HARMONIC NUMBER JUMP ACCELERATION IN SCALING FFAG RING

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Abstract

Constant RF frequency acceleration in non-isochronous rings by means of harmonic number jump is discussed in this paper. General considerations about harmonic number jump with many cavities distributed around a ring are thus set out. Application to fast acceleration of ultra-relativistic particles in scaling Fixed Field Alternating Gradient (FFAG) synchrotrons is then discussed. In this particular case, fundamental limitation on the maximum number of turns is presented, and a way to bypass this limitation using advanced scaling FFAG lattices is introduced.

INTRODUCTION

With their singular properties, FFAG rings are excellent candidates for the fast acceleration of unstable particles and large emittance beams, such as muon beams [1]. Because of their fixed guide field, the acceleration speed is only limited by the available accelerating gradient. Constant RF frequency operation is preferable to reach high accelerating gradient by means of high-Q cavities. Thus, several solutions have already been proposed to use constant RF frequency in either scaling [2] or non-scaling [3, 4] FFAG rings. This paper aims to go further in the study of the so-called harmonic number jump for ultra-relativistic particles acceleration in scaling FFAG rings. General considerations about harmonic number jump acceleration are presented in the first part. A fundamental limitation on the achievable turn number is there highlighted. In the next part, longitudinal and transverse motions are studied by means of stepwise tracking in the example of a 3 to 10 GeV muon ring. In the last part we show how the limitation on the achievable turn number mentioned in the first part can be bypassed using advanced scaling FFAG lattices.

GENERAL CONSIDERATIONS ABOUT HARMONIC NUMBER JUMP ACCELERATION

In a circular accelerator, the condition for a particle to cross an RF cavity with the same phase every turn is known as

$$f_{RF} = h \cdot f_{rev}, \tag{1}$$

where f_{RF} is the RF frequency, f_{rev} is the particle revolution frequency, and h is an integer called harmonic number.

If the revolution frequency varies with the particle energy, as it is the case for FFAG rings, it is still possible to satisfy the synchronization condition of Eq. 1 while using a constant RF frequency. To do so the harmonic number h must be changed of an integer number every turn. This is how works a microtron [5], and the purpose of this section is to discuss the use of this method in FFAG rings.

The mathematical formalism used in the following chapters is based on Berg's paper [6].

Single Cavity Case

We consider here a non-isochronous ring with a single RF cavity working at constant frequency f_{RF}. The harmonic number jump condition is

$$T_{i+1} - T_i = \frac{m_i}{f_{RF}}, \qquad m_i \in \mathbb{Z}, \tag{2}$$

where T_i and T_{i+1} are the time needed by the particle to cover the turn number i and $i+1$ respectively, and m_i is the number of harmonics jumped between turns i and $i+1$.

Linearizing the variation of T around the particle energy at turn i, one gets

$$T(E) = T(E_i) + (E - E_i) \cdot \left.\frac{\partial T}{\partial E}\right|_{E_i}. \tag{3}$$

Eq. 2 becomes

$$E_{i+1} - E_i = \frac{m_i/f_{RF}}{\left.\frac{\partial T}{\partial E}\right|_{E_i}}. \tag{4}$$

If T does not depend linearly on the particle energy, $\left.\frac{\partial T}{\partial E}\right|_{E_i}$ is function of the turn number. Disregarding the possible variation of m_i, Eq. 4 means that the energy gain has to vary from one turn to the next and be kept proportional to the inverse of $\left.\frac{\partial T}{\partial E}\right|_{E_i}$.

Limitation on the Maximum Turn Number

In a circular ring with a single RF cavity, the variation of revolution time between two turns is

$$T_{i+1} - T_i = \frac{2\pi \cdot \Delta R_i}{\beta_i c}, \tag{5}$$

where ΔR_i is the variation in average orbit radius between turn i and $i+1$, and $\beta_i c$ the particle velocity. Combining this equation with Eq. 2, one gets

$$\Delta R_i = \frac{m_i \beta_i c}{2\pi f_{RF}} = \frac{m_i \beta_i \lambda_{RF}}{2\pi}, \tag{6}$$

with $\frac{c}{f_{RF}} = \lambda_{RF}$, the RF wavelength. In the case of already ultra-relativistic particles ($\beta \approx 1$), if we want to accelerate over N_t turns, jumping exactly one harmonic every turn ($m_i = 1,\ \forall i$) we have to accept an average beam excursion of

$$average\ excursion = \frac{N_t}{2\pi} \cdot \lambda_{RF}. \tag{7}$$

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If the number of turn is larger that 6, the average excursion gets larger than the RF wavelength. In a ring with constant excursion, such as a scaling FFAG ring, RF cavity design issue limits then the achievable number of turns.

N Cavities Distributed Around the Ring

We now consider a ring with N RF cavities homogeneously distributed around (see Fig. 1). If the particle energy were not changed during the turn, the time needed to cover one turn would have been $T(E_i)$. But the particle crosses N cavities every turn, and its energy varies from one cavity to the next. T becomes function of the position around the ring. The time needed, from the cavity number k, to travel all over one turn is the sum of the times of flight between every single cavity:

$$\mathfrak{T}(E_{i,k}) = \sum_{j=0}^{N-1} \frac{T(E_{i,k+j})}{N} \tag{8}$$

$E_{i,k}$ is the particle energy after crossing cavity k the i^{th} turn. With this notation $E_{i,k+N} = E_{i+1,k}$. At the beginning of turn i, we assume that cavity number 0 works at constant frequency f_0, and that

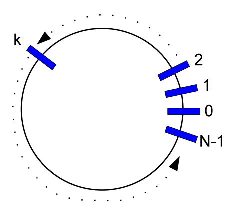

Figure 1: N cavities (numbered 0 to N-1) homogeneously distributed around a circular ring.

$$f_0 = \frac{h_i}{\mathfrak{T}(E_{i,0})} \qquad h_i \in \mathbb{Z}. \tag{9}$$

We also assume that during the turn i every cavity provides the same amount of energy gain ΔE_i, and that the total amount of energy gain follows Eq. 4:

$$\Delta E_i = \frac{E_{i+1,0} - E_{i,0}}{N} = \frac{m_i/f_0}{N \cdot \left.\frac{\partial T}{\partial E}\right|_{E_i,0}} \tag{10}$$

In this case the linearization of T around the energy $E_{i,0}$ gives

$$T(E_{i,k}) = T(E_{i,0}) + k \cdot \Delta E_i \left.\frac{\partial T}{\partial E}\right|_{E_{i,0}}. \tag{11}$$

Combining Eq. 8 and Eq. 11 one can get

$$\mathfrak{T}(E_{i,k}) = T(E_{i,0}) + \frac{2k+N-1}{2} \cdot \Delta E_i \left.\frac{\partial T}{\partial E}\right|_{E_{i,0}}. \tag{12}$$

If we would like to arrive every turn with the same RF phase in the cavity number k:

$$f_k = \frac{h_i}{\mathfrak{T}(E_{i,k})}, \tag{13}$$

which gives using Eq. 9, Eq. 10, and Eq. 12

$$f_k = \frac{f_0}{1 + \frac{m_i}{h_i} \cdot \frac{k}{N}}. \tag{14}$$

If we assume that h_i is large, and much larger than its variation during the acceleration cycle (i.e. $h_i \approx h_0$), we get

$$f_k \approx f_0 (1 - \frac{m_i}{h_0} \cdot \frac{k}{N}), \qquad k = \{0..N-1\}. \tag{15}$$

Disregarding the possible variation of m_i, f_k depends linearly on k and does not depend on the turn number. Every cavity can work at a constant frequency, but each cavity has to have a different frequency.

The fact that every cavity has to work at a frequency that is function of its position k around the ring implies that acceleration is only possible in one direction of rotation. If one wants to accelerate a particle and its anti-particle in the same time, theses two particles cannot be circulated in opposite directions.

EXAMPLE OF A 3 TO 10 GEV MUON SCALING FFAG RING

Double Beam Lattice Design

It exists a solution to circulate a particle and its antiparticle in the same direction in a scaling FFAG ring. In such a lattice two identical scaling FFAG magnets with reversed polarity are used. Figure 2 shows trajectories of a particle and its anti-particle going in the same direction in the lattice described Table 1.

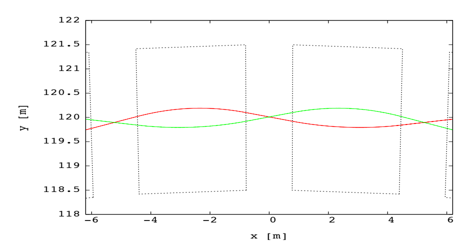

Figure 2: Closed orbits of μ^+ and μ^- circulating in the same direction. Results are obtained from Runge-Kutta stepwise tracking in hard-edge field.

4D Longitudinal + Horizontal Tracking

Simulation of harmonic number jump acceleration over 8 turns, with an initial beam emittance of 0.21 eV.s (longitudinal) $\times$ 10 000 π mm.mrad (normalized horizontal), are presented Figs. 3 and 4. Particle tracking is done with Runge-Kutta integration in soft edge fields (linear fringe field falloff) and thin RF cavity kicks. Lattice and RF parameters are described in Table 1. It is important to notice that the RF peak voltage is kept constant. Particle phase changes by itself turn after turn, as shown on Fig. 3. This phase change provides a variation in energy gain conparable to the one described in Eq. 4.

Table 1: 3 to 10 GeV Muon Ring Parameters

Lattice type	scaling FFAG - double beam
Mean radius	120 m
Number of cells	72
Field index k	145
Packing factor	0.7
B_{max}	2.6 T
Horiz. phase adv. per cell	93.2 deg.
Verti. phase adv. per cell	30.2 deg.
Mean RF frequency	$\sim$ 400 MHz
RF peak voltage	1.6 GV/turn
Number of RF cavities	72

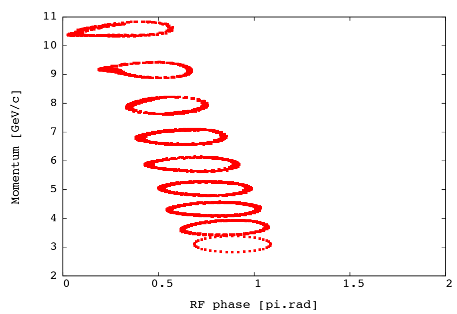

Figure 3: 8 turns acceleration cycle plotted in the longitudinal phase space, at the location of cavity number 0. Initial beam emittance is 0.21 eV.s $\times$ 10 000 π mm.mrad (normalized).

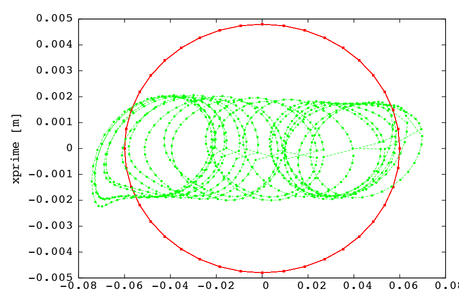

Figure 4: First turn (red squares) and last turn (green dots) of the 8 turns acceleration cycle plotted in transverse phase space. Beam emittance is 0.21 eV.s $\times$ 10 000 π mm.mrad (normalized). The horizontal beam spread of the last turn is due to the effect of dispersion (beam is not mono-energetic).

REDUCED EXCURSION AREA IN A SCALING FFAG RING

In the example described in the previous section, the total excursion is constant all around the ring and is about 0.98 m. As predicted in Eq. 7 this excursion is about 1.3 times the cavity wave length. To avoid cavity design problem related to such a large beam excursion, we would like to create reduced excursion areas in which RF cavities could be installed.

Principle of a possible excursion reducer insertion is presented in Fig. 5. Three scaling FFAG sections with different geometrical field index k are represented. P_0 is the reference momentum. For other momentums a betatron oscillation is exited. If the k_2 section has a total phase advance of 180 deg., and if

$$\frac{2}{k_2+1} = \frac{1}{k_1+1} + \frac{1}{k_3+1}, \tag{16}$$

the excursion is reduced around the momentum P_0. Particle tracking results in such an insertion are presented in Fig. 6.

Once the excursion is reduced, FFAG straight sections [7] can also be added to increase the available space for RF cavities.

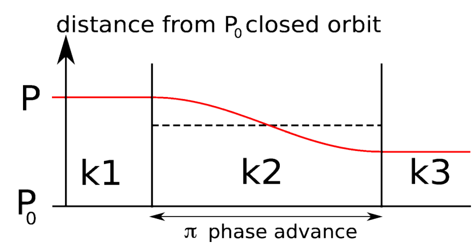

Figure 5: Principle of excursion reducer π section.

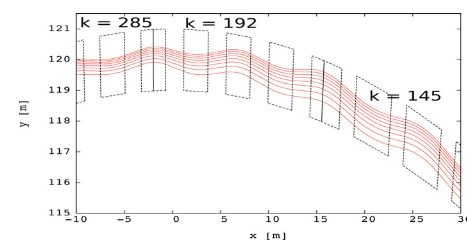

Figure 6: Here excursion is reduced from 0.98m to 0.50 m with the help of a π section with k = 192.

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