

# EPICYCLIC HELICAL CHANNELS FOR PARAMETRIC RESONANCE IONIZATION COOLING\*

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## Abstract

In order to achieve cooling of muons in addition to 6D helical cooling channel (HCC) [1], we develop a technique based on a parametric resonance. The use of parametric resonances requires alternating dispersion, minimized at locations of thin absorbers, but maximized in between in order to compensate for chromatic aberrations [2]. These solutions can be combined in an Epicyclic Helical Cooling Channel (EHCC) that meets requirements of alternating dispersion of beam periodic orbit with best conditions for maintenance of stable beam transport in a continuous solenoid-type field [3]. We discuss here basic features and new simulation results for EHCC.

## PARAMETRIC RESONANCE IONIZATION COOLING

Muon beam ionization cooling is a key element in designing next-generation high-luminosity muon colliders. To reach adequately high luminosity without excessively large muon intensities, it was proposed to combine ionization cooling with techniques using a parametric resonance [2]. In the linear approximation, a half-integer resonance is induced such that normal elliptical motion of  $x$ - $x'$  phase space becomes hyperbolic, with particles moving to smaller  $x$  and larger  $x'$  at the channel focal points. Thin absorbers placed at the focal points of the channel then cool the angular divergence of the beam by the usual ionization cooling mechanism where each absorber is followed by RF cavities. Further compensation for chromatic aberrations in this channel requires regions with large dispersion. On the other hand, the absorbers for ionization cooling have to be located in regions of small dispersion where emittance exchange can be applied to keep the beam momentum spread small. In order to satisfy both these requirements within a single transport line, we suggested [3] a design of a cooling channel characterized by alternating dispersion and stability provided by a (modified) magnetic field of a solenoid.

## BEAM TRANSPORT IN AN EPICYCLIC HELICAL SOLENOID

We demonstrate results of simulations for the muon trajectory for the proposed transport line. The magnetic

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field in the  $z$ -direction  $B_z$  is chosen uniform and constant, as provided by a straight long solenoid. The charged particle motion in this field is characterized by a cyclotron wave number  $k_c$ . Then we superimpose alternating transverse dipole fields  $B_{T1}$  and  $B_{T2}$  on the solenoid field  $B_z$ . Each transverse field is periodic as a function of  $z$ -position, with a period defined by wave numbers  $k_1$  and  $k_2$ :

$$B_T = B_{T1} + B_{T2} = |B_1| e^{ik_1 z} + |B_2| e^{ik_2 z}. \quad (1)$$

Such a structure of the magnetic field brings a new feature to the helical cooling channel, namely, a variable dispersion function that is modulated with a frequency proportional to  $(k_1 - k_2)$ . It can be shown (in a certain approximation) that if the following relation is satisfied,

$$\frac{|B_1|}{(k_1 - k_c)^2} = \frac{|B_2|}{(k_2 - k_c)^2}, \quad k_1 \neq k_2, \quad (2)$$

then the dispersion has periodic nodes with a frequency proportional to the difference  $(k_1 - k_2)$ .

Different ratios and relative signs  $k_1$  and  $k_2$  result in different geometry of transverse magnetic fields, producing different constraints on the particle trajectories, and in some cases resemble planet orbits in Ptolemy's system of epicycles. Examples of such magnetic field configurations are shown in Figure 2, where the same-sign (opposite-sign) commensurate wave numbers  $k_1$  and  $k_2$  produce the curves known as epitrochoids (hypotrochoids).

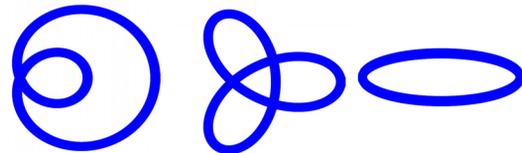


Figure 1: Transverse-plane projection of magnetic field lines in the epicyclic transport line as described by Eq.(1):  $k_1=2k_2$  (left plot);  $k_1=-2k_2$  (middle plot);  $k_1=-k_2$  (right plot).

We have solved numerically with *Mathematica* the equations of motion for a muon moving in such a double-periodic magnetic field and identified corresponding periodic trajectories.

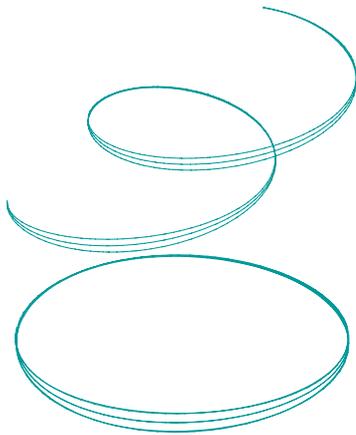


Figure 2: G4beamline simulation of muon trajectories in the epicyclic helical solenoid field Eq. (1) in the lab system (upper plot) and its projection on the transverse plane (lower plot). Kinematics are specified in the text.

To maximize efficiency of the epicyclic helical cooling channel, we choose wave numbers  $k_1$  and  $k_2$  compatible with the cyclotron wave number  $k_c$ . It appears possible to find a periodic orbit in this case that has a spatial period defined by wave numbers  $k_1$  and  $k_2$ . The results of analytic approximations and numerical calculations with Mathematica were further compared with simulations done with the GEANT4-based G4beamline code [4] and appeared to be in good agreement. Figure 1 demonstrates results of G4beamline simulations for 100 MeV/c muons (central line) with the parameters chosen as follows:  $k_1 = -k_2 = -k_c/2$ ,  $B_z = 7T$ ,  $B_1/B_2 = 9$ . Two additional lines show simulated particle trajectories with momenta 95 MeV/c and 105 MeV/c and demonstrate dispersion properties of this beam transport channel. The features of oscillating dispersion in EHSS satisfy requirements for the parametric-resonance ionization cooling channel [2,3]. In this case, absorbers for ionization cooling can be located in the areas of small dispersion in order to reduce the impact of straggling. The regions of large dispersion will be used to insert beam line elements in order to correct for chromatic and spherical aberrations.

Above examples are based on simplified magnetic fields used to demonstrate main features of this transport channel resulting in oscillating dispersion. We also used realistic magnetic fields with multipole expansions that confirmed the conclusions above.

### DESIGNING THE EPICYCLIC HELICAL COOLING CHANNEL

There are several technical possibilities for implementation of an epicyclic HCC. The most straightforward one would be a direct superposition of transverse helical fields, each having a selected spatial period. Another possibility would be along the lines suggested by V. Kashikhin and collaborators for single-periodic HCC, c.f. [5] and references therein. A series of close and parallel conducting rings centred on the parametric curves as shown in Figure 1 may produce the

desired double-periodic spatial structure of the epicyclic HCC. In this case the simplest solution is elliptic, as shown in Figure 3. Direct calculations corroborate this approach, as demonstrated in Figure 3.

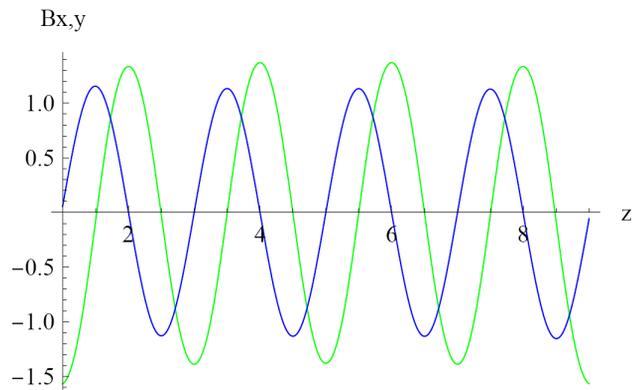
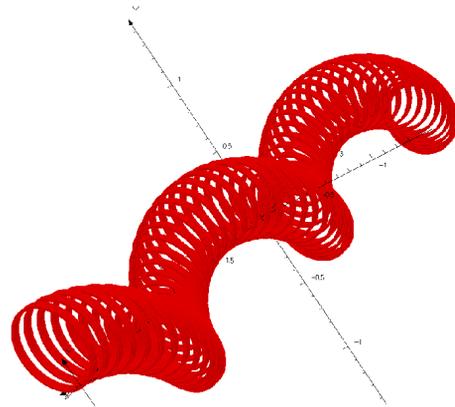


Figure 3: A series of current loops centred along an ellipse in the transverse plane (upper plot) and an example of magnetic fields (on central axis) that it generates (lower plot).

### REFERENCES

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