ENHANCED METHOD FOR CAVITY IMPEDANCE CALCULATIONS*

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Abstract

With the proposal of medium to high average current accelerator facilities, the demand for cavities with extremely low Higher Order Mode (HOM) impedances is increasing. Modern numerical tools are still under development to more thoroughly predict impedances that need to take into account complex absorbing boundaries and lossy materials. With the usually large problem size it is preferable to utilize massive parallel computing when applicable and available. Apart from such computational issues, we have developed methods using available computer resources to enhance the information that can be extracted from a cavity wakefield computed in time domain. In particular, this is helpful for a careful assessment of the extracted RF power and the mitigation of potential beam break-up or emittance diluting effects, a figure of merit for the cavity performance. The method is described as well as an example of its implementation.

INTRODUCTION

Numerical codes with wakefield solvers provide an elegant method to evaluate the broadband impedance spectrum of an RF cavity in a single step, for either longitudinal or transverse modes, by calculating the beam induced wake potential in time domain and applying a Fourier Transform (FT). This method has been utilized successfully in recent years for the development of single and multicell accelerating cavities with optimized HOM suppression (e.g. [1], [2]). The main benefit of wakefield solvers, when compared to non-complex Eigenmode solvers, is to deliver superior impedance results for heavily damped RF structures since absorbing boundaries (e.g. for waveguides, coaxial lines, beam tubes) can be handled adequately. However, to fully resolve the impedances of parasitic modes, their wake contributions need to be decayed sufficiently behind the exciting particle bunch before the calculation is aborted. Typically, at the initial design stage, the appropriate length to deliver a full resolution of all critical modes is yet unknown and/or prohibitively large. Impedances of high Q modes can be easily underestimated with too short wakes demanding sufficiently long wake lengths that may exceed 1 km. On the other hand, since the CPU time scales linearly with the wake length, it needs to be limited to facilitate an efficient, iterative cavity optimization within a reasonable time scale. We therefore have developed a method able to evaluate the resolved from a yet unresolved impedance spectrum with a single wakefield computation. The method enhances the impedance extrapolating scheme suggested in reference [3] carried out on a mode to mode basis. The new method is efficient and yet simple, so that the mathematics could be easily implemented in computer codes to provide an instantaneous impedance analysis tool, i.e. an online information whether modes are resolved or not already during the solver run. As a consequence one can reduce the CPU time significantly independent of available hardware resources.

IMPEDANCE SPECTRUM EXTRAPOLATION METHOD

When a source particle of charge q_1 traverses a cavity, it excites electromagnetic wakefields (**E**, **B**) that can act on a trailing particle of charge q_2 at a distance s and radial offset **r** behind the leading charge. This is described by the wake function:

$$W(s) = \frac{1}{q_2 q_1}\int_{-\infty}^{+\infty} dz\, q_2 \vec{E}(\vec{r}, z, \frac{z+s}{v}) + \frac{1}{q_2 q_1}\int_{-\infty}^{+\infty} dz\, q_2 \vec{v}\times\vec{B}(\vec{r}, z, \frac{z+s}{v}). \quad (1)$$

The complex cavity impedance in frequency domain is derived by applying the FT of the wake function:

$$\vec{Z}(\omega) = \int_{-\infty}^{+\infty} dt\, W(t) e^{-j\omega t}. \quad (2)$$

For an ultra-relativistic particle, the lower limit can be set to zero since the wakefield is solely left behind the bunch. For simplicity, we consider a particle with point charge q passing on the beam axis in z-direction (2^{nd} term in eq.(1) vanishes) in a damped cavity with only a single mode n present. This yields the longitudinal wake function [4]:

$$W_z(t) = 2\cdot\kappa\left(\cos(\omega_l t) - \frac{1}{2\omega_l \tau}\sin(\omega_l t)\right) e^{-\frac{1}{2}\frac{t}{\tau}}, \quad (3)$$

where $\kappa = \omega_0/4\cdot R/Q$ is the modes' (point charge) loss factor with R/Q the characteristic shunt impedance. The constant 2κ reflects the initial wake after the particle has left the cavity (readily determinable by an Eigenmode solver). Thereafter, the wake is damped by means of the natural time constant of the loaded system $\tau = Q/\omega_0$. Strictly, the loaded frequency (ω_l) is lower than the undamped frequency (ω_0) according to $\omega_l = \omega_0\ \mathrm{Sqrt}(1\text{-}¼\ Q^{-2})$. Insertion of eq.(3) in eq.(2) and integration yields the well-known expression:

$$\vec{Z}(\omega, t\to\infty) = \frac{R}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}. \quad (4)$$

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This result is true if the upper integration limit is set to infinity, so that the exponentially damped term is vanishing. However, when using a numerical wakefield solver, the integration is carried out up to a reasonable time limit only. Therefore a modes' shunt impedance R might not be fully resolved, unless the wake has fully decayed. For instance, consider a mode at 2 GHz with $Q = 1 \cdot 10^5$, a typical Q for a moderately damped mode in an L-band superconducting accelerating cavity. This yields a wake length as large as $c \cdot \tau_n \approx 2.4$ km to merely cover the time/distance until the modes' energy has decayed to 1/e. If we allow for a finite time limit t_{max}, eq.(4) takes the form:

$$\vec{Z}(\omega, t_{max}) = \frac{R}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}\left(1 - e^{-\frac{1}{2}\frac{t_{max}}{\tau}}\right). \qquad (5)$$

Herein we have approximated $\omega_1 \approx \omega_0$, legitimately for $Q >> 1$. The impedance is a real number only on resonance. Generally its amplitude is:

$$Z(\omega, t_{max}) = R'\left(1 - e^{-\frac{1}{2}\frac{t_{max}}{\tau}}\right) \approx \begin{cases} R' & \frac{t_{max}}{\tau} >> 1 \\ \text{partially resolved} & \frac{t_{max}}{\tau} \approx 1 \\ \frac{t_{max}\omega_0 R'}{2Q} & \frac{t_{max}}{\tau} << 1 \end{cases} \qquad (6)$$

where we have substituted

$$R' = \frac{R}{\sqrt{1 + Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}}. \qquad (7)$$

In the bracket of eq.(6) three regimes are characterized depending on the ratio t_{max}/τ. The second category is of major interest since one typically deals with multiple parasitic modes in a cavity yet unresolved at t_{max}. For such modes we like to achieve a better resolution without encountering prohibitively large computation times.

First, we follow the analyses carried out in reference [3] and define the ratio $\xi = Z(t_1)/Z(t_2)$ for impedances that have been calculated at two instants of time t_1 and t_2 respectively. It is advantageous to set $t_2 = 2t_1$. Then we can set up the following conditions:

$$\xi \approx \begin{cases} 1 & R' \text{ resolved at } t_1 \\ 0.5 \le \xi \le 1 & R' \text{ only partially resolved} \\ 0.5 & R' \text{ not resolved} \end{cases} \qquad (8)$$

For $\xi = 1$ the impedance is resolved already at time t_1. For any undamped mode, the impedance amplitude scales linear with the wake length. Particularly we have $\xi = 0.5$ when doubling the wake length. In such a case an Eigenmode solver provides an accurate solution of the shunt impedance not resolvable in time domain, e.g. for an accelerating mode. If the mode is an unwanted, parasitic HOM, the cavity design might have to be changed. This is potentially true also for partially resolved modes with $0.5 \le \xi \le 1$ with yet underestimated impedances at time t_2. However, by subtracting Z_1-Z_2 we can substitute the exponential function $-\exp(-½\, t_1/\tau)$ with $(\xi-1)/\xi$. This provides an expression to extrapolate the impedance of a mode from impedance values known at two different points prior in time.

$$R' = Z(t_1)\left(\frac{\xi}{2\xi - 1}\right) = \frac{Z(t_1)^2}{2Z(t_1) - Z(t_2)}. \qquad (9)$$

We have taken into account the impedance on and off resonance according to eq.(7). This is important, since eq.(9) serves as the basis expression for our subsequent analysis applicable to the whole impedance spectrum calculable by numerical wakefield solvers. One major benefit of wakefield solvers is, that the broadband coupling impedance of the cavity can be evaluated in a single step by means of eq.(2), though the wake potential has to be excited by a particle bunch of finite length. This limits the applicability to frequencies carried in the bunch spectrum. However, the bunch length can be generally chosen small enough to cover all narrowband cavity impedances of interest below - and even above - the corresponding beam tube cutoff frequencies.

In reference [3] two subsequent solver runs have been suggested to extrapolate the impedance on resonance on a mode to mode basis. This implies a search for resonant peaks, which might be tedious. We have developed an enhanced scheme to apply the extrapolation to the whole impedance spectrum according to eq.(9). The new scheme utilizes only a single existing wake potential. It can even be carried out instantaneously during the computation if the wake potential would be available online for analysis. Consider that numerical codes like MAFIA, CST Particle Studio (PS) [5] record the wake potential at equidistant time steps. By zero-padding the number of entries in the wake potential, high speed Fast Fourier Transform (FFT) computations are performed. This is advantageous especially for an FFT of radix-2 case, where 2^n data points are needed. In case we have less entries, we merely have to fill up zeros for the remaining data points, e.g. to the next smallest integer 2^n. The FFT applied to this wake potential delivers a discrete list of i complex impedances given at frequencies spaced by a fix interval $\Delta f = f_{i+1} - f_i$ $\forall$ i. The number of elements i - including the zeros - defines Δf, i.e. how well the spectrum is interpolated, though no better impedance resolution is achieved. With a wake potential abruptly truncated at a given time/distance, ringing effects in the impedance spectrum are produced. These can be minimized by multiplying a window function to the wake potential before the FFT is done (e.g. cosine^2 window in MAFIA). We are also able to artificially create a second wake potential of custom length by a correspondingly sized window function. For instance, let us assume a wake potential exists calculated up to a certain distance $c \cdot 2t_1$. It is windowed up to $c \cdot 2t_1$ and zero padded to a length 2^n for the FFT. With a

window function of length $c \cdot t_1$ we create a second wake potential from the existing one with half the length. Usually, we would zero-pad the data list to only 2^{n-1} elements for the quickest possible FFT. Instead we add more zeros to fill up the list with 2^n elements. Now we can apply eq.(2) to both wake potentials to evaluate two impedance spectra $Z(f_i,t_1)$ and $Z(f_i,2t_1)$. Since the integration time step for both wake potentials was left unchanged, both spectra possess frequency entries spaced by identical intervals Δf_i. Consequently, for the impedance magnitude we are able to readily apply eq.(8) to extrapolate the whole spectrum pointwise. For example, Figure 1 shows an excerpt of four impedance amplitude spectra of a five-cell 1497 MHz accelerating cavity optimized for high current applications [1]. The excerpt covers four quadrupole HOMs – two rather trapped inside the cavity - sampled at a radial offset of 10 mm from the beam axis using the MAFIA wakefield solver in 3D.

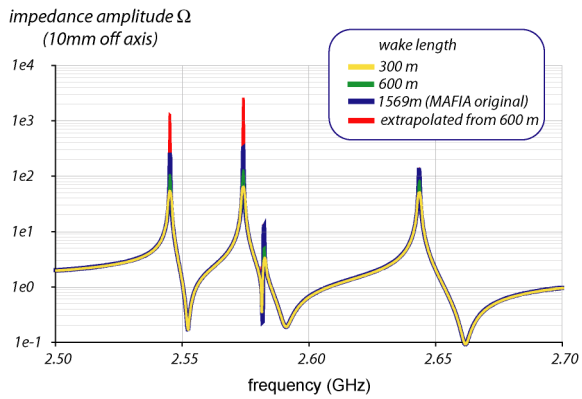

Figure 1: Excerpt of impedance amplitude spectra calculated with MAFIA for an HOM-damped cavity. Impedance spectra are normalized with the bunch spectrum to provide the actual cavity impedance.

The wake potential has been simulated originally for a length of 1569 m (blue curve). Since MAFIA is not supported for massive parallel computing, a CPU time of more than 13 days was needed on a single computer. From the same run we have artificially created a 300m wake ($c \cdot t_1$, yellow curve) and a 600 m wake ($c \cdot 2t_1$, green curve) as described above from which the extrapolated spectrum (red curve) has been calculated. From the 300 m and 600 m wakes it becomes obvious, that the first two modes are hardly resolved with ξ rather close to 0.5. We also notice, that without extrapolation a wake length multiple times larger than 1569 m would be required to fully resolve these modes. This leads to CPU times beyond acceptable limits. Instead we are able to restrict the wake length to about 600 m to resolve the modes, saving weeks (!) of CPU time. The method is easy to implement in existing computer codes. We have created a custom script using MathCad [6] that performs all necessary steps to automatically manipulate the wake potential and to evaluate the extrapolated impedance spectrum. It has been successfully benchmarked with MAFIA and CST Particle Studio. In fact, we encountered impedance resolution issues when testing the wakefield solver capabilities of CST Particle Studio (Version 2009 SP2). We had to export the wake potential and use our own MathCad script to achieve proper results. The mentioned codes unfortunately do not store the wakefield data to external files during the computation. This is desirable for an instantaneous impedance analysis, that could be carried out externally. It would enable us to stop the solver once all HOMs of interest are sufficiently resolvable. For instance we could analyze the rate of change in the impedance increase quasi in real time.

We eventually like to point out, that the wake potential, from which the extrapolation is performed, must not be too small in length. This depends on the damping efficiency. Otherwise, resonance peaks for comparably high Q modes are not resolved adequately. These unresolved peaks might appear at largely different frequencies depending on the wake length [7]. In this case, since we rely on correlating the impedance at identical frequency points, an over- or underestimation of the shunt impedance is possible.

SUMMARY

We have developed an impedance extrapolation method applicable to wakefield solvers to resolve cavity impedance amplitude spectra from a single existing wake potential. The method is applicable to a wake potential at any instance of time and can be easily implemented in numerical codes using FFT computations. It would provide a powerful scheme to analyze the HOM spectrum quasi in real time during the computation. The computation can then be aborted by the user, whenever a satisfying result is achieved. Consequently, a significant amount of CPU time can be saved independent on existing hardware resources.

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