

# A FOURTH ORDER RESONANCE OF A HIGH INTENSITY LINAC\*

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## Abstract

The  $4\sigma=360^\circ$  (or  $4\nu=1$ ) resonance of a linac is manifested when the depressed phase advance  $\sigma$  is close to and below  $90^\circ$ . It is observed that this fourth order resonance is dominating over the better known envelope instability and practically replacing it. Simulation study shows a clear emittance growth by this resonance and its stopband. Experimental measurement of the stopband of this resonance was proposed and conducted in 2008 using the GSI UNILAC confirming this resonance. The result is presented at a separate paper in this conference.

## INTRODUCTION

Recently many high intensity linacs have been designed or constructed like the SNS (USA) [1], J-PARC (Japan) [2], or people are trying to increase the intensity of existing linacs such as the UNILAC of GSI (Germany) [3]. For the high intensity linacs, it is the utmost goal to minimize the beam loss of halo particles by avoiding or minimizing contributions of various halo formation mechanisms. One such mechanism is the envelope instability [4]. So far the high intensity linac design such as the SNS linac has avoided the  $\sigma_0 = 90^\circ$  zero current phase advance because of the envelope instability [1].

Until 1998, mismatch was the primarily studied mechanism of halo formation. Late 1998, it was found that halo formation is induced by the  $2\nu_x-2\nu_y=0$  resonance from the space charge potential in the ring [5]. Further studies of halo formation and/or emittance growth by space charge and resonances are reported in [6] and by space charge coupling resonance studies of linac such as [7]. Besides these, halo formation by non-round beam was reported [8] and halo formation by rf cavity [9].

In this paper, we will report about a collaborative effort between SNS and FAIR-GSI concerning the  $4\sigma=360^\circ$  (or  $4\nu=1$ ) resonance of a high intensity linac. For a high intensity linear accelerator, tune  $\nu$  can be defined as  $\nu \equiv \sigma/360^\circ$ . Jeon proposed the beam experiment to measure the resonance stop-band using the UNILAC at GSI and the experiment confirmed this resonance (to be presented in a separate paper in this conference). Numerical simulation is performed with 50 000 to 100 000 macroparticles with the PARMILA code [10]. Space charge tune shift is about  $-20^\circ$ .

## THE LINAC FOURTH ORDER RESONANCE

The study shows that the  $4\sigma=360^\circ$  (or  $4\nu=1$ ) resonance occurs when the phase advance with space charge  $\sigma$  is lower than and close to  $90^\circ$  for a linac just like a ring

through the space charge octupole potential for a variety of beams including Gaussian, waterbag, etc. For the phase advance with space charge  $\sigma > 90^\circ$ , no resonance effect is observed, as shown in Fig. 1.

### RMS Emittance vs. Phase Advance

Simulation is done to study the equilibrium rms emittance as a function of the depressed phase advance  $\sigma$ . Figure 1 shows plots of the ratio of the final transverse emittance over the initial emittance as a function of the depressed phase advance  $\sigma$ . There is no emittance growth beyond  $90^\circ$ . Two cases are simulated and plotted; one is with a tune depression of  $-23^\circ$  and the other  $-20^\circ$ . For this simulation,  $\sigma$  is kept constant throughout the linac.

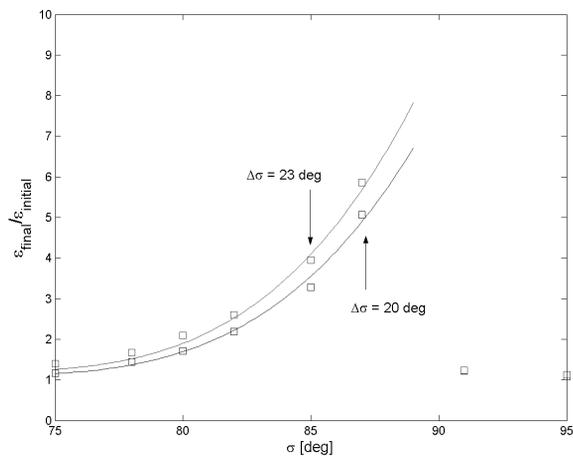


Figure 1: Plots of the rms emittance vs. the depressed phase advance  $\sigma$ . No emittance growth occurs for  $\sigma > 90^\circ$ .

### Crossing the Resonance From Below $90^\circ$

We performed a simulation of beam crossing the resonance from below  $90^\circ$ . Fig. 2 shows the normalized rms emittance growth as the beam crosses the  $4\sigma=360^\circ$  (or  $4\nu=1$ ) resonance together with the variation of the depressed phase advance  $\sigma$ . When the depressed phase advance with space charge  $\sigma$  reaches about  $75^\circ$ , emittance starts to grow. The initial beam is a well matched Gaussian beam to the linac and the emittance growth is solely due to the resonance crossing. The same phenomenon is observed with waterbag beam.

When the beam crosses the resonance from below  $90^\circ$ , the stable fixed points move from afar to the origin and beam particles are transported along the separatrices, not captured by the stable islands. This is well illustrated in Fig. 3.

\* See acknowledgement

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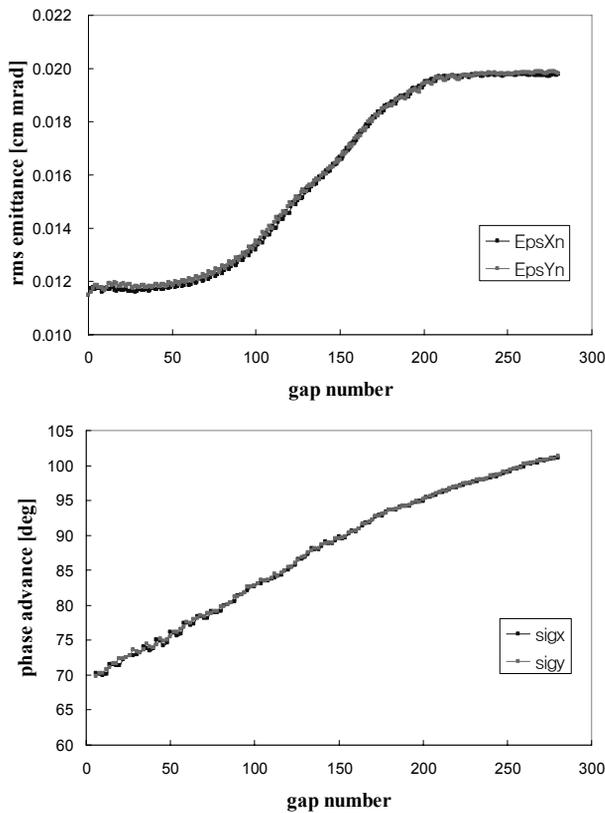


Figure 2: Top plots display rms emittance vs gap number and bottom plots the phase advance with space charge when the beam crosses the resonance from below.

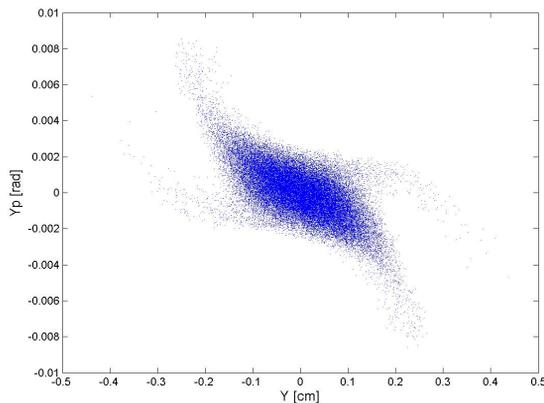


Figure 3: Plot of the beam distribution in Y phase space at the 96<sup>th</sup> gap. Transport of beam particles along the separatrices is observed.

### Crossing the Resonance from Above 90°

We performed a simulation of beam crossing the resonance from above 90°. Fig. 4 shows the normalized rms emittance growth as the beam crosses the  $4\sigma=360^\circ$  (or  $4\nu=1$ ) resonance along with the variation of the depressed phase advance  $\sigma$ . As the depressed phase advance  $\sigma$  crosses 90°, emittance starts to grow. The initial beam is a well matched Gaussian beam to the linac. For  $\sigma > 90^\circ$ , there is no resonance effect.

When the beam crosses the resonance from above 90°, stable fixed points move away from the origin to afar. Unlike rings, tune change rate is not slow enough for adiabatic capture of beam particles by the stable islands. So beam particles are not entirely captured as illustrated in Fig. 5.

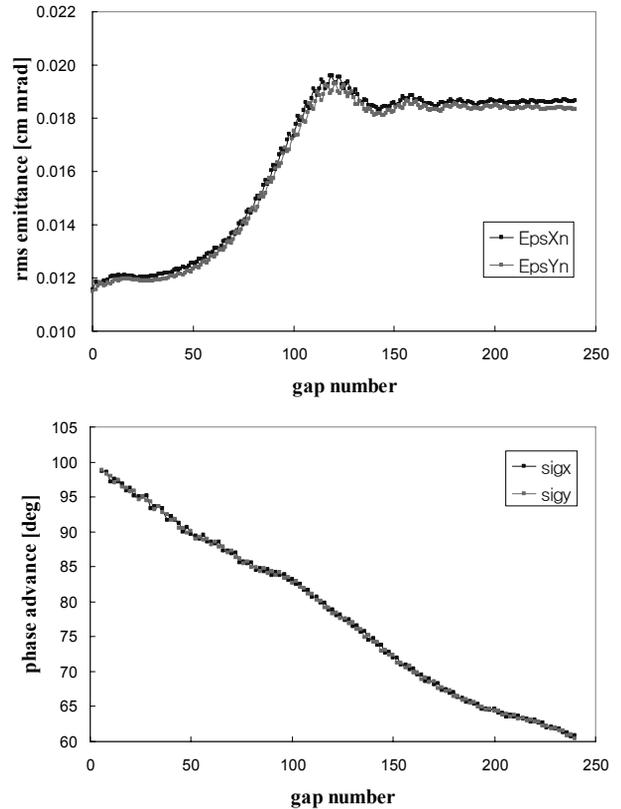


Figure 4: Top plots display rms emittance vs gap number and bottom plots the phase advance with space charge, when the beam crosses the resonance from above.

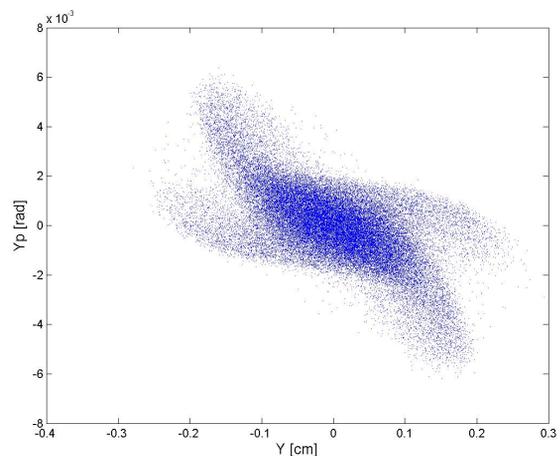


Figure 5: Plot of the beam distribution in Y phase space at the 88<sup>th</sup> gap.

Figure 6 shows the ratio of the output emittance over the input emittance for the two cases of crossing the resonance from below and from above. Two groups of

data show distinct difference due to the resonance characteristics.

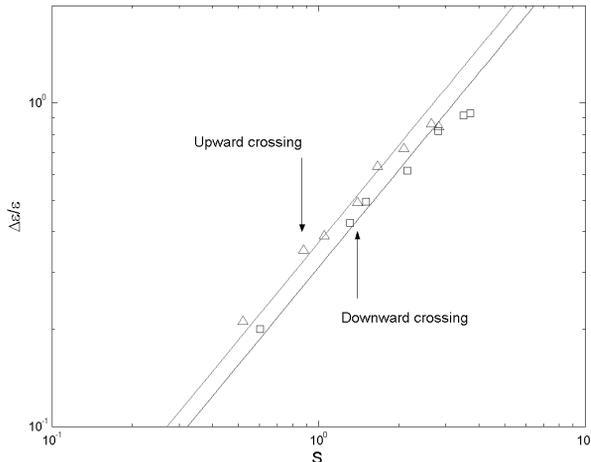


Figure 6: Plot of the beam distribution in Y phase space at the 88<sup>th</sup> gap.

### Measurement of the Stopband

One of the authors (Jeon) proposed a beam experiment in April 2007 to measure the stop-band of the  $4\sigma=360^\circ$  (or  $4\nu=1$ ) resonance using the GSI UNILAC. The experiment conducted in December 2008 confirmed this resonance. Figure 7 shows the simulation of  $(\epsilon_x + \epsilon_y)/2$  vs. zero current phase advance  $\sigma_0$  of this resonance for the UNILAC, which turned out very close to the experiment data. About 20% of rms emittance increase is shown due to the resonance peaked around  $\sigma_0 = 100^\circ$ . Experimental data is presented in a separate paper in this conference [11]

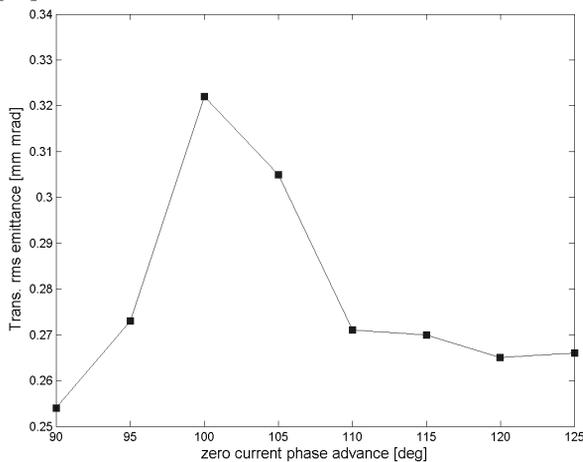


Figure 7: Plot of the sum of the normalized rms emittance  $(\epsilon_x + \epsilon_y)/2$  vs. zero current phase advance

### Envelope Instability??

High intensity linac design has avoided the  $90^\circ$  phase advance because of the well known envelope instability. Our study indicates that the  $4\sigma=360^\circ$  (or  $4\nu=1$ ) resonance is dominating over the envelope instability and practically replacing it. We did not observe the envelope instability during the simulation for the phase advance around  $90^\circ$ .

For instance, the beam distribution of Figs 3 and 5 show fourth order resonance structure and do not show a sign of the envelope instability. The envelope instability induces erratic coherent change in the entire beam. It should be noted that the initial beam is well matched to the linac.

Considering this, it should be stated that the high intensity linac design should avoid  $90^\circ$  phase advance because of the  $4\sigma=360^\circ$  (or  $4\nu=1$ ) resonance rather than the better known envelope instability. The effect of the envelope instability can actually be minimized – in theory - by nearly perfect envelope matching, whereas the  $4\sigma=360^\circ$  (or  $4\nu=1$ ) resonance is independent of the rms matching.

## CONCLUSION

The  $4\sigma=360^\circ$  (or  $4\nu=1$ ) resonance of a linac is demonstrated through space charge potential when the depressed tune is around  $90^\circ$ . It is observed that this fourth order resonance is dominating over the better known envelope instability and practically replacing it. It needs to be rephrased that the high intensity linac design should avoid  $90^\circ$  phase advance because of the  $4\sigma=360^\circ$  (or  $4\nu=1$ ) resonance rather than the better known envelope instability. This resonance was experimentally confirmed and presented at a separate paper in this conference.

## ACKNOWLEDGEMENT

This work is a result of the collaboration between GSI-FAIR and SNS. One of the authors (D.J) is grateful for the hospitality of GSI and the support through the EU-FP6 CARE-HIPPI (contract number RII3-CT-2003-506395). SNS is managed by UT-Battelle, LLC, under contract DE-AC05-00OR22725 for the U.S. Department of Energy.

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