

# WAKEFIELD SIMULATION OF CLIC PETS STRUCTURE USING PARALLEL 3D FINITE ELEMENT TIME-DOMAIN SOLVER T3P \*

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## Abstract

In recent years, SLAC's Advanced Computations Department (ACD) has developed the parallel 3D Finite Element electromagnetic time-domain code T3P. Higher-order Finite Element methods on conformal unstructured meshes and massively parallel processing allow unprecedented simulation accuracy for wakefield computations and simulations of transient effects in realistic accelerator structures. Applications include simulation of wakefield damping in the Compact Linear Collider (CLIC) power extraction and transfer structure (PETS).

## SIMULATION CODE T3P

T3P solves Maxwell's equations via the inhomogeneous vector wave equation for the time integral of the electric field  $\mathbf{E}$ :

$$\left( \varepsilon \frac{\partial^2}{\partial t^2} + \sigma_{\text{eff}} \frac{\partial}{\partial t} + \nabla \times \mu^{-1} \nabla \times \right) \int^t \mathbf{E} \, d\tau = -\mathbf{J}, \quad (1)$$

with permittivity  $\varepsilon = \varepsilon_0 \varepsilon_r$  and permeability  $\mu = \mu_0 \mu_r$ . For simplicity in the computations, a constant value of the effective conductivity  $\sigma_{\text{eff}} = \tan \delta \cdot 2\pi f \cdot \varepsilon$  is assumed by fixing a frequency  $f$ , and the losses are specified by the loss tangent  $\tan \delta$ . As is common for wakefield computations of rigid beams, the electric current source density  $\mathbf{J}$  is given by a one-dimensional Gaussian particle distribution, moving at the speed of light along the beam line.

The computational domain is discretized into curved tetrahedral elements and  $\int^t \mathbf{E} \, d\tau$  in Eq. (1) is expanded into a set of hierarchical Whitney vector basis functions  $\mathbf{N}_i(\mathbf{x})$  up to order  $p$  within each element:

$$\int^t \mathbf{E}(\mathbf{x}, \tau) \, d\tau = \sum_{i=1}^{N_p} e_i(t) \cdot \mathbf{N}_i(\mathbf{x}). \quad (2)$$

For illustration,  $N_2=20$  and  $N_6=216$ . Higher-order elements not only significantly improve field accuracy and dispersive properties [1], but they also generically lead to higher-order accurate beam-cavity coupling equivalent to, but much less laborious than, complicated higher-order interpolation schemes commonly found in finite-difference methods.

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The implicit Newmark-Beta scheme [2] is employed for numerical time integration. It is unconditionally stable, i.e. the largest allowable time step does not depend on the smallest mesh size in the overall computational domain. The resulting matrix is factorized either directly or iteratively using the conjugate gradient approach with a suitable preconditioner. Currently, iterative methods exhibit better parallel scalability and are the preferred method for solving large problems. At each time step, the electric field  $\mathbf{E}$  and the magnetic flux density  $\mathbf{B}$  are obtained from the solution vector  $\mathbf{e}$ :

$$\mathbf{E}(\mathbf{x}) = \sum_i (\partial_t \mathbf{e})_i \cdot \mathbf{N}_i(\mathbf{x}) \quad (3)$$

and

$$\mathbf{B}(\mathbf{x}) = - \sum_i (\mathbf{e})_i \cdot (\nabla \times \mathbf{N}_i(\mathbf{x})). \quad (4)$$

More detailed information about the methods used in T3P has been published earlier [3].

## PETS WAKEFIELD CALCULATIONS

The PETS is one of the two key components of the proposed CLIC two beam scheme for high RF transfer efficiency: A low energy, high current drive beam passes through the passive PETS and supplies RF power to the main beam. The PETS is a periodically loaded traveling wave structure with an active length of 21.3 cm (34 cells), a period of 6.253 mm (90°/cell) and an aperture of 23 mm [4]. Transverse impedance from loading of dielectric slots in the PETS is essential and numerical modeling is essential for design verification.

A Gaussian beam is driven along the beam pipe axis (the  $z$ -direction) with an  $x$ -offset of 2.5 mm and beam parameters  $\sigma_z=2$  mm,  $\pm 3\sigma_z$ . This becomes equivalent to a pure dipole current by enforcing electric boundary conditions on the  $yz$ -symmetry plane. For T3P simulations, the same basis order  $p$  is used for all mesh elements and the effective conductivity is calculated with  $f=12$  GHz for simplicity.

Figures 1 and 2 show the used mesh model and a snapshot of the excited wakefields as calculated with T3P.

Figure 3 shows the convergence of the transverse wake potential as a function of the order  $p$  of the Finite Element basis functions used in the T3P calculations. The transverse wake potential is obtained from the longitudinal wake potential by applying the Panofsky-Wenzel theorem [5]. Results from  $p=1$  calculations are the least accurate and show

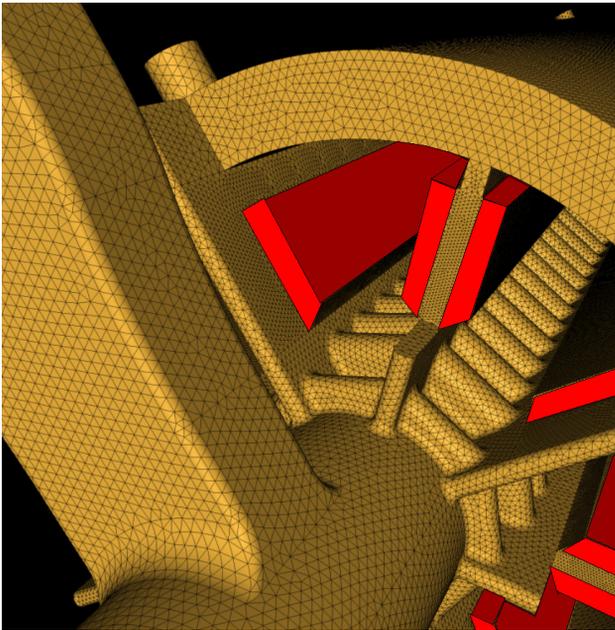


Figure 1: Unstructured conformal tetrahedral mesh model of the PETS for T3P calculations. The mesh for the dielectric loads (highlighted in red) is locally refined for improved resolution of the smaller wavelengths.

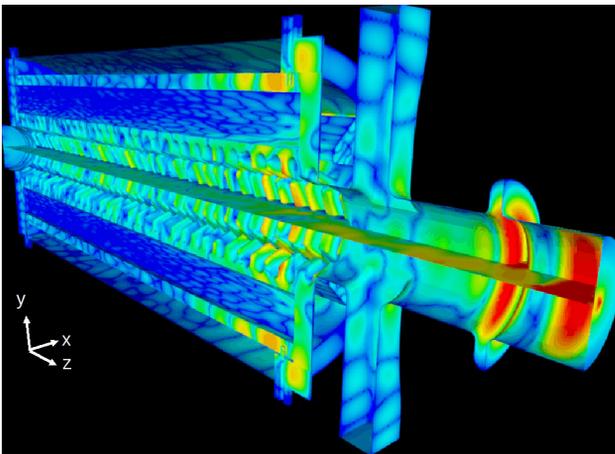


Figure 2: Snapshot of excited surface wakefields calculated by T3P as the beam is about to leave the PETS. Electric cut plane at  $x=0$  excludes monopole modes. The magnetic symmetry plane  $y=0$  is shown for visualization purposes, it includes the beam with an offset of  $x=2.5$  mm. Strong damping in the lossy dielectric loads ( $\epsilon_r=24$ ,  $\tan\delta=0.32$ ) can be seen, as well as some fields in the output coupler and choke.

differences to calculations with  $p=2$  and  $p=3$ , which are very close to identical. Results for  $p=1$  are obtained with a few hundred CPU hours, while  $p=3$  requires several tens of thousands of CPU hours on large supercomputers for extremely good field accuracy. Results with  $p=2$  require in the order of a few thousand CPU hours and can be con-

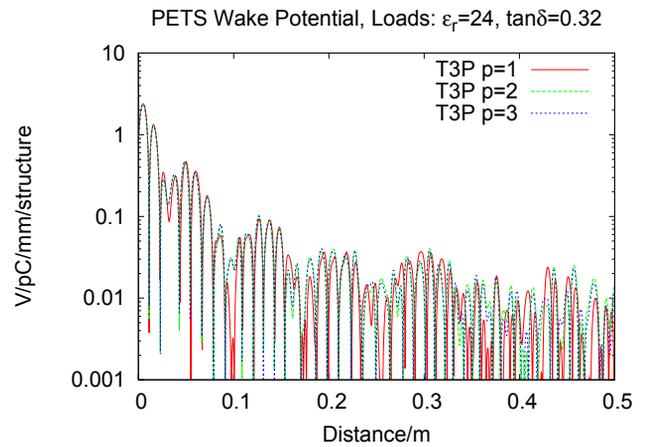


Figure 3: Convergence of transverse wake potential as a function of the order  $p$  of the Finite Element basis functions used in T3P calculations.

sidered well converged for purposes of this study. Consequently, all computations with T3P are performed with Finite Element basis order  $p=2$  in the following.

Convergence in the field integration time step has been observed as well, and a time step of 0.5 ps was required to reach convergence for  $p=2$ . This corresponds to a spatial resolution of hundreds of microns and allows modeling of the smallest features in the geometry with high accuracy.

Figure 4 shows the effect of the lossy dielectric loads by comparing the transverse wake potential with and without losses ( $\tan\delta=0.32$  vs  $\tan\delta=0$ , c.f. Eq. (1)). The dielec-

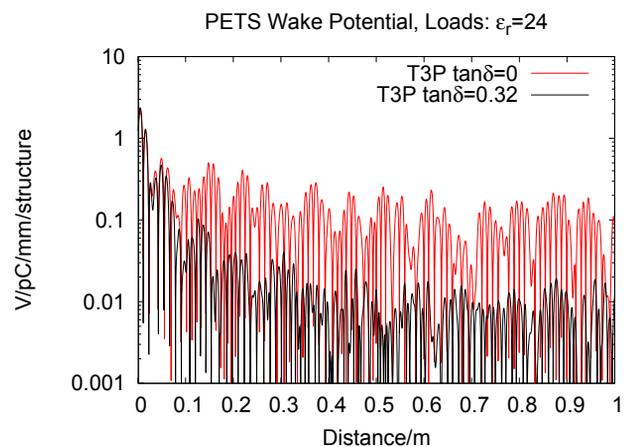


Figure 4: Impact of absorbers on the transverse wake potential as calculated with T3P. The lossless case ( $\tan\delta=0$ ) is compared to the lossy case with  $\tan\delta=0.32$ .

tric properties and permeability of the absorber material are kept constant, i.e.  $\epsilon_r=24$ ,  $\mu_r=1$ . A drop of about one order of magnitude in the wakefield amplitudes is observed after one active length if losses are enabled.

## COMPARISON OF T3P AND GDFIDL

T3P results are compared to GdfidL results [4]. GdfidL is a parallel electromagnetic time-domain code based on the conventional finite difference scheme on structured rectangular grids. GdfidL features first-order boundary approximation (cut-cells) and first-order field representation.

For simplicity, the output coupler and the outer tank are removed for the comparison simulations. All other parameters remain the same, including beam and material properties. T3P calculations are performed with Finite Element basis order  $p=2$ .

Figures 5 and 6 show the transverse wake potentials and impedances as calculated with T3P and GdfidL. Good gen-

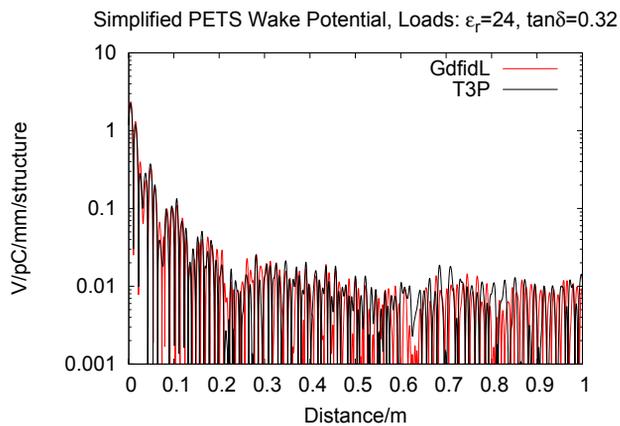


Figure 5: Comparison of transverse wake potential calculated with GdfidL and T3P for a slightly simplified PETS geometry (no output coupler, no outer tank).

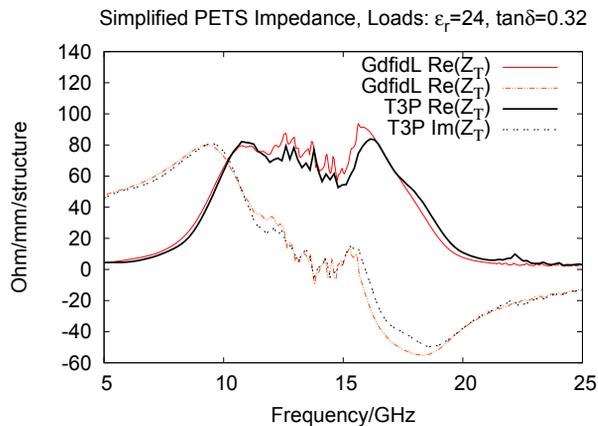


Figure 6: Comparison of transverse impedance calculated with GdfidL and T3P for a slightly simplified PETS geometry (no output coupler, no outer tank).

eral agreement between the codes is found. GdfidL results show relative deviations from the T3P results (with  $p=2$ ), in magnitude and features comparable to the ones between  $p=1$  and  $p=2$  in earlier T3P runs using the non-simplified

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model. This finding is consistent with GdfidL's general first-order approach – further corroborating the applicability of our methods.

## SUMMARY

SLAC's parallel Finite Element 3D electromagnetic time-domain code T3P employs state-of-the-art parallel Finite Element methods on curved conformal unstructured meshes with higher-order field representation. T3P allows large-scale time-domain simulations of complex, realistic 3D structures with unprecedented accuracy, aiding the design and operation of the next generation of accelerator facilities. T3P has been extensively benchmarked against established codes. In this study, T3P has been used to calculate transverse wakefield damping effects in the CLIC PETS, which involves lossy materials and complex geometric features. Thanks to access to large-scale computational resources, and due to the unique features of T3P, convergence of the results could be achieved.

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