BEAM TRANSPORT LINE WITH A SCALING TYPE FFAG MAGNET*

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Abstract

A scaling fixed field alternating gradient (FFAG) accelerator provides large momentum acceptance despite constant field in time. Optical functions are nearly the same for large momentum range. We have designed a straight beam transport line (BTL) using a scaling FFAG type magnet which has a field profile of y^k, where y is the horizontal coordinate and k is the field index. This FFAG-BTL has very large momentum acceptance and optical functions do not practically depends on momentum. We also designed a dispersion suppressor at the end by the combination of a unit cell with different field index k so that the momentum dependence of orbits should be eliminated at the exit. An obvious application of this design is the BTL line between FFAG accelerator and a gantry of a particle therapy facility. This could be an alternative to the conventional BTL with solenoid or quadrupole because of the strong focusing nature of quadrupole and the large momentum acceptance like solenoid.

INTRODUCTION

Since an accelerator has now the capability of fast switching beam momentum, the following beam transport line (BTL) should have a large momentum acceptance as well. Although the idea of using the scaling FFAG optics (the closed orbit shift depending on particle momentum is very small and the chromaticity is zero for a large momentum range) for a BTL has been around for years, there has been no design up to now. A BTL design with the nonscaling FFAG optics, for example by Trbojevic [1], is limited to certain geometry such as a gantry. This paper will describe a way of designing a BTL using the scaling FFAG optics. In the following, we will show a model of a magnet which is the essential element of the FFAG-BTL. We will also show a unit cell as the minimum focusing component and acceptance of a long BTL. We will then describe a way of obtaining the dispersion suppression.

RECTANGULAR SCALING TYPE MAGNET

By analogy to the magnet of a scaling FFAG [2, 3], we define a magnet for the FFAG-BTL with the field profile

$$B_z(x,y) = B_{z0}\left(\frac{y_0+y}{y_0}\right)^k F(x) \qquad (1)$$

where x, y, z are Cartesian coordinates, with x longitudinal, y horizontal and z vertical. In fact, this field profile turned out not to satisfy the scaling optics [4]. However, it practically gives the same optics as the more rigorous field profile when k is large.

Notice that the magnet has a rectangular shape and the constant field lines are straight along the BTL. The function $F(x)$ which describes the longitudinal position dependence of the fields is unity in the core of the magnets, zero far from the magnets, and has Enge function falloff in between, which is described by

$$F(x) = \frac{1}{1+\exp\left(\sum_{i=0}^{5} C_i (x/g)^i\right)} \qquad (2)$$

where C_i are the Enge coefficients and g is the gap of the magnet. We assumed that g is constant and independent of transverse coordinate. In fact, Eq. (2) only describes the fields at the median plane. The off plane fields are described as a series expansion

$$\begin{aligned} B_x(x,y,z) &= \sum_{i=0} B_{x,i}(x,y) z^i \\ B_y(x,y,z) &= \sum_{i=0} B_{y,i}(x,y) z^i \\ B_z(x,y,z) &= \sum_{i=0} B_{z,i}(x,y) z^i \end{aligned} \qquad (3)$$

The coefficients are obtained iteratively by imposing Maxwell's equations.

ORBIT AND OPTICS OF UNIT CELL

In order to make an alternating gradient focusing, magnets of this type with alternating signs, are aligned sequentially along a straight line. As the simplest example, we considered the FDDF configuration, where F is a horizontally focusing magnet and D is defocusing.

Using the parameters listed in Table 1, the orbit and optics with a periodic boundary condition were calculated. We took a reference momentum and momentum range according to the typical values of a hadron therapy facility. From the practical fabrication point of view, we expanded the field into multipoles.

$$B_{z0}\left(\frac{y_0+y}{y_0}\right)^k = B_{z0}\left(1+\sum_{n=1}\frac{1}{n!}\frac{k(k-1)\cdots(k-n+1)}{y_0^{\,n}}y^n\right) \qquad (4)$$

We found that the multipole field up to decapole (y^4 term) gives fairly good approximation of the y^k field profile. In the following, all the calculations were done with this multipole based field profile.

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Table 1: Parameters of a unit FDDF cell.

Parameter	Value	Unit
Magnet length	0.2	m
Drift length	0.2	m
Cell length	1.6	m
Reference radius	$1\text{x}10^6$	m
Field index/Reference radius	5	m^{-1}
Momentum range	0.25 to 1	GeV/c
Reference momentum	0.5	GeV/c
B_0 at F/D	2.0/-3.0	T

Figure 1 shows that the shape of the orbits for different momenta are identical and only slightly shifted. Figure 2 shows the beta function for different momenta. They are almost on top of each other.

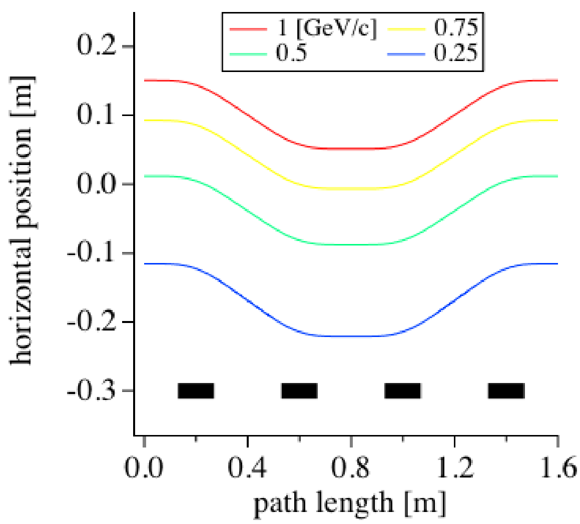

Figure 1: Different momentum orbits in a unit cell which satisfy the periodic boundary condition. Rectangles at the bottom show the position of FDDF magnets.

LONG BEAM TRANSPORT LINE

A long BTL is made up of many unit cells. We took, as an example, 100 cells and studied the acceptance.

Asymmetry of the field profile in the horizontal direction increases the restoring force on one side and decreases it on the other. When the particle amplitude exceeds a certain value, the restoring force is not enough and a particle does not come back to the beam axis. Even if there is the enough restoring force, nonlinearity coming from the field profile distorts beam emittance.

At the reference momentum, the largest amplitude particle which could be transported in the FFAG-BTL was calculated. A set of 5 initial phase space positions with the same horizontal amplitude was chosen uniformly in the azimuthal direction in horizontal phase space. At each horizontal phase space position, 5 particles are allocated which are also uniformly distributed in the azimuthal direction in vertical phase space. Therefore, each particle has the same horizontal and vertical amplitude, but the phase in phase space is different. The total number of particles is 25. If the amplitude of any one of the particles started growing, we considered that the initial amplitude to be outside the aperture. It turns out that the acceptance is much larger than beam emittance for typical proton beam such as 10 π mm mrad. For example, it is 570 π mm mrad (unnormalized) at the nominal operation point. No distortion of beam emittance is observed when emittance is 10 π mm mrad.

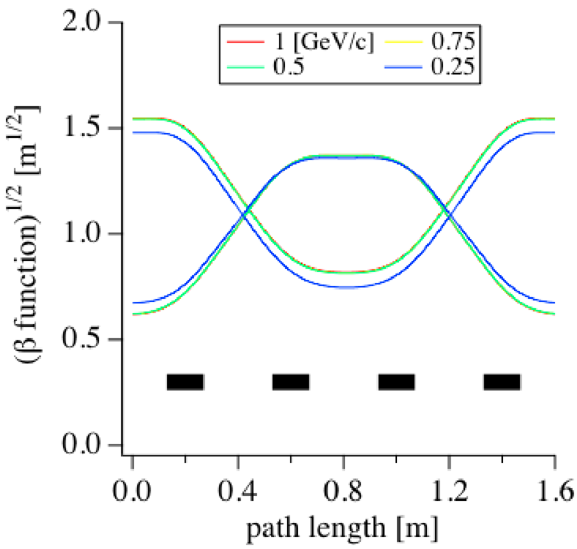

Figure 2: Beta functions with different momenta in a unit cell which satisfy the periodic boundary condition. Rectangles at the bottom show the position of FDDF magnets.

DISPERSION SUPPRESSOR

In the FFAG-BTL, a different momentum particle has a different central orbit. If we consider an application such as a BTL right after a FFAG accelerator, the orbits may be separated already at the entrance to the BTL because of the orbit separation depending on momentum in the ring. However, we want to suppress the dispersion at the end of the FFAG-BTL so that all the different momentum particles go to one single spot.

We know that if the dipole bending strength of the adjoining $-I$ section is halved, where I is the identity transfer matrix, the dispersion function will be zero at the end [5]. It is called a dispersion suppressor with the missing dipole scheme. In terms of scaling FFAG optics, a dispersion suppressor can be a $-I$ section with the field index k doubled with respect to a normal cell. In practice, it is easier to design a $-I$ section first. An example is two unit cells each with 90 degrees phase advance in the horizontal direction. Either the field index k or the ratio $B_{z0,D}/B_{z0,F}$ could be used to obtain the correct phase advance. Secondly one fixes the reference momentum and calculates a periodic solution. This gives an orbit position at the ends of a cell in a dispersion suppressor. We name it the reference position. Thirdly, one designs another unit cell with the field index k halved, which becomes a normal cell, and adjusts the field strength of F and D so that the reference momentum particle has the same orbit position at the ends of the cell as that of the dispersion suppressor. Finally, one connects the normal cell to the dispersion suppressor.

A reference momentum particle entering a normal cell at the reference position comes back to the reference position at the exit of the cell. Then, it enters the dispersion suppressor at the reference position comes back to the reference position at the exit of the cell but a smaller horizontal shift within the dispersion suppressor cell. A particle with different momentum has a shifted central orbit in the normal cell. It comes back to the same horizontal position at the exit as at the entrance in the normal cell. However, it goes to the reference position once it exits the dispersion suppressor if the $-I$ condition is preserved.

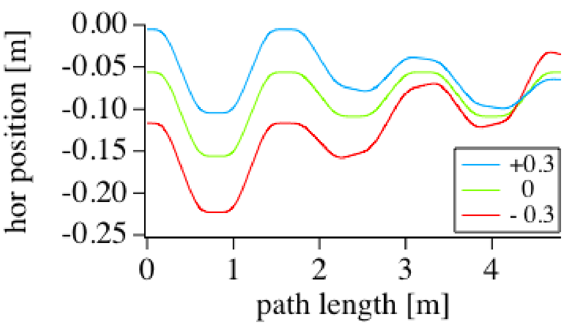

Figure 3: Different momentum orbits in a normal cell and the dispersion suppressor. Numbers in legend show the momentum deviation from the reference momentum. Normal cell is from 0 to 1.6 m and dispersion suppressor section from 1.6 to 4.8 m.

In fact, the $-I$ condition is only approximately satisfied for off momentum particles because the shape of orbits in the dispersion suppressor is not similar and the phase advance strongly depends on the orbit position as shown in Fig. 3. This shows the dispersion suppressor works only in the limited momentum range. It is still large enough for many purposes such as a hadron therapy facility. Failure to meet an exact condition of $-I$ also affects the beta functions in the dispersion suppressor. Figure 4 shows the beta function for different momenta. Because of the variation of the beta functions, the beam size at the end becomes a function of momentum. Within the momentum range of +-30%, the beam size varies by around +-30%. Nevertheless, for a target or a patient, the small variation of beam size should not be a problem.

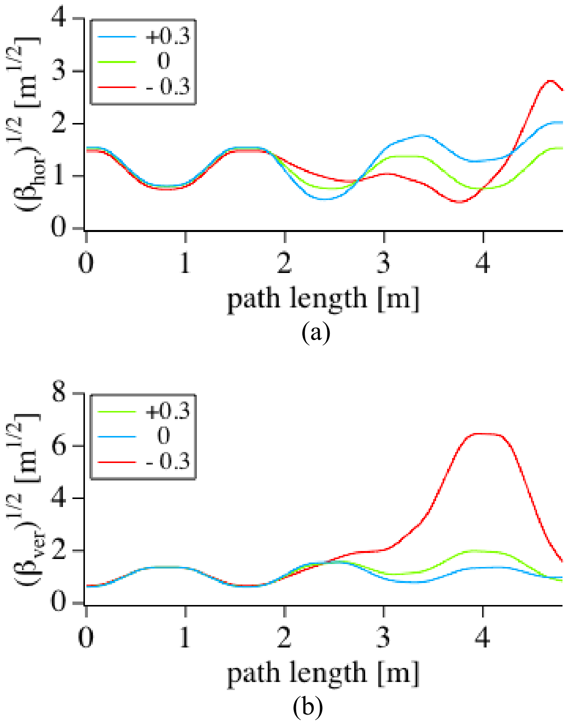

Figure 4: (a) Horizontal and (b) vertical beta functions in a normal cell and the dispersion suppressor. Number in the legend show the momentum deviation from the reference momentum.

SUMMARY

With a scaling FFAG accelerator type magnet, we have designed a beam transport line (FFAG-BTL). Although the magnetic field is constant in time, it has a large momentum acceptance. We studied orbit and optics of a unit FDDF cell and transverse acceptance of a long BTL. We also designed a dispersion suppressor section so that the shifted orbits depending on beam momentum can be merged into one spot at the end of the line.

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