

MATCHING WITH SPACE CHARGE #2

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Abstract

The required strengths of quadrupoles in a phase-space tomography section are significantly affected by the total charge per bunch. Finding settings at a high charge is challenging because of the non-linear nature of Coulomb interactions. This is further hindered by the inability to use thin-lens approximations and dependence on numerical simulations. Finally, one faces the problem that at some charge there simply is no solution at all. In this contribution we describe a simple procedure, implemented in the General Particle Tracer (GPT) code [1], which can be used to find optimal beamline settings in the presence of space-charge forces. The recipe 'transports' the settings for a zero-charge solution to those of the desired charge and it gives an indication what the maximum tolerable charge is.

INTRODUCTION

A proper way of doing matching with space charge is essential for several applications. For example, one may consider a simple tomography line [2, 3] where the beam being diagnosed is a low energy one, but the charge may vary. Equivalently, as is the case in [2], the charge remains the same but the energy varies. Therefore, a series of pre-determined settings at which to put the quadrupoles would be helpful. Further, it would be even better if the different settings did not deviate significantly from the nominal solution (the solution with zero charge).

Finding optimal settings for beamline components is typically a complicated problem because of the large number of free parameters and a corresponding large number of constraints. Where matching Courant-Snyder parameters (by varying focusing elements) is in practice already complicated without space charge, it becomes notoriously hard when non-linear space charges dominate the trajectories. In that case it is crucial to have some procedure, even if it is computationally not the most efficient one, which is *a priori* known to get the desired result. Without such procedure, much time is wasted in trial-and-error mode. This paper describes a numerical procedure to systematically find beam line settings in a space charge dominated regime. The proposed procedure has the additional benefits that it can easily be automated and that it informs the user about a potential non-existence of the desired solution.

EXAMPLE SET-UP

Shown in Fig. 1 is a simple transport section containing three triplets (top), and corresponding rms transverse beam sizes in the two transverse planes (bottom). Assumed is a 10 MeV input beam with a top-hat transverse distribution with a radius of 1 mm, and a Gaussian longitudinal distribution with an rms-length of

1 mm. Desired, in this specific example, is a focus in both transverse planes at $z=1$ m, a parallel beam at $z=2$ m and a focus again at $z=3$ m. As long as space charge effects are of no importance, the required settings for the quadrupoles $Q_1\dots Q_6$ can easily be found and turn out to be $Q_1=Q_3=Q_5=3.04$ T/m and $Q_2=Q_4=Q_6=2.84$ T/m.

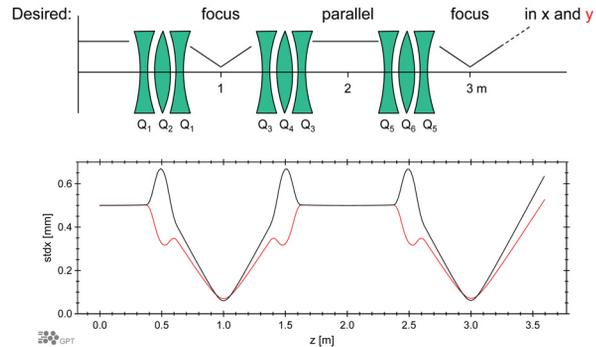


Figure 1: Example set-up.

Obviously the zero charge settings are not optimal when space charge forces start to alter the trajectories. Clearly we need more focusing, but how much more?

FINDING SETTINGS

The example set-up has six variables: $Q_1\dots Q_6$. There are also six constraints $C_1\dots C_6$: Focus, Parallel, Focus at $z=1,2,3$ m respectively, in both transverse planes. Symbolically we can write that the task at hand is to find the values for the variables \mathbf{Q} such that:

$$\mathbf{C} = \mathbf{f}(\mathbf{Q}) \quad (1)$$

where \mathbf{f} is evaluated by particle tracking including space-charge effects, and processing the results such that it can be compared with the desired constraints. To evaluate \mathbf{f} we can use a space charge tracking code such as GPT, ASTRA or PARMELA.

Linear Approach

A typical set of \mathbf{Q} 's will not match the constraints \mathbf{C} . However, in the linear world—when space charge effects are of no importance—any initial guess \mathbf{Q}_0 can be used to find the desired \mathbf{Q} by first-order Taylor expansion of equations (1):

$$\mathbf{C} \approx \mathbf{f}(\mathbf{Q}_0) + \mathbf{M}(\mathbf{Q} - \mathbf{Q}_0) \quad \text{with} \quad M_{ij} = \frac{df_i}{dQ_j} \quad (2)$$

$$\mathbf{Q} \approx \mathbf{M}^{-1}(\mathbf{C} - \mathbf{f}(\mathbf{Q}_0)) + \mathbf{Q}_0$$

where the Jacobian matrix of derivatives M_{ij} can easily be obtained numerically. Unfortunately, this recipe fails miserably in practice when non-linear space charge is taken into account.

Iterative Procedure: Newton-Raphson

A small improvement over the recipe outlined in equation (2) is to use it iteratively:

$$\mathbf{Q}_{n+1} \approx \mathbf{M}_n^{-1}(\mathbf{C} - \mathbf{f}(\mathbf{Q}_n)) + \mathbf{Q}_n \quad (3)$$

This procedure is known as Multidimensional Newton-Raphson, known to converge quadratically: The number of correct digits of the solution doubles at each iteration, if \mathbf{Q}_n is sufficiently close to the solution. That may sound perfect, but again in practice it fails more often than it succeeds due to the difficulty in finding good starting values.

Solution: Use Bunch Charge as Free Parameter

The solution to the problem of finding good starting values proposed in this paper is to use the bunch charge itself as free parameter, even though the goal is finding optimal values at fixed charge. The reasoning is that the variables Q that match the constraints are all a continuous function of charge. Hence one can find the solution at zero charge, use that as the starting point for a solution with hardly any charge, use that solution in turn for slightly more charge, and so on and so forth. In the end one arrives at the solution for the desired charge.

RESULTS

Shown in Fig. 2 are the settings obtained for the quadrupole strengths $Q_1 \dots Q_6$ of our sample set-up. The charge has been increased in small steps of 100 pC, starting from zero. At each such step, a set of Newton-Raphson iterations is performed according to equation (3) starting from the result found in the previous step. Each function evaluation \mathbf{f} is one obtained by one GPT run and the Jacobian \mathbf{M} is obtained numerically by six additional GPT runs with slightly varying Q settings.

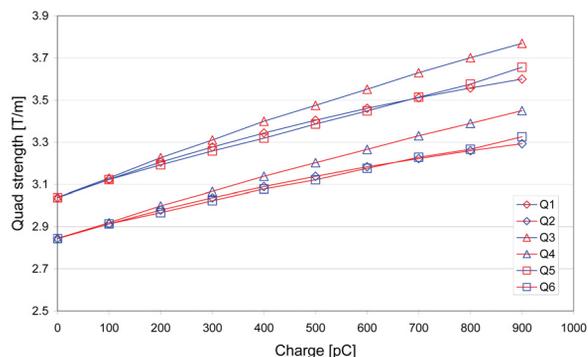


Figure 2: Quadrupole gradients obtained as function of charge, where the optimal values at $N \cdot 100$ pC are used as starting conditions for the $(N+1) \cdot 100$ pC iteration.

The corresponding transverse rms beam sizes are shown in figure 3. As expected, smooth behaviour as function of bunch charge is observed.

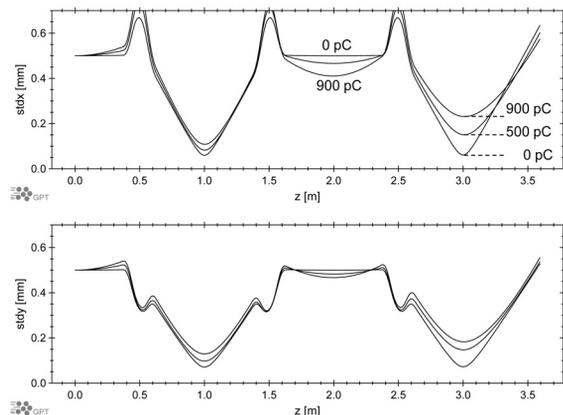


Figure 3: Trajectories at stepwise increasing charge, all matching the required constraints.

Limitations

It is clear that the above procedure is not always applicable. For example, it will fail at the point where space-charge forces result in loss of laminarity. In order to see this, we look at the condition for laminarity as derived, for example, in [4, 5, 6] which we summarize here. For a coasting beam with an elliptical cross section, the electric field inside the beam ($x^2/a^2 + y^2/b^2 < 1$) may be given by:

$$\begin{aligned} E_x(x, y) &= \frac{4I}{v} \frac{x}{a(a+b)} \\ E_y(x, y) &= \frac{4I}{v} \frac{y}{b(a+b)} \end{aligned} \quad (4)$$

where a and b are the transverse beam sizes, $I=I(s)$ is the current, s the longitudinal coordinate and $v=\beta c$ the longitudinal velocity. If we assume that the average kinetic energy of the transverse particle motion (or temperature) is much less than the electrostatic potential energy, then we have a laminar beam and can approximate the linear space charge effect by a defocusing quadrupole field (in both x and y planes) whose strength is given by:

$$\frac{1}{F_x} = -\frac{4I(s)}{(\beta\gamma)^3 I_0} \frac{L}{a(a+b)} \quad (5)$$

where L is the drift length, $I_0 \approx 17$ kA (for electrons) is the Alfvén current, and with a similar expression for the vertical plane. The condition for laminarity is derived from the Kapchinsky-Vladimirsky (KV) (applicable to our example) envelope equations which are given by:

$$\begin{aligned}
 a'' + K_x(z)a - \frac{4I(s)}{(\beta\gamma)^3 I_0} \frac{1}{(a+b)} - \frac{\epsilon_x^2}{a^3} &= 0 \\
 b'' + K_y(z)b - \frac{4I(s)}{(\beta\gamma)^3 I_0} \frac{1}{(a+b)} - \frac{\epsilon_y^2}{b^3} &= 0.
 \end{aligned}
 \tag{6}$$

where ϵ_x and ϵ_y are the effective emittances in both planes. From the KV equations it follows that, for a round beam ($b=a$, $\epsilon_x=\epsilon_y=\epsilon$) and for the space charge term to dominate, we must have:

$$\frac{\epsilon}{\beta_x} \ll \frac{2I(s)}{(\beta\gamma)^3 I_0}
 \tag{7}$$

or, in more practical units and using $a^2 = \epsilon_N \beta_x$

$$\frac{\epsilon_N [\mu m]}{\langle x [mm] \rangle^2} \times 10^{-3} \ll \frac{2I [A]}{\beta\gamma I_0 [kA]}
 \tag{8}$$

where ϵ_N is now the normalised emittance.

Therefore, regarding the laminarity criterion, the idea to slowly increase the charge for every match gives good results as shown below. For most practical cases one wants the zero charge solution, but then with all settings (slightly) changed to accommodate space charge effects. In that case, the algorithm makes perfect sense and any failure is a useful indication that the desired solution simply doesn't exist.

If we look at equation (8) we find an explanation for the behaviour of the beam in Fig. 3, namely, for certain bunch charges and beam sizes we do see a loss of laminarity in the beam as shown in Fig. 4.

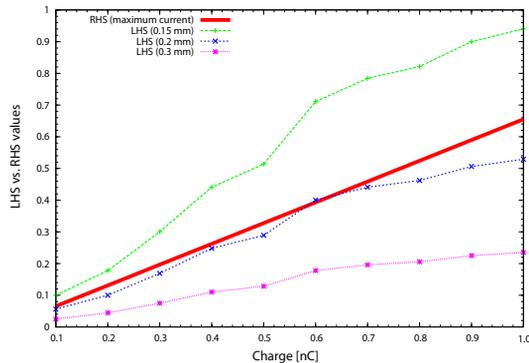


Figure 4: Comparison of thresholds for loss of laminarity, for different beam sizes.

Simulation Considerations

The procedure described in this paper relies on Newton-Raphson iterations that in turn rely on derivatives. Consequently, the simulation results should

be of sufficient quality to be able to numerically determine the derivative. In our experience with the GPT [1] code, this is not a serious limitation.

DISCUSSION

The procedure outlined in this paper provides a method to find optimal beam line settings in a space charge dominated regime. Basically it ‘transports’ the zero charge solution into a regime where space charge significantly affects the trajectories. The procedure eliminates the need for a time-consuming trial-and-error approach, at the expense of requiring more computing power in a simple scheme that can be fully automated.

The procedure outlined in this paper makes the assumption that the solution vector is a continuous function of bunch charge. It should be noted that this is not always the case. The parameter space where the recipe is applicable seems to be relatively large in practice, but clearly this needs further study.

It is interesting to note that the recipe is not at all limited to stepwise increasing bunch charge: As long as a solution is a continuous function of some ‘constant’, that constant can be used to ‘transport’ the solution from an easy to obtain solution into a difficult regime. For example, the same effect could be achieved by starting with a solution for a fixed charge and an energy at which space charge is no longer problematic. Then it should be possible to reduce this energy in incremental steps until one has the desired solution which includes space charge. Alternatively, one can even find solutions of a mildly relativistic case—in simulation—by gradually decreasing the speed of light from infinity to c .

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