

OPTIMAL CONTROL OF ELECTRON BEAM PARAMETERS AND MACHINE SETTING WITH A NEW NONLINEAR PROGRAM*

Martin J. Lee and Juhao Wu[†], SLAC National Accelerator Laboratory, Menlo Park, CA 94025, USA

Abstract

An x-ray Free-Electron Laser (FEL) calls for a high brightness electron beam. Generically, such a beam needs to be accelerated to high energy on the GeV level and compressed down to tens of microns, if not a few microns. The very bright electron beam required for the FEL has to be stable and the high quality of the electron beam has to be preserved during the acceleration and bunch compression. With a newly developed model independent global optimizer [1], here we report study for the optimization of such a generic machine. In this paper, we focus on the electron bunch longitudinal parameters: the peak current, the centroid energy and the RF cavities' setting. Applicability is detailed for the LINAC Coherent Light Source, an x-ray FEL project being commissioned at SLAC.

TWO-STAGE COMPRESSION SCHEME

We model the compression scheme for a generic X-ray Free-electron Laser (XFEL) via two-stage magnetic chicanes. This two-stage compression scheme is adopted in the LINAC Coherent Light Source (LCLS) at SLAC.

Nominal Values

We assume that the electron bunch centroid energy out of the RF-gun is E_0 with an energy chirp (defined later) of h_0 . The LINAC L_1 between the gun exit and the first magnetic chicane BC1 is running at phase φ_1 with amplitude of V_1 . Hence the centroid energy at BC1 is then

$$E_{1,0} = E_0 + V_1 \cos(\varphi_1). \quad (1)$$

For electrons in the electron bunch, they see the RF wavefront at different phase, hence with the internal coordinate z , the electron z -dependent energy is

$$E_1 = E_0(1 + h_0 z) + V_1 \cos(\varphi_1 + kz), \quad (2)$$

where k is the RF wave number. Therefore along the electron bunch, the energy has a slope, which is referred to as an energy chirp in this paper, and defined as $h \equiv d\delta(z)/dz$, where the relative energy deviation is defined as $\delta(z) \equiv [E(z) - E_n]/E_n$, with E_n as the nominal energy. Hence, after the first acceleration sections L_1 , the energy chirp on the electron bunch evaluated at $z = 0$ is

$$h_1 = \frac{E_0 h_0 - kV_1 \sin(\varphi_1)}{E_{1,0}}. \quad (3)$$

The resulting compression factor is then $C_1 = (1 + R_{56,1} h_1)^{-1}$, where $R_{56,1}$ is the transport matrix (5,6) element in BC1. Similarly, at the second chicane BC2, we have

$$E_{2,0} = E_{1,0} + V_2 \cos(\varphi_2), \quad (4)$$

the energy chirp

$$h_2 = \frac{E_{1,0} h_1 C_1 - kV_2 \sin(\varphi_2)}{E_{2,0}}, \quad (5)$$

and the resulting compression factor as $C_2 = (1 + R_{56,2} h_2)^{-1}$, where we assume that the LINAC L_2 between BC1 and BC2 is running at phase φ_2 with amplitude V_2 . At the undulator entrance

$$E_{3,0} = E_{2,0} + V_3 \cos(\varphi_3), \quad (6)$$

and the energy chirp is

$$h_3 = \frac{E_{2,0} h_2 C_2 - kV_3 \sin(\varphi_3)}{E_{3,0}}. \quad (7)$$

At this stage, the final rms bunch length is

$$\sigma_{z2} = \frac{L_0}{2\sqrt{3}C_1 C_2}, \quad (8)$$

where L_0 is the initial bunch length out of the gun.

The Jitter

In reality, the machine has jitter, therefore, besides considering the nominal designed values, we need study how the jitter affects the machine performance. Because of this, to optimize the machine, the jitter has to be taken into consideration. Here, we only study the LINAC phase jitter, which is assumed to have a normal distribution as

$$f(\delta\varphi) = \frac{1}{\sqrt{2\pi}\sigma_{\delta\varphi}} e^{-\delta\varphi^2/(2\sigma_{\delta\varphi}^2)}. \quad (9)$$

For simplicity, we further assume that the jitter is the same everywhere in the accelerator cavities. This can be extended for real machine where jitter can be located by studying the singular value decomposition (SVD) of a large scale response matrix.

The Object Function

The nominal values which we need to look at are $E_{3,0}$, $\sigma_{z,2}$, and h_3 . Since the phases in the LINAC are jittering

*Work supported by the US Department of Energy under contract DE-AC02-76SF00515.

[†] jhwu@slac.stanford.edu

with the normal distribution in Eq. (9), we will minimize the following object function

$$\begin{aligned}
I &= \left\langle (\sigma_{z2} - \sigma_{z2n})^2 \right\rangle + \left\langle (E_3 - E_{3n})^2 \right\rangle \\
&\quad + \left\langle (h_3 - h_{3n})^2 \right\rangle \\
&\quad + (\sigma_{z2} - \sigma_{z2n})^2 + (E_3 - E_{3n})^2 \\
&\quad + (h_3 - h_{3n})^2. \tag{10}
\end{aligned}$$

To clarify the notation and to simplify the calculation, let us work out one example in details as follows

$$\begin{aligned}
\left\langle (\sigma_{z2} - \sigma_{z2n})^2 \right\rangle &\equiv \iint (\sigma_{z2} - \sigma_{z2n})^2 d(\delta\varphi_1) d(\delta\varphi_2) \\
&\approx (\sigma_{z2} - \sigma_{z2n})^2 + \left[\frac{\partial^2(\sigma_{z2}^2)}{\partial\delta\varphi_1^2} - 2\sigma_{z2n} \frac{\partial^2(\sigma_{z2})}{\partial\delta\varphi_1^2} \right] \sigma_{\delta\varphi_1}^2 \\
&\quad + \left[\frac{\partial^2(\sigma_{z2}^2)}{\partial\delta\varphi_2^2} - 2\sigma_{z2n} \frac{\partial^2(\sigma_{z2})}{\partial\delta\varphi_2^2} \right] \sigma_{\delta\varphi_2}^2. \tag{11}
\end{aligned}$$

For terms related to E_3 and h_3 , the independent variables are φ_1 , φ_2 , and φ_3 , therefore, we can approximate the object function as

$$\begin{aligned}
I &\approx W_{1,0}(\sigma_{z2} - \sigma_{z2n})^2 \\
&\quad + \sum_{i=1}^3 W_{1,i} \left[\frac{\partial^2(\sigma_{z2}^2)}{\partial\delta\varphi_i^2} - 2\sigma_{z2n} \frac{\partial^2(\sigma_{z2})}{\partial\delta\varphi_i^2} \right] \sigma_{\delta\varphi_i}^2 \\
&\quad + W_{2,0}(E_3 - E_{3n})^2 \\
&\quad + \sum_{i=1}^3 W_{2,i} \left[\frac{\partial^2(E_3^2)}{\partial\delta\varphi_i^2} - 2E_{3n} \frac{\partial^2(E_3)}{\partial\delta\varphi_i^2} \right] \sigma_{\delta\varphi_i}^2 \\
&\quad + W_{3,0}(h_3 - h_{3n})^2 \\
&\quad + \sum_{i=1}^3 W_{3,i} \left[\frac{\partial^2(h_3^2)}{\partial\delta\varphi_i^2} - 2h_{3n} \frac{\partial^2(h_3)}{\partial\delta\varphi_i^2} \right] \sigma_{\delta\varphi_i}^2, \tag{12}
\end{aligned}$$

where $W_{i,j}$ for $i = 1, 2, 3$ and $j = 0, 1, 2, 3$ are the weight functions. It is worth to emphasize that by introducing quantities like $(E_3 - E_{3n})^2$ in the object function, we in fact release the constraints on the designed values so that the optimized final centroid energy, energy chirp, and bunch length can vary near the designed values with subscript n . This is indeed necessary, since for example, in real LCLS design the energy chirp after BC2 is taken out by the LINAC geometric wakefield, which is not included in the model calculation in this paper.

INVERSE MATRIX ITERATIVE GLOBAL OPTIMIZATION

In accelerator community, model-based electron accelerator control is used for the maintenance of optimal parameters of an electron beam such as its orbit, size, and shape, as well as machine parameters such as tunes. A common approach for optimizing the machine is to form a positive defined object function. Minimizing this object function will lead to the optimal solution for operating the machine. In practice, finding the global-minimum

solution is a difficult task. Here, a global-minimum solution refers to the bounded multi-variable point having the overall lowest objective function value. Normally, a nonlinear program has to be specifically developed to find the global-minimum. Existing nonlinear programs can be classified into two basic types: One uses an analytical iterative method and the other relies on a stochastic search method such as a genetic algorithm. The inherent difficulty of using an iterative method to find the global-minimum solution is well known. In general, an iterative method requires an initial guess solution. If this start solution is too far from the global-minimum solution, the program will find only a local-minimum solution. A common approach to address this shortcoming is to use an ‘exhaustive search’ or a genetic algorithm. However, there are notable limitations when using such methods—they are often difficult and time consuming to use, particularly when used to find the global-minimum solution to a large scale problem as is the case for accelerator modeling. How to find a way to overcome these limitations in the use of conventional nonlinear programs remains to be a challenge. Recently, Lee proposed a nonlinear programming method which is an attempt to mitigate these limitations [1]. We apply this newly developed approach—Inverse Matrix Iterative Global Optimization (IMIGO)—in this paper to find the optimal solution of a two-stage bunch compression scheme with a jitter model in the LINAC accelerating cavity phase. We ask the reader to refer to Ref. [1] for details of this approach.

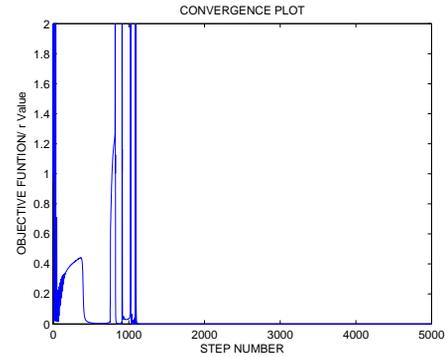


Figure 1: The object function evolves with iteration steps with LINAC phase jitter.

OPTIMIZATION OF THE TWO-STAGE SYSTEM

According to the LCLS FEL requirement, the final electron bunch centroid energy is set at 13.64 GeV and bunch rms length at $10 \mu\text{m}$ entering the LCLS undulators. The initial parameters are $E_0 = 6 \text{ MeV}$, $h_0 = 0$, and $L_0 = 1.5\sqrt{3} \text{ mm}$ with flattop current profile. As we describe in the previous section, the object function is a function of at least eight variables: namely, the phase and amplitude of L_1 , the L_2 , and the L_3 ; and the R_{56} transport matrix element of two chicanes, BC1 and BC2. Hence, minimizing the object

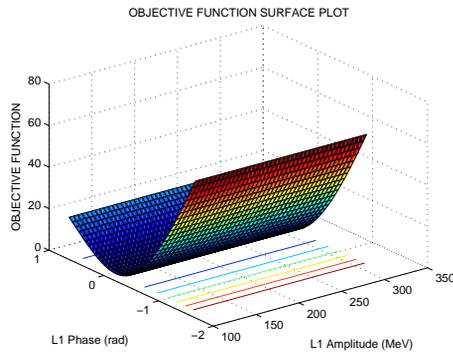


Figure 2: The object function vs. L_1 phase and amplitude with LINAC phase jitter.

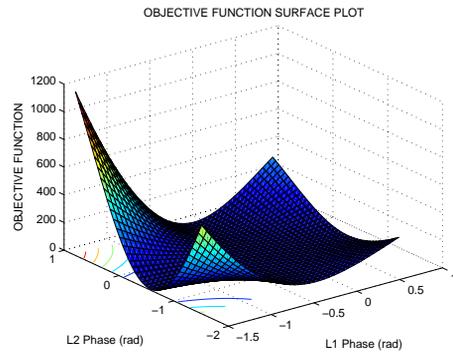


Figure 5: The object function vs. L_1 phase and L_2 phase with jitter in the system.

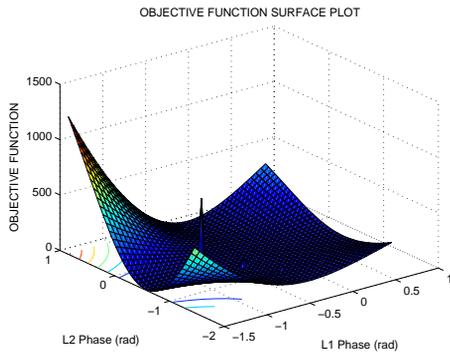


Figure 3: The object function vs. L_1 phase and L_2 phase with LINAC phase jitter.

function is a multi-dimension search. In Fig. 1, we show how the object function evolves as a function of the iteration steps. For example, shown in Fig. 2 is the dependence of the object function on L_1 phase and amplitude. Similar dependence is found for L_2 phase and amplitude. As yet another example, in Fig. 3, the object function is shown as a function of two phases: L_1 phase and L_2 phase. In the above example, we set $\sigma_{\delta\varphi} = 0.1$ S-band degree according

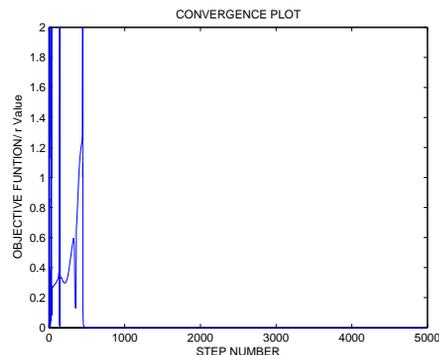


Figure 4: The object function evolves with iteration steps with no jitter in the system.

to LCLS LINAC system operation status; $R_{56,1} = -45.5$ mm and $R_{56,2} = -24.7$ mm. The optimized solution is to set L_1 at -19.4 degree and L_2 at -16.6 degree, this sets BC1 to have $C_1 = 14.7$ and BC2 to have $C_2 = 5.1$. The electron bunch final centroid energy is 13.64 GeV and bunch rms length is $10 \mu\text{m}$ as required. To see how this 0.1 degree LINAC phase jitter affects the optimal values, we zero the jitter and re-optimize. The corresponding object function evolves as in Fig. 4. The object function as a function of L_1 phase and L_2 phase is shown in Fig. 5, where the sharp peak is missing. The optimized solution is to set L_1 at -19.0 degree and L_2 at -18.8 degree, this sets BC1 to have $C_1 = 12.4$ and BC2 to have $C_2 = 6.0$. Further this study, we artificially increase the jitter to be $\sigma_{\delta\varphi} = 1$ S-band degree, then IMIGO suggests that we set $C_1 = 16.2$ and $C_2 = 4.6$, which sets the final centroid energy at 13.64 GeV and bunch rms length at $10.1 \mu\text{m}$.

In above studies, we do not take out the residual energy chirp on the electron bunch after BC2, since in real LCLS accelerator system, the LINAC after BC2 has a strong wakefield, which can dechirp the electron bunch, so that the electron bunch enters the undulator with no residual energy chirp for FEL operation. Therefore, collective effects, such as coherent synchrotron radiation, space charge, and various wakefields are also important in real machine optimization and operation. They can be incorporated in a parametric approach [2]. Besides, the electron bunch transverse qualities can be optimized similarly with IMIGO. In that regard, the magnetic elements, the electron beam trajectory, and electron bunch transverse rms size will be treated similarly as for the RF cavities, the electron bunch centroid energy, and peak current in this paper. All these will be reported in a later publication.

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