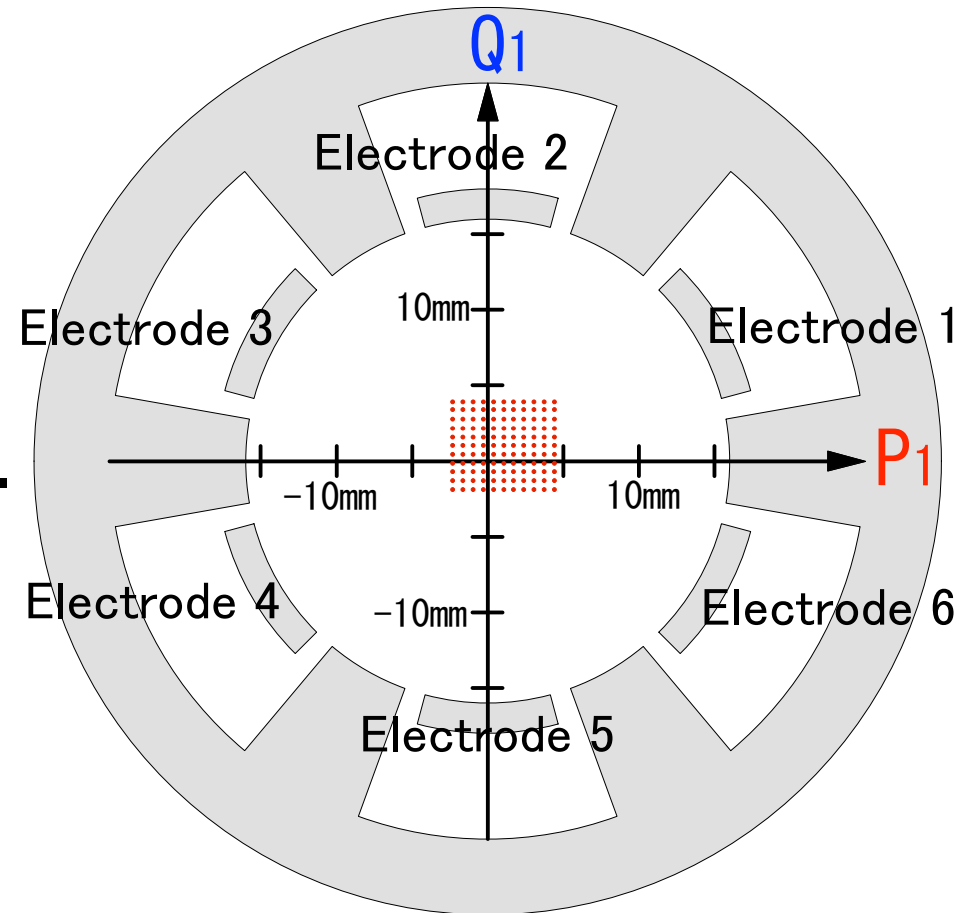
R_{S1P4} : 19.953 [mm]

R_{S1Q5} : 19.531 [mm]



Roughly $-1 < \Delta P_{g2} < 1$ [mm²] ($-5 < P_1, Q_1 < 5$ [mm])

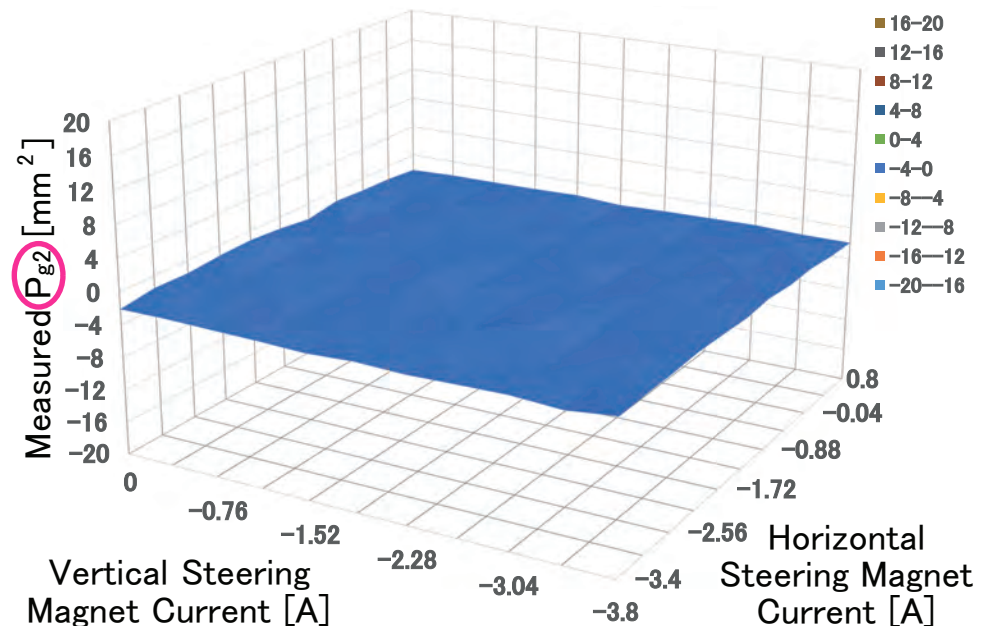
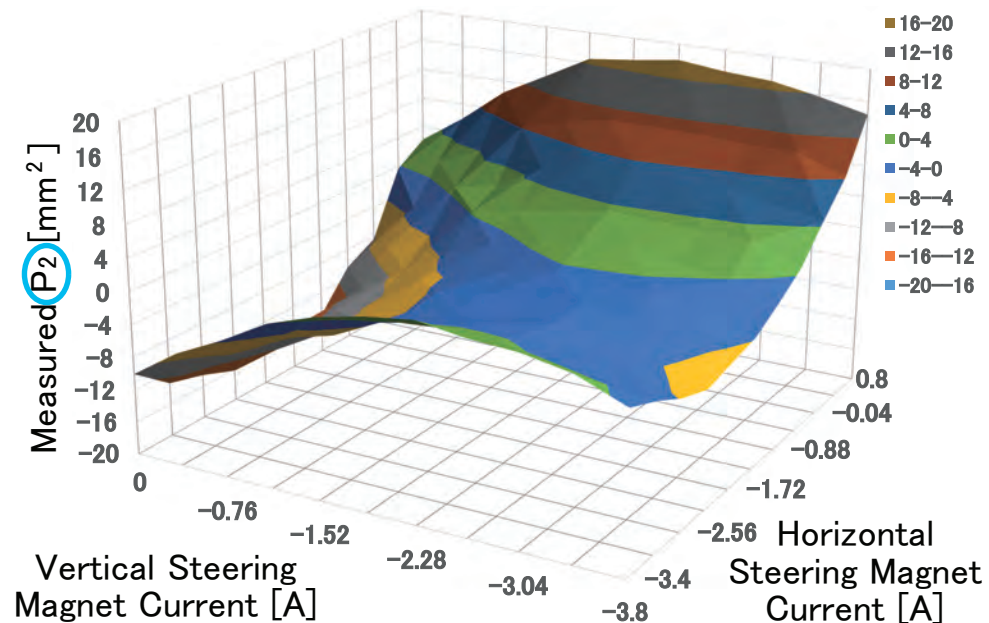
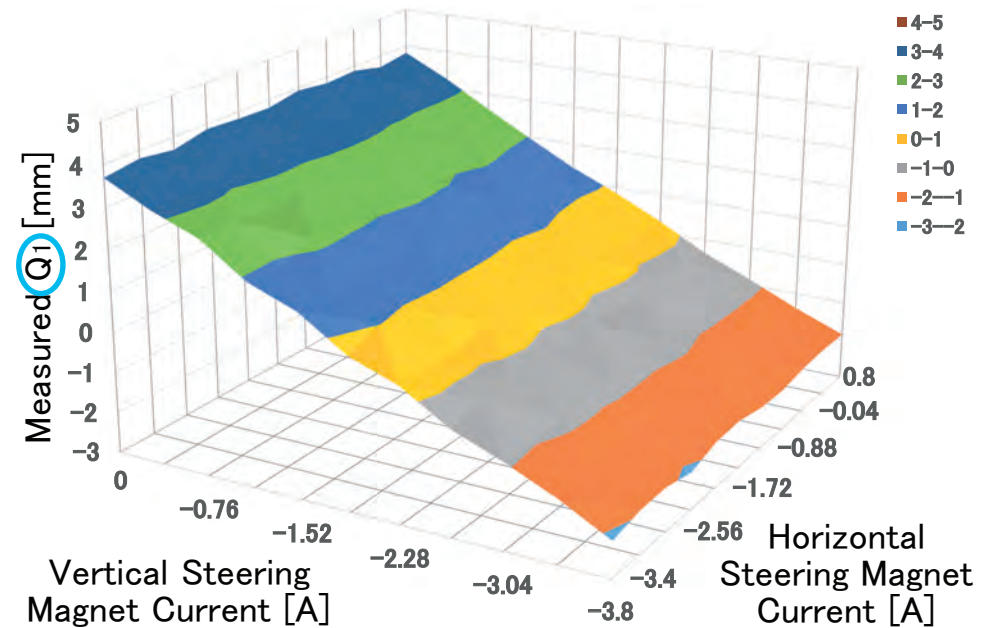
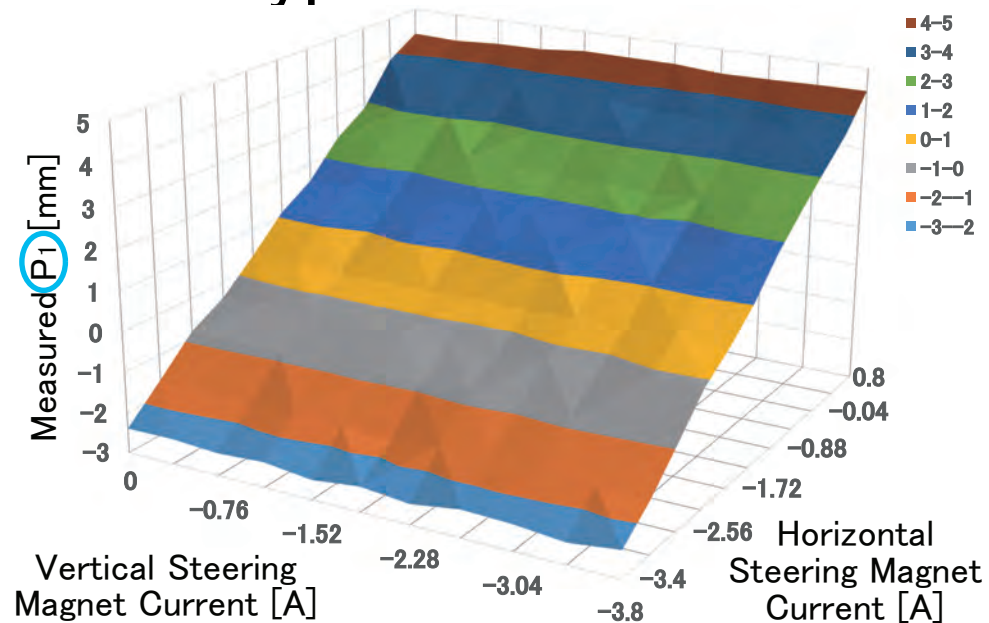
- The concept was presented at LINAC2012.
- Absolute moment was **changed** by steering magnet.
- Relative moment was **not** **changed** by steering magnet.
- We deduce the relative attenuation factors between electrode channels.



Red circles :
Scanned beam
positions P_1 , Q_1

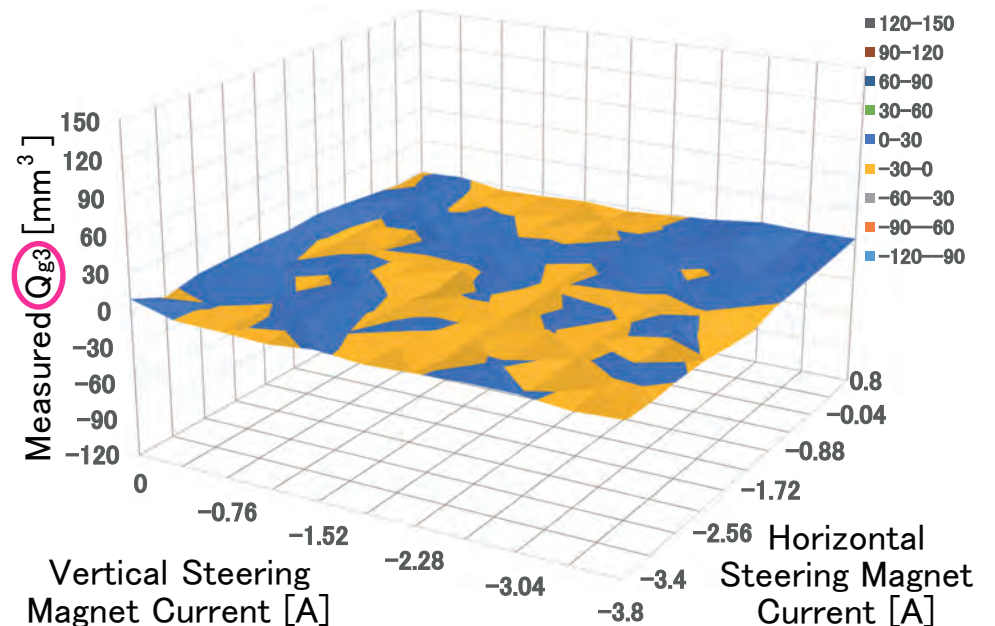
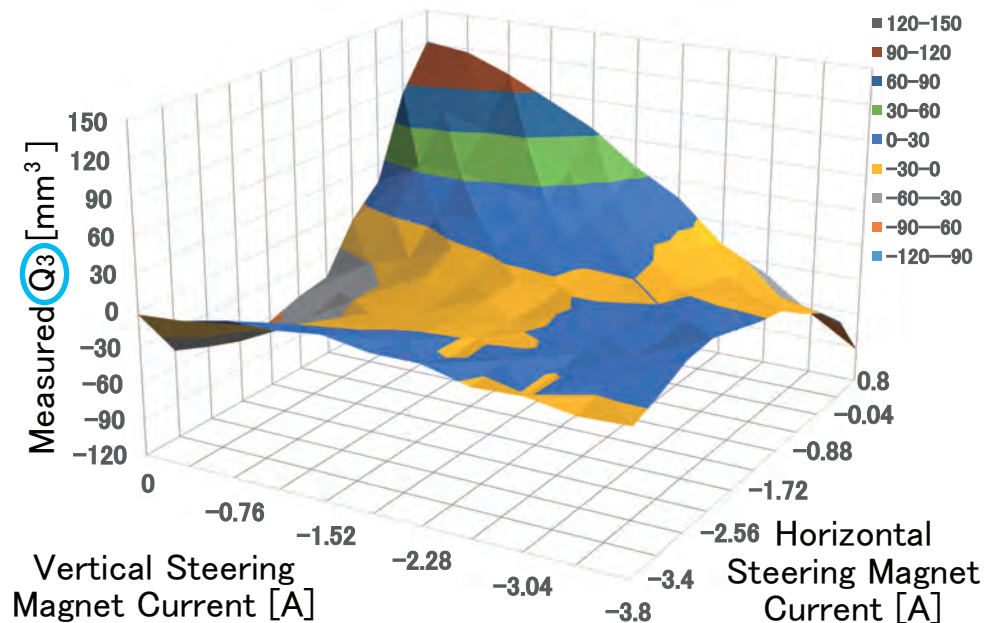
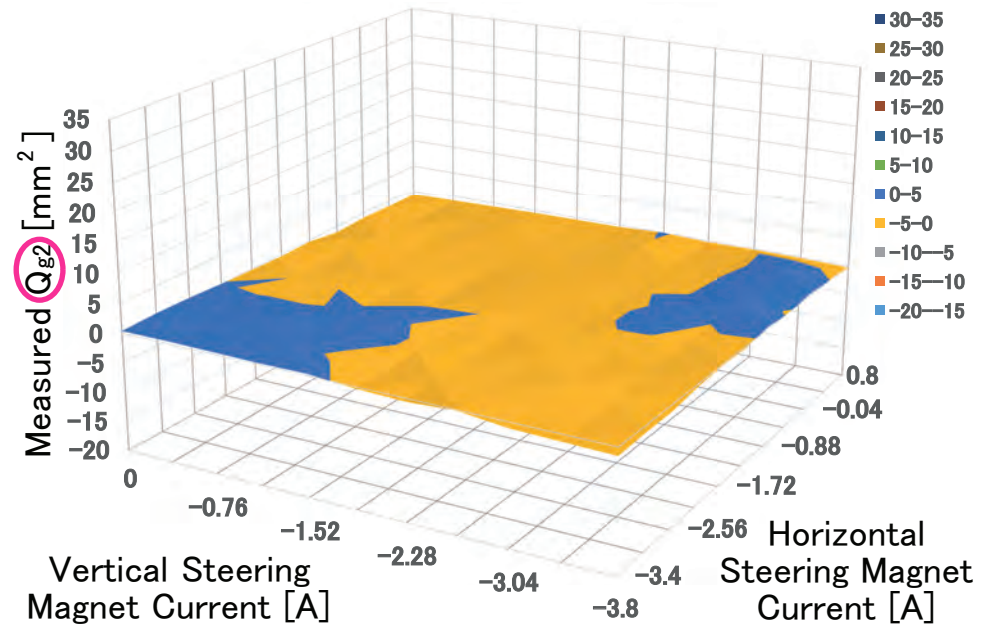
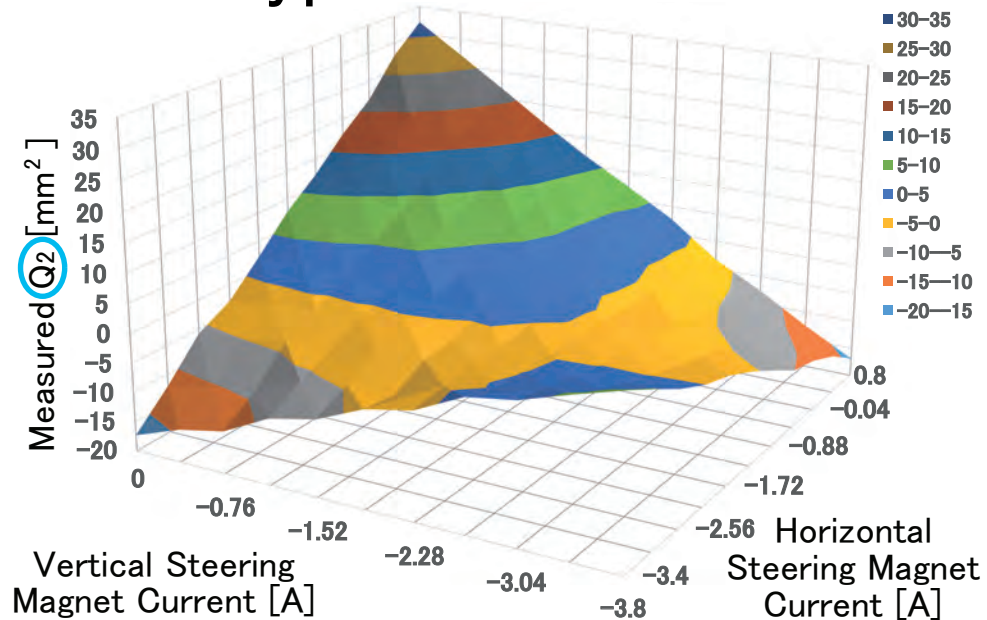
Entire Calibration

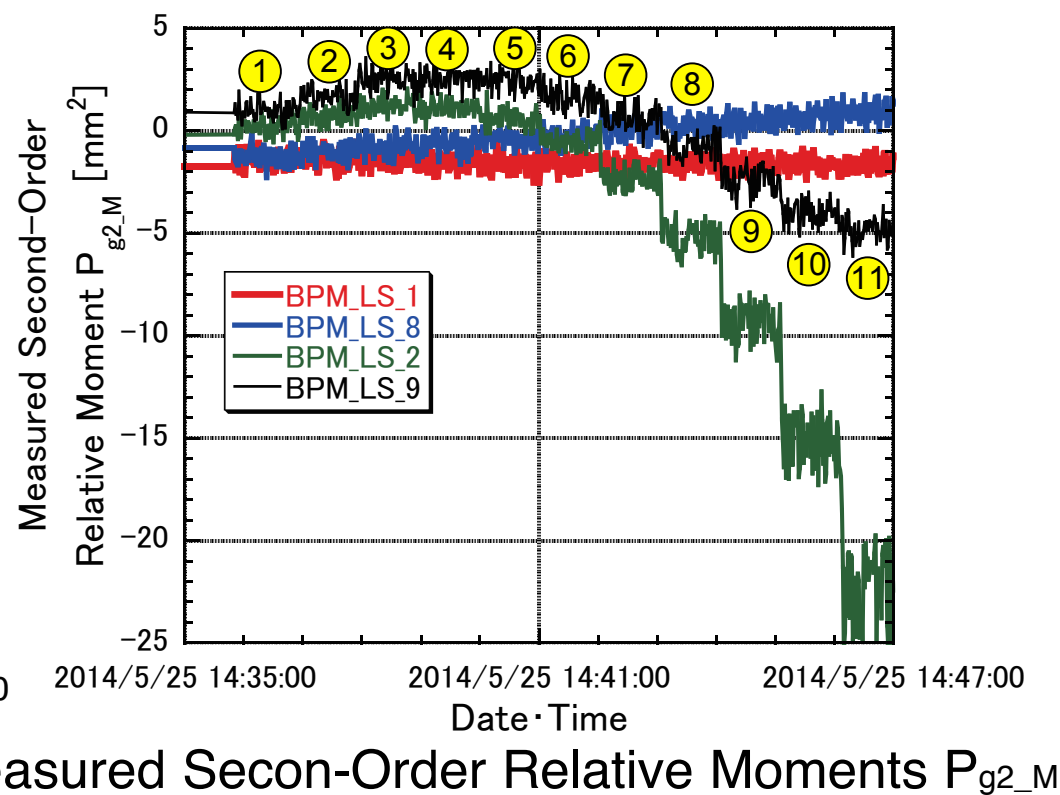
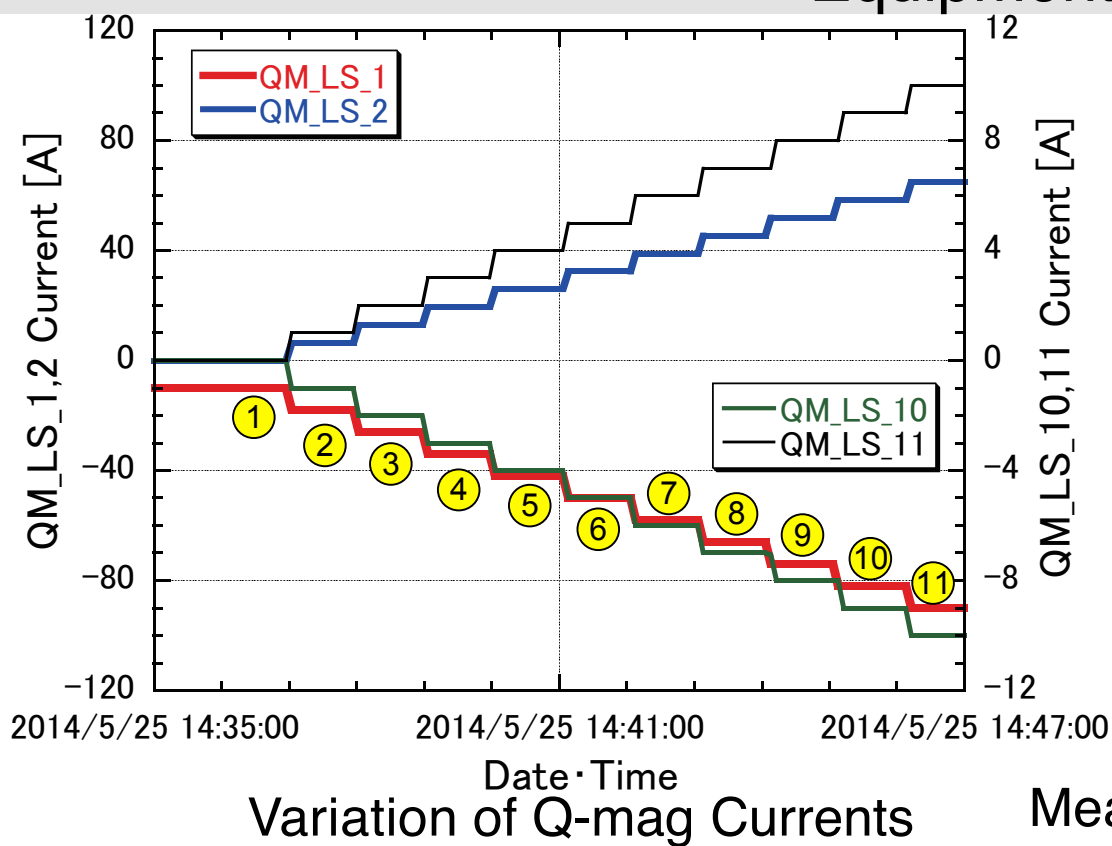
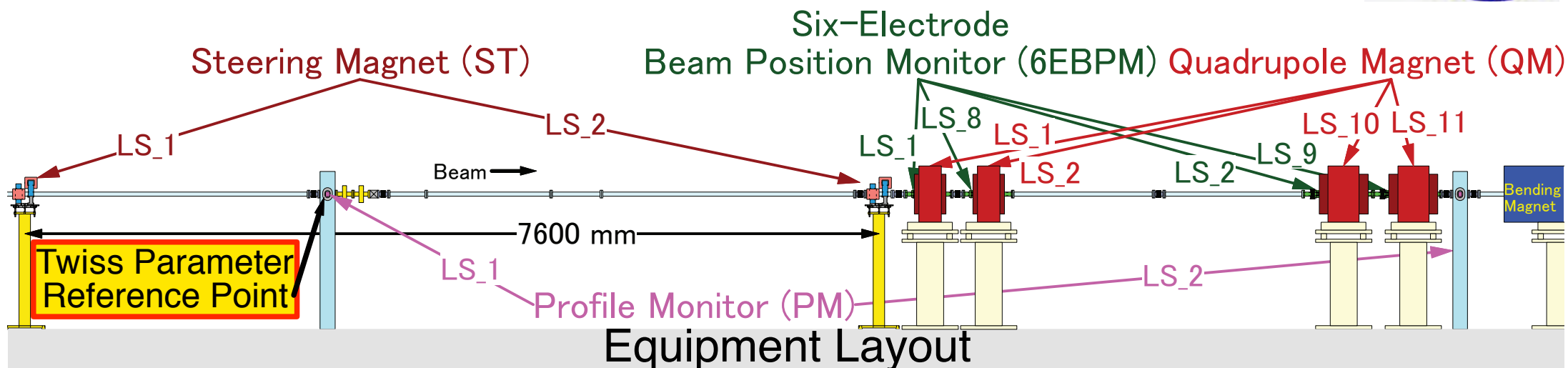
- Typical measured moments in entire calibration.



Entire Calibration

- Typical measured moments in entire calibration.





Deduced Emittances and Twiss Parameters

Parameter	Horizontal	Vertical
ε [$\pi\text{mm} \cdot \text{mrad}$]	0.168 ± 0.002	0.299 ± 0.001
β [m]	14.7 ± 0.1	5.7 ± 0.2
α	2.25 ± 0.04	0.50 ± 0.03

All P_{g2_C} and P_{g2_M} are agree within error bars.

Twiss Parameter
Reference Point

$$P_{g2_C} = \sigma_{H_C}^2 - \sigma_{V_C}^2$$

