

## EFFECT OF WAKE FIELDS IN AN ENERGY RECOVERY LINAC\*

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### Abstract

Wake fields arising from the discontinuities in the vacuum chamber produce energy spread. In an energy recovery linac (ERL), a spent beam is decelerated before it is dumped in order to use its energy for the acceleration of new beam. While the energy spread accumulated from wakes before deceleration is small compared to the beam's energy, it becomes more important relatively as the beam's energy decreases. Therefore, in an ERL, wakes can produce very significant energy spread in the beam as it is decelerated to the energy of the beam dump so that beam transport to the dump may become impractical. This effect can place a limit either on the maximum charge per bunch or on the wake field-budget for the ERL. As an example of these wake field effects, this paper discusses their impact for the present design of the Cornell ERL and estimates the effects for typical vacuum chamber components being considered.

### EFFECTS OF WAKEFIELDS IN AN ERL

As a charged particle beam passes through the accelerator's vacuum chamber, its electro-magnetic (E-M) fields interact with discontinuities in the chamber's cross-section producing two basic effects. The first is power flowing from the beam's fields heating the region around the discontinuity. If the energy loss  $\Delta U$  per bunch passage is not resonant, the power  $P$  loss is

$$P = \Delta U f_b = k(\sigma_z) q_b^2 f_b \equiv K_0 \left\{ \frac{k(\sigma_z)}{1 \text{ V/pC}} \right\}$$

where  $f_b$  is the average bunch repetition frequency,  $q_b$  is the charge per bunch,  $k(\sigma_z)$  is the loss parameter associated with the shape of the discontinuity for a bunch of length  $\sigma_z$  and  $K_0$  is the beam current-dependant scaling factor. If the frequency of a trapped, i.e non-propagating, TM mode of the vacuum structure is harmonically related to  $f_b$ , then it is possible to resonantly enhance the power dissipation proportional to  $Q$ , the quality factor of that mode. For typical bunch frequencies the enhancement can approach 100 to 1000 times the non-resonant power loss. This effect is particularly important when choosing the vacuum pipe's wave-guide cutoff frequency, which is close to the lowest trapped mode's frequency for small discontinuities to the chamber's shape. To put a scale to this effect, for a storage ring with a charge per bunch of 25 nC, an average repetition frequency of 36 MHz,  $K_0$  equals 22 KW, while for an ERL with 77 pC per bunch with the an average frequency of 1300 MHz,  $K_0$  is 77 W.

Assuming a vacuum structure with modest cooling can tolerate 200 W of dissipated EM beam power, then this structure has a maximum limit for the loss parameter of  $9 \times 10^{-3}$  V/pC for this storage ring and 26 V/pC for this ERL. For typical ERLs as long we avoid resonant excitation of trapped modes, the power lost into wake fields should not pose much of a limitation.

The second basic interaction is through the induced wake voltage seen by a reference particle traveling with the bunch displaced a distance  $s$  from a point moving along with the bunch. If this reference particle's position stays fixed with the respect to the bunch, then the induced voltage for one traversal of the ERL is

$$V_{||}(s) \Big|_{\text{entire ERL}} = q_b W_{||}(s) \Big|_{\text{entire ERL}}$$

where  $W_{||}(s)$  is the longitudinal wake at position  $s$ . This effect increases the energy spread  $\Delta E$  of particles within the bunch; for the highest energy beam in the ERL this is not serious. However, in an ERL as the beam decelerates the relative energy spread is magnified by the inverse of the beam's energy. Since the beam dump has a certain maximum energy acceptance  $\max\{\Delta E|_{\text{dump}}\}$ , there is a maximum acceptable total wake field for the ERL,

$$\max\{eV_{||}(s) \Big|_{\text{entire ERL}}\} \leq \kappa \max\{\Delta E|_{\text{dump}}\}$$

where  $\kappa$  is bounded by  $0 \leq \kappa \leq 1$ , and  $\kappa=0.5$  is a reasonable choice to allow for other sources contributing to a bunch's energy spread. So the maximum permissible wake field for the ERL having a charge per bunch  $q=77$  pC is then

$$\begin{aligned} \max\{W_{||}(s) \Big|_{\text{entire ERL}}\} &= \frac{1}{q_b} \max\{eV_{||}(s) \Big|_{\text{entire ERL}}\} \\ &\leq 0.5 \frac{1}{q_b} \max\{\Delta E|_{\text{dump}}\} \approx (0.5) \frac{5 \text{ MeV}}{77 \text{ pC}} \approx 32 \text{ kV/pC} \end{aligned}$$

where this example has a 10 MeV beam dump with an energy acceptance of 5 MeV. This places an impedance budget limit on the total ERL wake field of 32 kV/pC. If the impedance budget were to be exceeded for the actual vacuum system chamber components, then possibly either not all of the particles in the bunches will be able to decelerate to the planned beam dump's energy or the charge per bunch must be reduced.

For a single bunch the self-wake is generally strongest within the charge distribution and then decays after the bunch's passage. If any of the trapped modes within the vacuum structure are resonant, then the wakes from later bunches will cause the amplitude to grow over the filling time of that trapped mode, enhancing the total wake field.

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In order to determine whether a practical ERL would be affected by this wake field impedance budget limit, we chose a particular ERL design and a set of typical vacuum chamber components. To estimate the scale of the wake field, we examined vacuum chamber components for a 1.3 GHz CW ERL design under study at Cornell[1]. We also assume that there are no resonant trapped modes so that only the self-wake needed to be considered.

### EXAMPLES OF COMPONENTS

A number of common accelerator structures were considered in this study. To simplify the calculations cylindrical symmetry was assumed for almost all cases, generally permitting the use of ABCI[2] for wake field computations. All of the wake field results reported here assume a 1300 MHz CW beam of 77 pC bunches having a Gaussian shaped bunch with  $\sigma_z=0.6$  mm. Unless otherwise noted, the ERLs vacuum chamber was taken to be a round pipe of 12.7 mm inside radius. In each case the wake function was computed per component or per meter and those results were summed over the estimated number (or length) of the component. The difference of the positive and negative peak values within  $\pm 3 \sigma_z$  of the bunch's center indicates the contribution to the energy spread from the self-wake field.

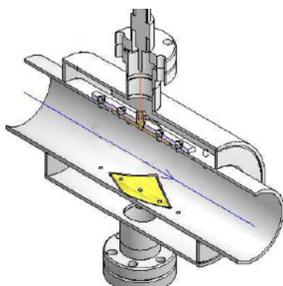


Figure 1: Ion clearing electrode vacuum chamber.

The RF accelerator structures for this design utilize superconducting cavities. The shape of the cavity cells are close to the shape of the TESLA accelerator design and, since the number of cells in a cryo-module is large in both cases (allowing the average wake field per cell to reach a limiting functional form), we scaled the TESLA wake field by the number of cells in the ERL design[3]. The ERL cryo-modules employ HOM loads attached to the entrance and exiting beam ports of the cavities, each of which is shared with the adjacent seven-cell RF structure. The two beam ports for each seven-cell module have different beam pipe diameters so the loads on the larger diameter beam pipes can couple to the lowest dipole passband of the cells. The wake fields for the vacuum chamber discontinuities for these loads were calculated using NOVO[4,5]. However, this wake field does not include the contribution from the ferrites on the surface of the HOM load, which is also the beamline. It is expected that this additional contribution will largely be similar to an enhanced resistive wall wake. The beam passes twice through the 400 seven-cell RF structures as it is first accelerated and then decelerated.

The model for the vacuum chamber wall roughness wake included two contributions, a resistive wake coming from an effective dielectric layer on the surface and an inductive wake as the wall currents flow over the rough surface[6,7]. For Gaussian bunches the wake function per unit length of chamber may be written as

$$\begin{aligned} \frac{d W_{\parallel}(\tau)}{dz} &= \frac{c}{\sqrt{2\pi} \sigma_z} \left[ L' \frac{c^2 \tau}{\sigma_z^2} \exp\left(-\frac{c^2 \tau^2}{2 \sigma_z^2}\right) \right. \\ &= \int_0^{\infty} dt \left\{ -R' \cos[k_0 ct] \exp\left(-\frac{c^2 (t-\tau)^2}{2 \sigma_z^2}\right) \right\} \end{aligned}$$

where  $\tau$  is the time relative to the center of the bunch,

$$R' = \frac{Z_0 c}{\pi b^2}, \quad k_0^2 = \frac{2 \epsilon}{(\epsilon-1) b \delta}, \quad L' = \frac{Z_0 \delta^2 \kappa_0}{8 \pi^2 c b} \left( \frac{q-2}{q-3} \right)$$

and where  $b$  is the chamber radius,  $\epsilon$  is the effective dielectric constant for the surface layer,  $\delta$  is thickness of the layer,  $\kappa_0$  is the correlation length of the features of the surface roughness and  $q$  is the inverse power law for the wave number for a fractal landscape model of the surface. For the calculations we used  $\epsilon=1.4$ ,  $\kappa_0=1/\delta$ ,  $q=3.75$ , and surface roughness layers of  $\delta=3 \mu\text{m}$  and  $0.5 \mu\text{m}$  for pipes of radius  $b=12.7$  mm (normal chambers) and 3 mm (undulator chambers), respectively. The resistive wall wake fields of the beam pipes were approximated with a simple model having a surface resistance characteristic of stainless steel with a skin depth set by the frequency,  $f=c/2\pi \sigma_z$ . Although this not the correct wake function, it sets an approximate peak value for the actual wake field.

For expansion joints we modeled a telescoping structure with tapers at both the transitions. This was modeled for both the compressed and expanded configurations and the larger wake field was used for the full ERL model. A flange joint with a copper gasket and knife-edge seals, having two gaps along the vacuum chamber wall, was the geometry employed for the wake field simulation. Gate valves were modeled as structures, which tapered outward to the moveable slider and tapered back to the beam pipe's diameter. Two types of BPMs were studied. The button type, which has a 1 mm gap between the electrode and the wall, was simulated as a single ring electrode encompassing the entire beam circumference. The second type of BPM, a stripline having a 2 mm gap at one end and being shorted to the beam pipe wall at the other end, was also modeled as a discontinuity over the entire chamber's circumference. To make these approximations closer to the real geometry of a BPM, whose electrodes produce discontinuities to the chamber wall only over a small fraction of the chamber's circumference, the wake fields were scaled by the ratio of the total electrodes' transverse extent to the chamber's circumference. Shown in Figure 1, a preliminary design for a diamond-shaped ion-clearing electrode was modeled by Yie using MAFIA [8,9]. The beam in the ERL is

expected to pass through a number of undulators requiring a beam pipe radius of 3 mm. Cylindrical tapered sections at the transitions to the undulator beam pipe were also modeled.

### SUMMARY OF RESULTS

The results of the wake field and loss parameter calculations are summarized by vacuum chamber component in Table 1. This includes a typical number of components, the peak negative and positive going wake fields and the total loss parameter for all components. Since in general the maxima and minima of the wake fields for different components do not occur at the same positions, the individual wake functions must be summed to compute the total ERL wake field, which is seen in Figure 2. The peak-to-peak variation of the wake field over the bunch distribution is 30 KV/pC, essentially at the budgeted wake field impedance limit determined above. We see that the major contributors to the wake fields are the roughness of the 2.5 km of 12.7 mm radius pipe, the RF cavities, then the roughness of the 144 m of undulator chambers and the resistive impedance of the 2.5 km of beam pipe. The calculations gave a value of 24 KV/pC for the ERL loss parameter  $k_{total}$  and a predicted parasitic mode power of 180 KW for the 100 mA circulating beam.

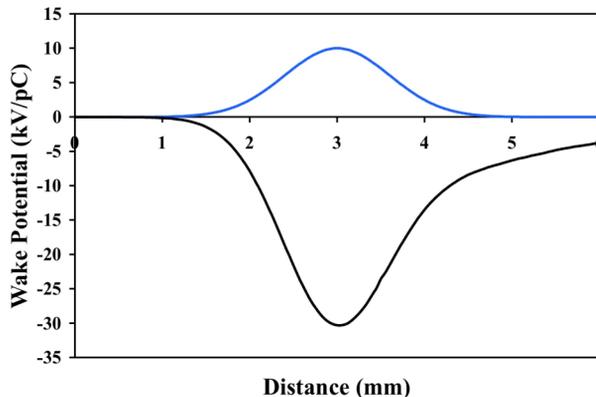


Figure 2: Total wake field (lower trace) for the components in Table 1 for the Gaussian-shaped bunch (upper trace.) The peak-to-peak wake field is 30 KV/pC.

### CONCLUSIONS

From these calculations it is clear that ERL designs need to consider the effect that wake fields from the short bunches at high repetition rates have on the parameters of the beam dumps. For the typical set of ERL vacuum components considered above, the total expected wake field would produce a 2.5 MeV energy spread in the beam for 77 pC bunches. This alone would require a 25% energy acceptance for a 10 MeV beam dump, effectively limiting the charge per bunch. One method to increase the bunch charge would be to increase the bunch length, which generally decreases the wake function. Alternately the energy of the beam dump could be raised, but this would increase the incident beam power, creating other problems. The shape of the total wake field in Figure 2 suggests third alternative: using a very high frequency RF

Table 1. Wake field and loss parameter contributions for the total number (or length) of each component in an ERL. The radius of the beam pipe is b.

Component	Number or Length	Total - Wake (KV/pC)	Total + Wake (KV/pC)	Total $k_0$ (KV/pC)
7 Cell RF Cavity	2x400	-6.47	0	5.81
HOM Load (b = 78 mm)	400	-0.89	0	0.64
HOM Load (b = 106 mm)	400	-0.50	0	0.36
Expansion Joint	356	-0.74	0.10	0.53
BPM (Button)	664	-0.35	0	0.24
BPM (Stripline)	20	-0.01	0	0.01
Flange Joint	356	-0.90	0	0.64
Clearing Electrode	150	-0.18	0.14	0.04
Gate Valve	68	-0.71	0.69	0.42
Resistive Wall (b = 12.7 mm)	2500 m	-4.00	0	2.75
Roughness (b = 12.7 mm)	2500 m	-14.00	0.50	8.75
Undulator Taper (b = 3 mm)	18	-0.61	0.37	0.36
Resistive Wall (b = 3 mm)	144 m	-0.98	0	0.69
Roughness (b = 3 mm)	144 m	-3.60	0.12	2.52

accelerator structure could substantially reduce energy variation within the bunch.

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