

# POSSIBLE EMITTANCE GROWTH DUE TO NONUNIFORM PARTICLE DISTRIBUTION IN BEAMS WITH THERMAL EQUILIBRIUM CONDITION

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## Abstract

Possible emittance growth due to a nonuniform particle distribution can be analyzed with a thermal equilibrium state in various space-charge potential beams. The possible emittance growth is given by a function of a space-charge tune depression and a nonlinear field energy factor. The nonlinear field energy factor is estimated in the thermal equilibrium distribution on a cross-section in a beam. The nonlinear field energy factor changes with space-charge potential for the thermal equilibrium distribution. It is expected that the possible emittance growth will be decreased effectively to consider in the thermal equilibrium.

## INTRODUCTION

To generate a localized high energy density condition, a particle beam with high current and rather low kinetic energy must be focused into a small spot on a target. As the beam with large volume in phase space is not able to be focused into a small area, production of an intense and low emittance beam is of great significance for various applications. Therefore transport of space-charge-dominated beams with a low-emittance condition is key issue in applications such as warm dense matter (WDM) science and heavy ion inertial fusion (HIF) [1, 2, 3].

The particle distribution becomes nonequilibrium state during the beam manipulations, and it sometimes has a large free energy. A nonuniform charge distribution of a beam can cause significant emittance growth, especially in the space-charge-dominated regime. Wangler et al. derived an equation for emittance evolution along the transport [4], and Reiser summarized the relation between free energy and the emittance growth in nonstationary beams [5]. Lund et al. showed the emittance growth caused due to the thermal relaxation of the initial extreme nonuniform distribution [6].

In this study, we analyze the possible emittance growth due to a nonuniform distribution with a thermal equilibrium. A nonlinear field energy factor is derived for the thermal equilibrium distribution on a cross-section in a beam. The results show that the emittance growth will be decreased effectively to consider in the thermal equilibrium.

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## THEORY

### Emittance Growth and Free Energy

The possible emittance growth  $\varepsilon_f/\varepsilon_i$  in nonstationary condition is calculated by [5]

$$\frac{\varepsilon_f}{\varepsilon_i} = \left[ 1 + \frac{1}{2} \left( \frac{1}{\sigma^2/\sigma_0^2} - 1 \right) \frac{U}{w_0} \right]^{1/2}, \quad (1)$$

where  $\varepsilon_f$  is the emittance at the final state,  $\varepsilon_i$  is the emittance at the initial (nonuniform) distribution,  $\sigma/\sigma_0$  is the tune depression given by the depressed and undepressed phase advances ( $\sigma$  and  $\sigma_0$ ) per lattice period, and  $U/w_0$  is the nonlinear field energy factor written with  $U = w - w_u$ . Here the field energy  $w$  per unit length is given as

$$w = \pi \epsilon_0 \int_0^{r_p} E^2 r dr, \quad (2)$$

where  $\epsilon_0$  is the permittivity of free space,  $r_p$  is the pipe (chamber) radius, and  $E$  is the self-field in the radial direction  $r$ . The field energy per unit length within the actual beam volume is  $w_0 = \lambda^2/16\pi\epsilon_0$ , where the line charge density  $\lambda$  is calculated by

$$\lambda = 2\pi q \int_0^\infty n(r) r dr, \quad (3)$$

with  $q$  as the charge of the beam particle, and  $n(r)$  is the number density along the radius. For the estimation of the possible emittance growth due to the nonuniform particle distribution, the factor  $U/w_0$  should be given in the arbitrary nonuniform distribution.

### Thermal Equilibrium Distribution

The thermal equilibrium in a beam is described by [7, 8]

$$n(r) = \hat{n} \exp \left[ -\frac{\gamma_b m \beta_b^2 c^2 k_{\beta 0}^2}{2k_B T} r^2 - \frac{1}{\gamma_b^2} \frac{q\phi(r)}{k_B T} \right], \quad (4)$$

where  $\hat{n}$  is the number density at the axis,  $\gamma_b$  is the relativistic factor,  $m$  is the mass of the beam particle,  $\beta_b$  is the beam velocity divided by the light speed  $c$ ,  $k_{\beta 0}$  is the wavenumber of the betatron oscillation without the space charge effect,  $k_B T$  is the beam temperature, and  $\phi(r)$  is the space charge potential. The  $k_{\beta 0}$  is determined by  $k_{\beta 0} = \sigma_0/L_p$ , where  $L_p$  is the one lattice period length.

When the number density is rewritten by  $n(r) = \hat{n} \tilde{n}(r)$ , the normalized number density  $\tilde{n}$  can be expressed as  $\tilde{n}(r) = e^{-\tilde{\Psi}}$ , where  $\tilde{\Psi} \equiv \tilde{\Psi}(r)$  is obtained by

$$\tilde{\Psi} = \frac{\gamma_b m \beta_b^2 c^2 k_{\beta 0}^2}{2k_B T} r^2 + \frac{1}{\gamma_b^2} \frac{q\phi(r)}{k_B T}. \quad (5)$$

Using the Poisson equation, the dimensionless potential  $\tilde{\Psi}$  is calculated as [9]

$$\left( \gamma_b \frac{\lambda_D}{r_b} \right)^2 \frac{r_b^2}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \tilde{\Psi}}{\partial r} \right) = 1 + \Delta - e^{-\tilde{\Psi}}, \quad (6)$$

where  $\Delta$  is the dimensionless parameter determined by the tune depression [9],

$$\frac{\sigma}{\sigma_0} = \left\{ 1 - \frac{[\int_0^\infty \rho \tilde{n} d\rho]^2}{(1 + \Delta) \int_0^\infty \rho^3 \tilde{n} d\rho} \right\}^{1/2}, \quad (7)$$

$r_b = \sqrt{2 \langle r^2 \rangle}$  is the rms edge radius of the beam, and  $\lambda_D$  is the Debye length at the axis. The scaled radius  $\rho = r/\gamma_b \lambda_D$  can give the scaled Poisson equation [9, 10],

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \tilde{\Psi}}{\partial \rho} \right) = 1 + \Delta - e^{-\tilde{\Psi}}. \quad (8)$$

Here

$$\gamma_b \frac{\lambda_D}{r_b} = \frac{1}{\sqrt{2}} \left( \frac{\int_0^\infty \rho \tilde{n} d\rho}{\int_0^\infty \rho^3 \tilde{n} d\rho} \right)^{1/2}. \quad (9)$$

By using  $\Delta$  obtained for  $\sigma/\sigma_0$  given, the normalized number density is approximately calculated by [10]

$$\tilde{n} \sim \frac{(1 + \Delta/2 + \Delta^2/24)^2}{\{1 + I_0(\rho)\Delta/2 + [I_0(\rho)\Delta]^2/24\}^2}, \quad (10)$$

where  $I_0(x)$  is a modified Bessel function of 0th order.

### Beam Radius

The rms edge radius of the beam  $r_b$  is calculated by

$$r_b^2 = 2 \frac{\int_0^\infty r^3 \tilde{n}(r) dr}{\int_0^\infty r \tilde{n}(r) dr}, \quad (11)$$

while in the case of the uniform density the beam radius  $a$  is given by

$$a = \left( \frac{\varepsilon_b}{k_{\beta 0} \sigma / \sigma_0} \right)^{1/2}, \quad (12)$$

where  $\varepsilon_b$  is the rms edge emittance. Also the ratio of the Debye length to the beam radius is obtained by [8]

$$\frac{\lambda_D}{a} = \left[ 8 \left( \frac{1}{\sigma^2 / \sigma_0^2} - 1 \right) \right]^{-1/2}, \quad (13)$$

for the uniform density profile in the beam core.

### Beam Dynamics and Electromagnetic Fields

#### D03 - High Intensity - Incoherent Instabilities, Space Charge, Halos, Cooling

### Self-Electric Field

The self-electric field is calculated by

$$E(r) = \frac{q}{\epsilon_0} \frac{1}{r} \int_0^r n(r) r dr = \frac{q\hat{n}}{\epsilon_0} \frac{1}{r} \int_0^r \tilde{n}(r) r dr. \quad (14)$$

When the electric field can be written by  $E(r) = \hat{E} \tilde{E}(r)$ , the  $\tilde{E}(r)$  is given by

$$\tilde{E}(r) = \frac{1}{r} \int_0^r \tilde{n}(r) r dr. \quad (15)$$

### Field Energy

According to the above discussions, the field energy per unit length can be obtained by

$$w = w_0 \frac{4}{(\int_0^\infty \tilde{n} r dr)^2} \int_0^{r_p} \tilde{E}^2 r dr. \quad (16)$$

While the field energy in the uniform particle distribution is described by  $w_u = w_{ui} + w_{uo}$ , where  $w_{ui} = w_0$  and  $w_{uo} = 4w_0 \log(r_p/r_b)$  are the field energies for inside and outside of the beam core. As a result, the field energy in the uniform particle distribution can be written as

$$w_u = w_0 \left( 1 + 4 \log \frac{r_p}{r_b} \right). \quad (17)$$

Finally, the nonlinear field energy factor is given by

$$\frac{U}{w_0} = \frac{4}{(\int_0^\infty \tilde{n} r dr)^2} \int_0^{r_p} \tilde{E}^2 r dr - \left( 1 + 4 \log \frac{r_p}{r_b} \right). \quad (18)$$

## RESULTS

Figure 1 shows the nonlinear field energy factor  $U/w_0$  as a function of tune depression  $\sigma/\sigma_0$ . The nonlinear field energy factors are calculated in cases for the Kapchinskij-Vladimirskij (KV), waterbag (WB), parabolic (PA), Gaussian (GA), and the thermal equilibrium (TE) distributions as shown in Fig. 1. Since in the case with the strong space-charge regime the TE distribution becomes uniform shape, the  $U/w_0$  closes to 0. On the other hand, the  $U/w_0$  for the weak space-charge regime approaches that for GA, because the TE distribution comes up to the GA distribution in the regime.

Figure 2 shows the possible emittance growth  $\varepsilon_f/\varepsilon_i$ , which can be solved by using Eqs. (1) and (18), as a function of tune depression  $\sigma/\sigma_0$ . Figure 2 indicates that the possible emittance growth strongly depends on the particle distribution and even in strongly space charge dominated regime the possible emittance growth will be decreased in case of the beam with TE distribution.

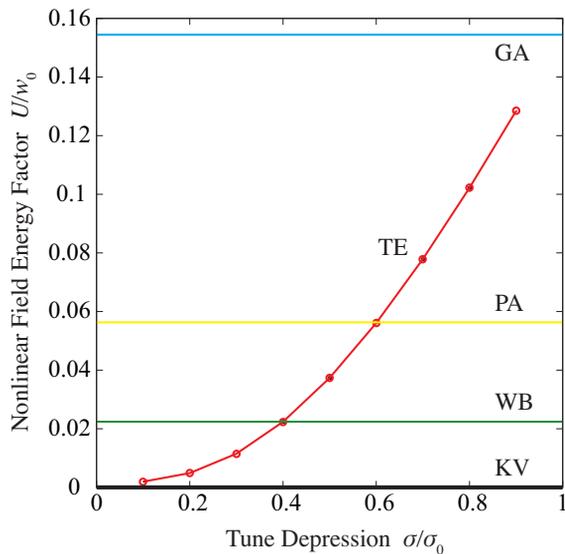


Figure 1: Nonlinear field energy factor as a function of  $\sigma/\sigma_0$ , for  $U/w_0 = 0$  as the KV (black), for WB (green), for PA (yellow), for GA (cyan), and for TE (red with circle) distributions.

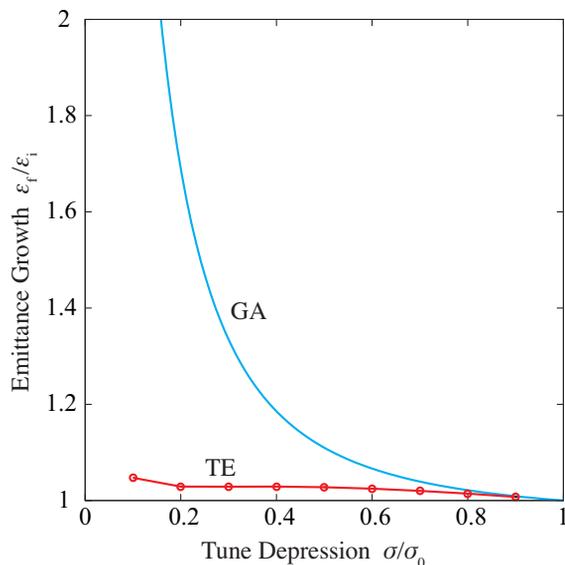


Figure 2: Possible emittance growth as a function of  $\sigma/\sigma_0$  for GA (cyan) and TE (red with circle) distributions.

## DISCUSSION

Consider the situation in the final bunch compression in the HIF accelerator complex. During the final bunch compression, the tune depression of the beam bunch is changing along with the beam current increase. The tune depression history depends on the compression schedule.

In our previous study, the tune depression during the bunch compression changes from 0.9 to 0.15 [11]. Multiparticle simulation shows the  $U/w_0$  changes during the bunch compression [12], and the reconstruction of the dis-

tribution is occurred by the space-charge effect. This means that even though the beam has a thermal equilibrium distribution at injection of the buncher, it can produce significant emittance growth during the transport.

In the case of the assumption for the beam as a GA distribution, the possible emittance growth is estimated as a function of the tune depression as indicated in Fig. 2. While as shown in Fig. 2, the possible emittance growth can be estimated as almost constant for the TE distributed beam, even in the different compression schedule. For this reason, the estimation of the emittance growth during the bunch compression becomes easy for the TE beam, and it is a more realistic situation. Because it is indicated in the previous numerical result [13] that the beam keeps an equilibrium condition during the bunch compression.

## CONCLUSION

The transverse particle distribution and the nonlinear field energy factor depended on the tune depression for the TE distribution. Although  $U/w_0$  of TE beam increases when the tune depression is near unity, the  $\sigma/\sigma_0$  term compensates the increase of possible emittance growth as shown in Eq. (1). It was shown by the static analysis that with keeping the beam at thermal equilibrium the possible emittance growth will be able to be decreased effectively.

This is considered to be particularly important for the applications such as the bunch compression for the HIF driver and ion-beam driven WDM experiments.

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