LONGITUDINAL SPACE CHARGE EFFECTS NEAR TRANSITION *

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Abstract

Experimental and numerical studies of the longitudinal beam dynamics in the Small Isochronous Ring (SIR) at Michigan State University revealed a fast, space-charge driven instability that did not fit the model of the negative mass instability. This paper proposes a simple analytical model explaining these results. Also, the paper compares the model to results of experimental and numerical studies of the longitudinal beam dynamics in SIR.

INTRODUCTION

Experimental and numerical studies of space charge effects in SIR conducted at Michigan State University in 2003-2004 revealed a fast, space-charge driven longitudinal instability [1],[2]. The instability was observed in the isochronous regime and also below transition (with a negative slip factor). In the isochronous regime, the instability growth rate scaled linearly with the beam current. This beam behavior contradicted predictions by the conventional model of the negative mass instability and originally was not well understood.

The instability observed in SIR can be explained if the transverse horizontal component of the coherent space charge field and its effect on the coherent longitudinal motion are included in the model. This transverse field, caused by a deformation of the beam shape, effectively increases the dispersion function and enhances the negative mass instability. In this paper, using a simplified model, we demonstrate that the transverse space charge field can noticeably affect the longitudinal beam dynamics in the isochronous regime. Also, we describe results of experimental and numerical studies of the beam dynamics in SIR and compare them to the model predictions.

ENHANCEMENT OF NEGATIVE MASS INSTABILITY

The space charge impedance has a maximum at a wavelength comparable to the beam diameter. In the short wavelength limit, when the effect of image charges can be neglected, the field on the beam axis can be obtained by simple integration. For a round beam with a uniform transverse distribution, this yields:

$$Z_{||}(k) = i\frac{2Z_0R_0}{ka^2\beta}\left(1-\frac{ka}{\gamma}\cdot K_1\left(\frac{ka}{\gamma}\right)\right), \tag{1}$$

where Z_0 is the characteristic vacuum impedance, R_0 is the average machine radius, γ and β are the relativistic factors, a is the beam radius, K_1 is the modified Bessel function of the second kind. The impedance reaches its maximum value at the wavelength approximately equal to 2.5 beam diameters.

An energy perturbation with a short wavelength causes a horizontal offset of the beam centroid. Accompanying these horizontal wiggles is a transverse horizontal electric field (see Fig. 1). This field is generated by two sources: a) interaction between radially shifted parts of the beam and b) image charges induced on the vertical (side) walls of the vacuum chamber. In calculating the coherent field, we assume that: (i) both the perturbation of the linear charge density and the distortion of the horizontal beam shape are small; (ii) the coherent transverse force is caused only by the interaction between shifted parts of the beam; (iii) the beam cross-section is axially symmetric (round), which is a good first approximation for SIR [1]; (iv) the beam is non-relativisitic ($\gamma = 1$), which is a valid assumption for SIR.

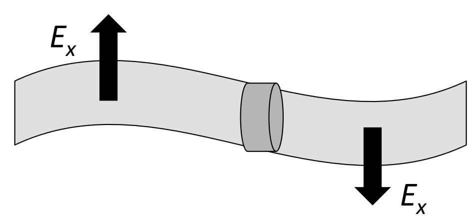

Figure 1: Schematic drawing of a horizontal beam shape distortion caused by energy modulation.

Assuming that the horizontal offset of the beam centroid is given by $x_c(z) = a_c\cos(kz)$, we can write the x-component of the field at an arbitrary point (x, z) as

$$\begin{aligned} E_x(x,z) &= \pi\rho a^2\int_{-\infty}^{\infty}\frac{(x-a_c\cos(k\bar{z}))\mathrm{d}\bar{z}}{(a^2+(z-\bar{z})^2)^{3/2}} \\ &= 2\pi\rho(x-x_c(z)ka\cdot K_1(ka)). \end{aligned} \tag{2}$$

The instability in SIR develops on a time scale of ten turns, which is an order of magnitude slower than the period of betatron oscillations. Therefore, the particle motion can be represented as a sum of a betatron term and a term representing the slow motion of the beam centroid: $x = x_c + x_\beta$. Substituting this and (2) into the equation of particle motion yields the equation of motion for the beam centroid:

$$x_c'' + \frac{\nu^2}{R_0^2}x_c = \frac{1}{R_0}\frac{\delta p}{p} + \frac{2e\pi\rho(1-ka\cdot K_1(ka))}{m\beta^2c^2}x_c, \tag{3}$$

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where we used the axially-symmetric "smooth" approximation. The expression $2e\pi\rho x_c(1 - ka \cdot K_1(ka))$ in the second term on the right hand side of (3) is the electric field on the beam centroid, induced by the coherent distortion of the beam shape.

An off-momentum closed orbit solution of Eq.(3) easily can be found:

$$x_{\rm ss} \approx \frac{R_0}{\nu^2}\left(1 + 2\left(\frac{-\delta\nu}{\nu}\right)_{\rm sc}(1 - ka \cdot K_1(ka))\right)\frac{\delta p}{p}, \tag{4}$$

where we formally expressed the uniform charge density ρ via the incoherent space charge tune shift to simplify formulas:

$$\frac{\pi e \rho}{m\beta^2 c^2} = \frac{\nu^2}{R_0^2}\left(\frac{-\delta\nu}{\nu}\right)_{\rm sc}. \tag{5}$$

Thus, the coherent space charge field changes the dispersion function D by

$$\delta D \approx 2D\left(\frac{-\delta\nu}{\nu}\right)_{\rm sc}(1 - ka \cdot K_1(ka)) \tag{6}$$

This dispersion function variation changes the slip factor:

$$\begin{aligned}\eta &= \frac{p}{T}\frac{{\rm d}T}{{\rm d}p} = \eta_0 + \frac{\delta D}{R_0} \\ &\approx \eta_0 + 2\left(\frac{-\delta\nu}{\nu}\right)_{\rm sc}(1 - ka \cdot K_1(ka)),\end{aligned} \tag{7}$$

where η_0 is the bare slip factor (without space charge).

The growth rate of the microwave instability is given by the formula [3]

$$\tau^{-1}(k) = \omega_0\sqrt{-i\frac{\eta e I_0 k R_0 Z_{||}}{2\pi\beta^2 E}}, \tag{8}$$

where ω_0 is the revolution angular frequency, I_0 is the average peak current, and E is the beam full energy. Substituting (7) into (8) yields the growth rate of the negative mass instability enhanced by the transverse coherent field. If the machine bare optics is exactly isochronous ($\eta_0 = 0$), the growth rate is given by

$$\tau^{-1}(k) \approx 2\sqrt{2}\omega_0\left(\frac{-\delta\nu}{\nu}\right)_{\rm sc}(1 - ka \cdot K_1(ka)) \tag{9}$$

Thus, the growth rate of this instability at the transition is proportional to the beam peak current.

If the bare machine optics is set below the transition ($\eta_0 < 0$), the instability has a threshold current corresponding to the condition:

$$2\left(\frac{\delta\nu}{\nu}\right)_{\rm sc} \approx \eta_0. \tag{10}$$

Below transition, the maximum growth rate depends on the beam current I as

$$\tau_{max}^{-1} \sim I\sqrt{1 - \frac{I_{\rm th}}{I}} \tag{11}$$

where $I_{\rm th}$ is the threshold corresponding to the condition (10).

The Hamiltonian term $\epsilon x/R$, where ϵ is the energy deviation and R is the curvature of the reference beam trajectory, couples the betatron and the longitudinal motions. Thus, particles executing betatron oscillations also oscillate longitudinally within the beam. Therefore, the field of harmonics with a wavelength shorter than the transverse size of the beam is averaged out. Using this argument and Eq. (9), we can expect that harmonics with a wavelength approximately equal to the beam diameter will exhibit the strongest growth.

SIMULATION OF BEAM DYNAMICS IN SIR

We used the code CYCO [1] to simulate the beam dynamics in SIR. First, the beam dynamics in SIR was simulated in the isochronous regime. Fig. 2 shows the growth rate of five harmonics with the wavelength λ=0.6, 0.75, 1, 1.5, and 3 cm for four different beam intensities: 5, 10, 15 and 20 μA. The growth rates are normalized to the beam current. The normalized curves practically overlap in the region λ=1.5 cm and are close to each other in other regions. Thus, Fig. 2 demonstrates that (i) harmonics with λ=1-1.5 cm exhibits the strongest growth and (ii) the growth rate of the instability in the isochronous regime scales linearly with the beam current. This behavior can be explained if the x-component of the coherent space charge field is included.

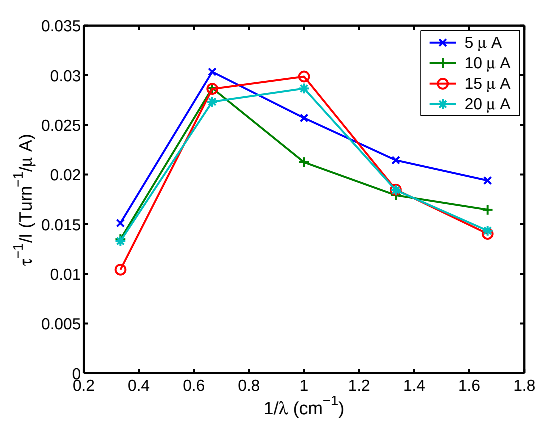

Figure 2: Amplitude growth rate for harmonics with the wavelength λ=0.6, 0.75, 1, 1.5, and 3 cm normalized to the peak beam current. The normalized growth rate shown for four different peak intensities: 5, 10, 15 and 20 μA.

Below transition ($\eta_0 = -0.04$), bunches with a peak current of 5 μA exhibited no instability. The other three simulated cases with a peak current of 10, 15 and 20 μA exhibited the unstable behavior. Fig. 3 shows the growth rate of the harmonic with λ=1.5 cm for the isochronous optics and the optics with the negative slip factor -0.04. The simulated data (solid lines with points) is fitted with corresponding fits: $y = 0.028x$ and $y = 0.028x\sqrt{1 - 7/x}$. Additionally, growth rates calculated directly from Eq. (9) and (11) for $a = 0.5$ cm and $\lambda = 1.5$ are shown. The growth rates predicted by the model exceed the simulated growth rates by approximately 20%.

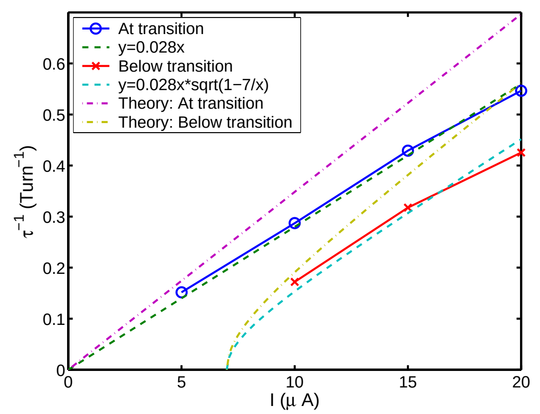

Figure 3: Growth rate of the harmonic with λ=1.5 cm for the isochronous optics and the optics with the negative slip factor -0.04.

EXPERIMENTAL RESULTS

Results of experimental studies of the instability in SIR presented in [2] show a very good agreement with simulation results [1], [2]. These experimental results show that the growth rate of the instability scales linearly with the beam current in the isochronous regime.

Fig. 4 depicts the spectrum of the linear charge density for a bunch with a peak beam current of 5 μA. Similarly to the simulations presented in the previous section, harmonics with $\lambda \approx 0.7 - 2$ cm exhibit the strongest growth.

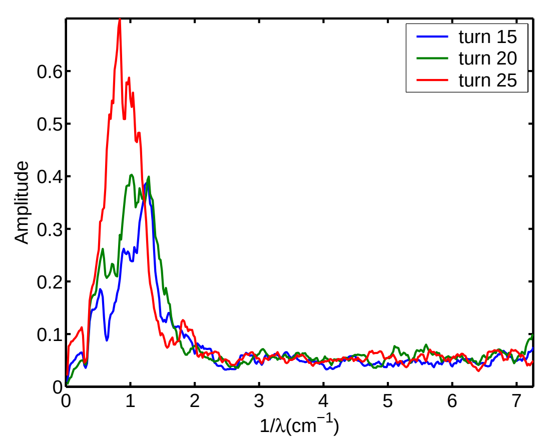

Figure 4: Spectra of the bunch linear charge density for three different turns. The initial bunch peak current is 5 μA.

To demonstrate a finite threshold of the instability below the transition, the slip factor η was lowered to -0.04. In these conditions, bunches with a peak beam current of 5 μA did not exhibit unstable behavior up to 100 turns (see Fig.5. Also, compare to Fig. 3). After the beam current was increased to 15 μA, the beam has become unstable again. (No other current values have been tried so far.) In the isochronous regime, the instability could be easily observed in the 5μA beam after 10-15 turns.

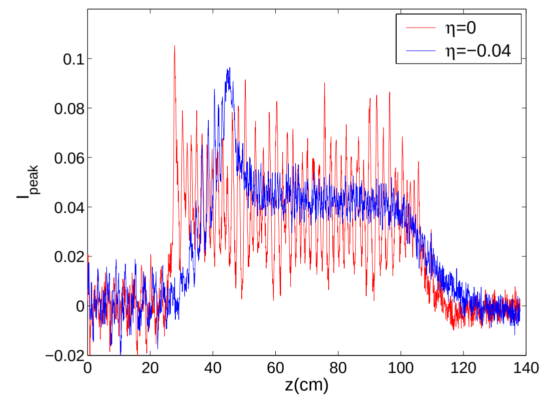

Figure 5: Longitudinal profiles measured by a fast Faraday cup after 50 turns. The bunch exhibits the instability in the isochronous regime (red curve) but shows no instability with the slip factor of -0.04 (blue curve). The beam peak current was 5 μA.

SUMMARY

The model presented in the paper proposes an enhancement mechanism of the negative mass instability. According to the model, (i) the transverse coherent electric field effectively increases the dispersion function and the slip factor, enhancing the negative mass instability; (ii) the instability growth rate in the isochronous regime scales linearly with the beam peak current; (iii) the instability has a threshold given by (10) if the slip factor is negative. In this case, the dependence of the growth rate on the beam peak current is given by (11); (iv) harmonics with a wavelength approximately equal to or somewhat lager than the beam diameter exhibit the strongest growth. The model describes the simulated and experimentally observed beam behavior in SIR with a reasonable accuracy. To be applicable to large scale, high energy machines the described model has to be extended to include relativistic effects and Landau damping.

REFERENCES

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