ANALYSIS OF DECOHERENCE SIGNALS AT THE SLS STORAGE RING

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Abstract

An online measurement of the beam energy spread is based on the analysis of the decoherence/recoherence signals obtained from the beam position monitors after a single turn beam excitation by a pinger magnet. Furthermore the analysis allows calibration of the model in terms of higher order chromaticities and amplitude dependant tune shifts. An analytical model including 1st and 2nd order chromaticities and amplitude dependant tune shift will be presented.

INTRODUCTION

When a beam is kicked transversely from the closed orbit, it performes betatron oscillations about the closed orbit. The oscillations can be observed with beam position monitors (BPM), which give the centroid of the beam. If the particles all have the same betatron tune the centroid motion is harmonic. However, if the beam contains a spread of tunes, the motion will decohere as the individual betatron phases of the particles disperse.

Here we will consider decoherence due to two sources of betatron tune spread: The beam may have an intrinsic betatron tune spread due to transverse nonlinearity, and there may be an additional tune spread due to the energy spread of the beam which is coupled to betatron tune through chromaticity.

For solving these problems exactly we make some assumptions. In the case of transverse nonlinearity, we assume that the transverse distribution is Gaussian. This implicitly assumes that the distortion of phase space trajectories due to the nonlinearity is small. We also assume that the tune shift with amplitude is a linear function.

For the case of decoherence due to chromaticity, we assume that the synchrotron motion is linear and the energy distribution is Gaussian. Also assume that the energy distribution is uncorrelated with the transverse distribution. Under this assumption the decoherence due to chromaticity is completely independent of the transverse distribution. Note that in this case we take into account the first and the second order chromaticities.

ANALYTICAL TREATMENT

A particle in longitudinal phase space propagates in time by

$$\begin{pmatrix}\delta(n)\\ \tau(n)\end{pmatrix}=\begin{pmatrix}\cos 2\pi\nu_s n & \sin 2\pi\nu_s n\\ \sin 2\pi\nu_s n & \cos 2\pi\nu_s n\end{pmatrix}\begin{pmatrix}\delta_0\\ \tau_0\end{pmatrix}, \tag{1}$$

where $\delta=\dfrac{\Delta E/E}{\sigma_e}$, $\tau=\dfrac{\Delta s}{\sigma_s}$ are normalized longitudinal coordinates, ν_s is the synchrotron tune and time is measured by turn numbers.

For a single particle the transverse displacement time evolution is given by the betatron phase

$$x(n)=\sigma_x\, r\cos\varphi(n). \tag{2}$$

Here the oscillation amplitude has been scaled to the transverse rms beam size σ_x.

In the presence of nonlinearity in lattice structure and non zero value of first and second order chromaticities the electrons with different energies and betatron amplitudes will execute betatron oscillations with different tunes

$$\nu(n)=\nu_0-\mu r^2+\xi_1\sigma_e\delta(n)+\xi_2\sigma_e^2\delta^2(n), \tag{3}$$

where ν_0 is the betatron tune, ξ_1 and ξ_2 are the first and the second order chromaticities respectively and

$$\mu=-\frac{\varepsilon}{2}\frac{\partial\nu}{\partial J}, \tag{4}$$

where ε is the beam emittance, $2J$ is the betatron amplitude.

According to our assumptions we obtain for the phase advance after N turns

$$\varphi(N)-\varphi(0)=2\pi\int_0^N\nu(n)dn=2\pi\nu_0N-2\pi\mu r^2N+ \\ +A\delta_0^2+B\tau_0^2+C\delta_0\tau_0+2a\delta_0+2b\tau_0. \tag{5}$$

Here we introduce several time dependant functions

$$T:=2\pi\nu_sN,\ s:=\sin T,\ c:=\cos T,\ K_1:=\frac{\sigma_e\xi_1}{\nu_s}, \\ K_2:=\frac{\sigma_e^2\xi_2}{\nu_s},\ a:=\frac{K_1s}{2},\ b:=\frac{K_1(1-c)}{2},\ A:=\frac{K_2(T+sc)}{2}, \\ B:=\frac{K_2(T-sc)}{2},\ C:=\frac{K_2s^2}{2}. \tag{6}$$

We are interested in the signal an ensemble of particles will generate at a beam position monitor operating in single turn mode, so we have to weight the single particle's trajectory with the density of the 4 dimensional phase space element $drd\varphi d\delta d\tau$.

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In longitudinal phase space the distribution of particles is invariant in time and has the following form:

$$\rho_s(\delta,\tau)=\frac{1}{2\pi}e^{-\frac{1}{2}\left(\delta^2+\tau^2\right)}. \qquad (7)$$

In transverse phase space the situation is different. Before the kick we have the following distribution of particles:

$$\rho_\beta(x,x')=\frac{1}{2\pi\varepsilon}\exp\left[-\frac{1}{2}\left(\frac{\gamma x^2+2\alpha xx'+\beta x'^2}{\varepsilon}\right)\right] \qquad (8)$$

Introducing normalized variables

$$\frac{x}{\sigma_x}=r\cos\varphi,\quad \frac{\alpha x+\beta x'}{\sigma_x}=-r\sin\varphi. \qquad (9)$$

where $\sigma_x=\sqrt{\beta\varepsilon}$, we obtain

$$\bar\rho_\beta(r,\varphi)=\frac{1}{2\pi}re^{-\frac{r^2}{2}}. \qquad (10)$$

After application of the $\Delta x'$ the transverse distribution becomes

$$\rho_\beta(x,x'-\Delta x')=\frac{1}{2\pi\varepsilon}e^{-\frac{1}{2}\left(\frac{\gamma x^2+2\alpha xx'+\beta(x'-\Delta x')^2}{\varepsilon}\right)}, \qquad (11)$$

or in normalized variables

$$\bar{\bar\rho}_\beta(r,\varphi)=\frac{1}{2\pi}re^{-\frac{1}{2}\left(r^2+z^2+2rz\sin\varphi\right)}. \qquad (12)$$

Here $z=\beta\Delta x'/\sigma_x$ is the normalized kick.

For the distribution in a 4-dimensional phase space we can write

$$\begin{aligned}\rho(r,\varphi,\delta,\tau)&=\bar{\bar\rho}_\beta(r,\varphi-2\pi\nu_0N+2\pi\mu Nr^2-\\&-A\delta^2-B\tau^2-2a\delta-2b\tau-C\delta\tau)\cdot\rho_s(\delta,\tau).\end{aligned} \qquad (13)$$

From this distribution for the centroid displacement we get (using some formulas from [1] and [2])

$$\langle x(N)\rangle=\frac{\sigma_xA(N)H(N)\cos\left(P(N)+Q(N)+2\pi\nu_0N-\frac{\pi}{2}\right)}{e^{M(N)}} \qquad (14)$$

Here the functions $A(N)$, $H(N)$, and $e^{-M(N)}$ are shaping the envelope of the beam signal:

$$H(N)=\left[1+2K_2^2\left(T^2+s^2\right)+K_2^4\left(T^2-s^2\right)^2\right]^{\frac{1}{4}},$$

$$M(N)=\frac{K_1^2(1-c)}{1+K_2^2(T+s)^2},$$

$$A(N)=\frac{z}{1+(4\pi\mu N)^2}e^{-\frac{z^2}{2}\frac{(4\pi\mu N)^2}{1+(4\pi\mu N)^2}}. \qquad (15)$$

The functions $A(N)$ and $H(N)$ describe a monotonous pseudo-damping of the beam centroid signal due to tune shift with amplitude and second order chromaticity. The exponential function $e^{-M(N)}$ describes a modulation of the centroid displacement due to decoherence and recoherence driven by linear chromaticity, with the modulation depths slowly decreasing due to nonlinear chromaticity.

The functions $P(N)$ and $Q(N)$ describe a slow phase modulation of the fast betatron oscillation driven by second order chromaticity and tune shift with amplitude.

$$Q(N)=-\frac{z^2}{2}\cdot\frac{4\pi\mu N}{1+(4\pi\mu N)^2}-2\arctan(4\pi\mu N),$$

$$P(N)=\frac{1}{2}\arctan\frac{\lambda}{f}-\frac{\lambda\gamma^2-\lambda h^2+2f\gamma h}{f^2+\lambda^2}+V. \qquad (15)$$

Here we again introduce several functions for notational convenience

$$g:=\frac{1}{4}+A^2,\ f:=\frac{1}{2}+\frac{C^2}{2g},\ h:=\frac{aC}{2g},\ \lambda:=B-\frac{AC^2}{g},$$

$$\gamma:=b-\frac{aAC}{g},\ V:=\frac{1}{2}\arctan 2A-\frac{a^2A}{g}. \qquad (16)$$

Note that in the limiting case of $\xi_2=0$ equation (14) gives the result from ref. [1].

Figure 1 shows the function according to equation (14) with typical parameters from the SLS storage ring:

ε_x	$5.7\cdot10^{-9}\ m\cdot rad$
ν_{x0}	20.43
ν_s	$6.25\cdot10^{-3}$
σ_e	$8.6\cdot10^{-4}$
β_x	$4.51\ m$
ξ_{x1}	+5
ξ_{x2}	−148
$\Delta x'$	$8\cdot10^{-5}\ rad$
μ_x	$3.5\cdot10^{-6}$

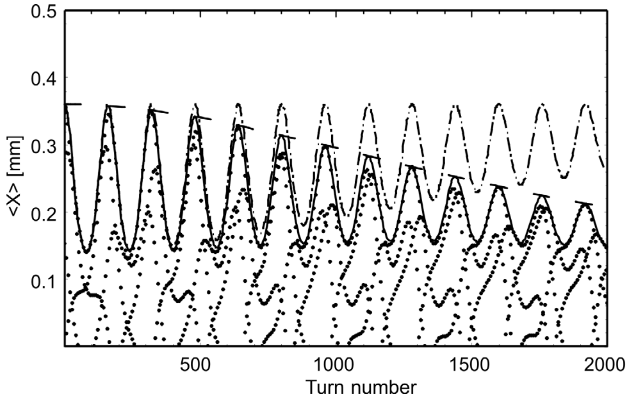

Figure 1: Analytic formula according to eq. (14).

The envelope is shown as solid red line. The nonlinear decoherence alone is plotted by the dashed blue line, the modulation alone by the dash dotted green line. Only the positive side of the signal is shown, since it is symmetric.

In Figure 2 it is shown the results from a TRACY [4] simulation tracking 5000 particles over 2000 turns and the envelope calculated by analytical formula (14) for the same parameters mentioned above.

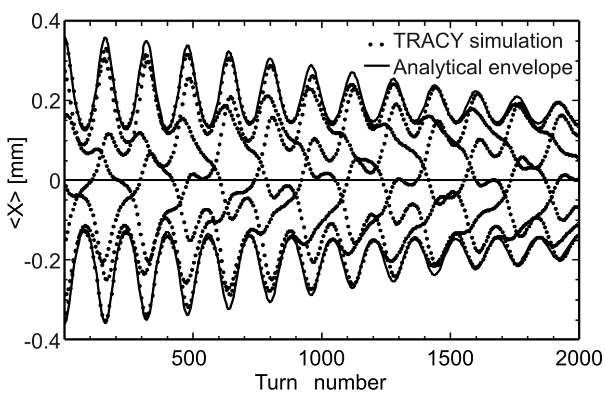

Figure 2: TRACY simulation (dots) and envelope calculated by the analytical formula (14) (solid line).

CONCLUSIONS AND OUTLOOK

An analytical model for the decoherence/recoherence signals obtained from the beam position monitors after a single turn beam excitation by a pinger magnet has been developed including amplitude dependant tune shifts and 1st and 2nd order chromaticities.

Development of a fit procedure to analyse turn by turn BPM data obtained at the SLS storage ring is in progress.

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