

# BENCHMARKING MULTIPACTING SIMULATIONS IN VORPAL\*

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## Abstract

We will present the results of benchmarking simulations run to test the ability of VORPAL to model multipacting processes in Superconducting Radio Frequency structures. VORPAL is an electromagnetic (FDTD) particle-in-cell simulation code originally developed for applications in plasma and beam physics. The addition of conformal boundaries and algorithms for secondary electron emission allows VORPAL to be applied to multipacting processes.

We start with simulations of multipacting between parallel plates where there are well understood theoretical predictions for the frequency bands in which multipacting is expected to occur. We reproduce the predicted multipacting bands and demonstrate departures from the theoretical predictions when a more sophisticated model of secondary emission is used. We will also present simulations of existing cavity structures being developed at Jefferson National Laboratory.

## VORPAL MULTIPACTING MODELS

VORPAL was developed as a general simulation framework for the simulation plasma and beam physics. The computational models used for this work are a finite difference time domain (FDTD) model for electromagnetics and a particle-in-cell (PIC) model for the dynamics of charged particles interacting with the electromagnetic fields. Two other models were added to allow for simulations of multipacting processes in superconducting radio frequency cavities (SRF). The first is a cut-cell model used to improve the FDTD model at curved surfaces such as those encountered in SRF cavities. The second is a model to predict the number, energy and trajectory of secondary electrons emitted when an electron impacts the metal surface of the cavity.

In general, when modeling complex metal structures with FDTD methods the boundary is approximated by stair-stepping the cells of the computational mesh. Even if the FDTD model is 2nd order accurate, the stair-step boundary will introduce a 1st order error to global quantities like the mode frequencies of the cavity. We use the cut-cell boundary method developed by Dey and Mittra[2] to return to 2nd order accuracy for global quantities. This method modifies the finite difference operator in the cells that are cut by the boundary to account for the reduced size and shape of these cells.

The physics involved in the emission of secondary electrons produced by impacting electrons is extremely com-

plex. However, the average number of secondary electrons produced by an impacting electron follows a general curve which can be seen in figure 1. This curve is usually referred to as the Secondary Electron Yield (SEY). Key points on this curve are the peak value of the curve and the region in energy over which the curve is greater than one.

Often in multipacting simulations the SEY curve is parameterized to capture the key elements of the curve. VORPAL can model secondary electron emission in this manner, emitting a secondary electron when the energy is in the range where the SEY is greater than one. We have also incorporated a more advanced model that splits secondary electrons into true secondaries, diffusely reflected secondaries and elastically scattered secondaries [3]. This model uses experimental data to make phenomenological fits to generate the SEY curve that also gives information about the energy and emission angle of the secondary electrons. The SEY curve for this model fitted to data for copper is shown in figure 1 along with the contributions from the three different types of secondaries to the SEY curve.

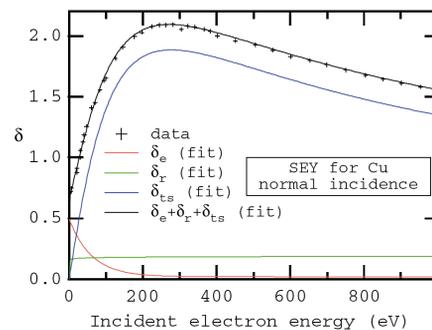


Figure 1: Secondary electron yield for copper taken from Ref [3]. The crosses represent experimental data and lines represent fits from the various aspects of the phenomenological model.

## PARALLEL PLATE BENCHMARKING SIMULATIONS

To test the multipacting modeling capability of VORPAL we ran a series of simulations of multipacting between two parallel plates. In each simulation a traveling wave of fixed frequency and amplitude was driven between the two plates. A sheet of electrons was introduced and driven to walls by the wave. Simulations were run using the full phenomenological model and a simpler model where a secondary electron is emitted when the SEY given by a standard parameterization is above unity. For both models a

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series of simulations were run where the amplitude and frequency of the wave are varied. When the number of electrons in the simulation were found to be stable or increasing we identified that amplitude and frequency as a multipacting resonance.

We can calculate analytically where the multipacting resonances will occur for the parallel plate geometry with a traveling wave by determining the electron energy at impact and comparing this energy with SEY curve. If the energy falls in the range where the SEY is greater than one, then one or more secondary electrons will be produced. If the frequency and phase of the wave are such that the electron's impact energy is in the same range when it returns to the opposite plate, then these parameters describe a multipacting resonance. In figure 2 we show a plot of typical multipacting resonances determined using this method.

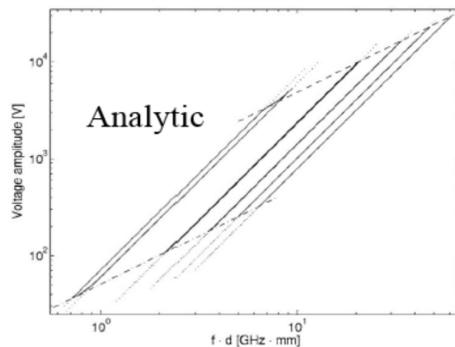


Figure 2: Analytically determined multipacting resonances for the parallel plate configuration. This plot was taken from Ref[4]

In figure 3 we show the resonances found from our simulations using VORPAL with both the phenomenological model[3] and simpler model based off the SEY curve. The blue points are the resonances from the phenomenological model, and the red points are the resonances from the SEY curve. Both models find resonances that closely match the analytically predicted resonances. However, there are some regions where the simple computational model and the analytical predicted resonances are missing from the predictions of the phenomenological model. Also, the resonances found by the phenomenological model are often broader than the other predictions. The reason for these results is that the phenomenological model contains more physics than either the analytical or simple computational model. Elastically and diffusely scattered electrons can alter the width and location of the multipacting resonances.

## SIMULATIONS OF CAVITY MULTIPACTING

Multipacting continues to be an issue in SRF structures. Simulations can play an important role in evaluating cavity designs for potential multipacting problems. We ran simulations of the APS elliptical crab cavity design being

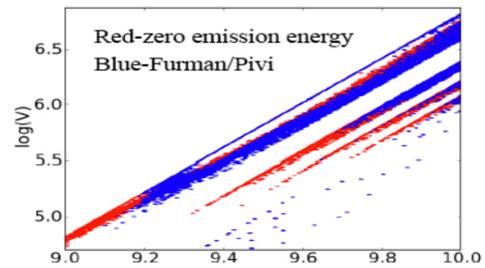


Figure 3: Multipacting resonances determined by VORPAL simulations using the phenomenological model from Ref[3] (show in blue) and a simpler emission model based off a standard SEY parameterization (show in red).

developed at Jefferson National Laboratory [5]. We took an approach similar to the one taken for the parallel plates. First the cavity was driven at the frequency of the operating mode, in this case it was a TM110 mode at 2.8 GHz. The cavity was driven over several mode periods to insure only the operating mode was excited. After the cavity mode was rung up electrons were introduced in one quadrant of the cavity. The left hand image in figure 4 shows the initial electron distribution in the cavity. The full phenomenological model was used for secondary emission. After the electrons were introduced the simulation was run for approximately 25 mode periods.

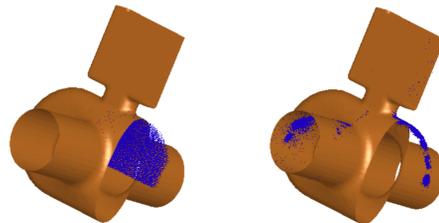


Figure 4: Initial and final distribution of electrons in crab cavity.

In figure 5 we have plotted the number of electrons in the simulation as a function of time. After an initial drop, we see an exponential growth in the number of electrons in the cavity. This is a clear sign that multipacting is occurring. In the right hand image in figure 4 we see the spatial distribution of the electrons at the end of the simulation. The electrons are clumped near the equator of the cavity. We also see some electrons in the beam pipe, although these electrons may be coming from multipacting sites inside the cavity.

In figure 6 we apply our tracking algorithms to three different multipacting trajectories from the simulation. The dark shades on the trajectories represent the location of the electron at later times. We see two trajectories being pulled into the beam pipe and multipacting there. The remaining trajectory stays near the equator of the cavity. A close up

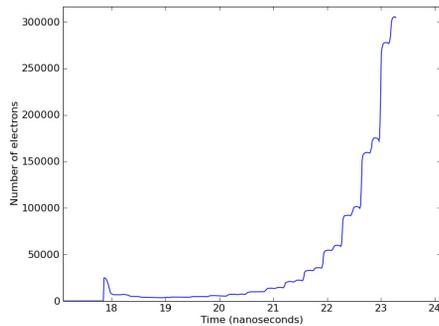


Figure 5: The growth of the electron population.

of this trajectory is seen in the right hand image of figure 6. The particle growth along the last trajectory is much greater than the two trajectories in the beam pipe.

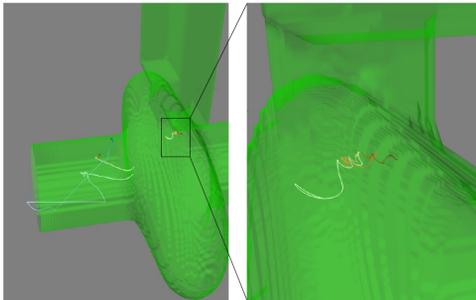


Figure 6: Multipacting trajectories from simulation.

By expanding on these initial simulations to do more detailed parameter scans over the possible operating regime VORPAL can help determine whether multipacting is a potential problem for this and other cavity designs.

## ACKNOWLEDGMENTS

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