MULTI-BUNCH SIMULATIONS WITH HEADTAIL

G. Rumolo, E. Métral, CERN, Geneva, Switzerland

Abstract

The HEADTAIL code has been used for many years to study the interaction of a single bunch with a localized or lumped source of electromagnetic perturbation, usually self-induced (impedance, electron cloud or space charge). It models the bunch as macroparticles and at each turn slices up the bunch into several adjacent charged disks, which are made to subsequently interact with the perturbing agent.

A first step toward the extension of HEADTAIL to multi-bunch simulations is presented in this paper. In this case, the bunches themselves are modeled as charged disks and are not sliced, which makes us lose information on the intra-bunch motion but can describe a zero mode interaction between different bunches in a train. The interaction of an SPS bunch train of 72 bunches with the resistive wall is studied as an example.

INTRODUCTION

The HEADTAIL code was originally developed to study the interaction of a single bunch with an electron cloud lumped in one or more locations in an accelerator ring [1]. Subsequently, it was extended to simulate also the interaction of a single bunch with a lumped wake field (modeling either a localized impedance source, like that of a specific equipment, or a kick approximation for a distributed source, such as the resistive wall from the beam pipe or a broad-band resonator including the integrated contributions of many elements) [2]. Although HEADTAIL is a versatile code in its range of applicability, it was conceived to study in detail only single bunch effects. In other words, its validity is limited to intra-bunch and single turn effects. In fact, the code defines the beam as an ensemble of macroparticles with a 6D Gaussian distribution in phase space (with different distributions optionally available in the longitudinal phase space) and longitudinally subdivided into a variable number of slices across which the macroparticles are free to move while they are executing synchrotron oscillations. This bunch interacts at every turn with the electron cloud or the chosen impedance source, and each macroparticle feels the integrated effect of the wakes left behind by all the slices previous to the one to which the macroparticle belongs. However, the code only tracks one bunch, as if it was the only one circulating in the accelerator. No information exists on the possible effects of the passage of previous bunches through the same impedance source over the same turn. Furthermore, the "wake" memory is lost over subsequent turns. No information is retained on the possible trailing effects of the passages of the bunch itself through the same impedance source on previous turns. The current version of the HEADTAIL code can easily allow for a pure multi-bunch mode, if we adopt a simplified description of the bunches as flat disks (i.e., neglecting the longitudinal extension of each bunch). The multi-turn feature would still be not included, which implies that this model is suited to machines operating with trains of bunches and long gaps, and whose wake fields are important over one train length, but have significantly decayed over one turn. Section 2 explains how this simple multi-bunch extension of HEADTAIL is obtained from the present code, while in Section 3 an application to SPS with an LHC train under the effect of resistive wall is discussed. In this section, also results from an analytical approach are compared to the simulation results. The conclusions are drawn in Section 4.

HEADTAIL IN MULTI-BUNCH MODE

The idea of converting the current HEADTAIL code into a simple multi-bunch code, capable of simulating the zero mode oscillations of a bunch train, is based on two existing features of the code, i.e. the possibility to simulate longitudinally flat bunches and to freeze the longitudinal motion. The only coding step needed for the construction of a simple multi-bunch model was to create a new option which would merge the flat bunch (rectangular distribution in the longitudinal direction) with frozen motion in the longitudinal plane. This new option can be seen as the generation of a train of point-like bunches instead of a single bunch. In fact, by simply setting the bunch length to the train length and the number of slices to the number of bunches in the train, we automatically generate a sequence of transversely Gaussian disks separated by the interbunch gap. What was before the total number of particles in the bunch will be in this mode the total number of particles in the train. The motion of macroparticles across slices (namely, bunches in this case) will be naturally inhibited by having frozen the longitudinal motion. The price to pay with this use of HEADTAIL is that we lose the intra-bunch description of the beam. Each uniform disk, as is created at the moment of the beam generation, is a delta function in the longitudinal direction and the effect of synchrotron motion on the coupled-bunch phenomena we study cannot be assessed.

APPLICATION TO THE SPS

An LHC-type bunch train is known to suffer from coupled bunch resistive wall instability in the SPS, if no stabilization is applied through a transverse feedback system [3]. A possible set of SPS parameters, under which the resistive wall instability has been observed during machine studies, is given in Table 1. We will refer to these number both for the analytical calculation and the simulation study presented below.

Table 1: SPS parameters used in our study

Parameter	Symbol	Value
Momentum	p_0 (GeV/c)	26
Norm. transv. emitt.	$\epsilon_{x,y}$ (μm)	3.0, 3.0
Bunch length	σ_z (ns)	0.8
Bunch spacing	ΔT_b (ns)	25
Bunch population	N	5×10^{10}
Number of bunches	N_b	72
Tunes	$Q_{x,y}$	$26.13, 26.18$
Average beta functions	$\beta_{x,y}$ (m)	$41, 41$
Momentum compaction	α_p	0.00192
Circumference	C (m)	6911
Period	T_0 (μs)	23
Pipe half height	b (cm)	2.1
Chamber conductivity	σ ($\Omega^{-1}\mathrm{m}^{-1}$)	10^6

A model including the resistive wall effect over the train length but not over subsequent turns can be justified for this case because the ratio between the bunch-to-bunch wake and the tail-to-head wake (the closest wake neglected in the computation), i.e. $|W_\perp(cT_b)/W_\perp[cT_0-(N_b-1)cT_b]|$, is about 30.

An Analytical Approach

The theory of the transverse coupled-bunch instability in circular machines is usually discussed using Sacherer's formula in the frequency domain [4], which extended the results of Ref. [5]. It holds for any wake field in the case of equipopulated and uniformly spaced bunches. However, when the gap is much larger than the train, it is better to make a time-domain analysis, as shown in the following. This approach was already used in the past to predict transverse coupled-bunched instability rise times in the SPS [3]. It neglects the intra-bunch motion, as HEADTAIL does for the moment, but takes into account the wake field from all the preceding bunches and from all the previous turns. In the case of the resistive-wall impedance, the equation of motion for the bunch l (at azimuthal coordinate s), subjected to forces exerted by all the preceding bunch k (at azimuthal position $(s+z_k)$) and all the bunches on previous turns, can be written as

$$\frac{d^2x_l(s)}{ds^2} + \left(\frac{Q_0}{R}\right)^2 x_l(s) = \sum_{k=1}^{M} f_{lk}x_k(s) \qquad (1)$$

where f_k are defined

$$f_{lk} = \chi^{l-k-1}\sum_{m=0}^{\infty} \mathrm{e}^{\left(i\frac{Q}{R}z_{km}\right)}\cdot$$

$$\cdot\left\{\frac{k_{RW}}{\sqrt{z_{km}}} - k_{IB}\mathrm{e}^{\alpha_{IB}z_{km}}\left[1-\mathrm{Erf}\left(\sqrt{\alpha_{IB}z_{km}}\right)\right]\right\} +$$

$$\chi^{k-l}\sum_{m=1}^{\infty} \mathrm{e}^{\left(i\frac{Q}{R}z_{km}\right)}\cdot$$

$$\cdot\left\{\frac{k_{RW}}{\sqrt{z_{km}}} - k_{IB}\mathrm{e}^{\alpha_{IB}z_{km}}\left[1-\mathrm{Erf}\left(\sqrt{\alpha_{IB}z_{km}}\right)\right]\right\}$$

and

$$k_{RW} = \frac{N_b e^2 F\sqrt{Z_0\rho}}{p\pi^{3/2}b^3} \quad k_{IB} = \frac{2N_b e^2 F\rho}{p\pi b^4} \quad \alpha_{IB} = \frac{4\rho}{b^2 Z_0}$$

Here, $x_l = X_l \mathrm{e}^{isQ/R}$ is the transverse position of bunch l, Q_0 is the unperturbed transverse betatron tune (the smooth approximation has been used in Eq. (1)), R is the machine radius, M is the total number of bunches in the beam (which might be composed of several batches or trains spaced by different gaps), $\chi(n)$ is equal to 1 if $n \geq 0$ and 0 otherwise, Q is the transverse betatron tune we are looking for, N_b is the number of particles per bunch, e is the elementary charge, F is the (dipolar) Yokoya factor for asymmetric structures, p is the beam momentum, b is the (smaller) half gap, Z_0 is the free space impedance, $\rho = 1/\sigma$ is the resistivity, and $z_{km} = (l-k)s_b + m2\pi R$ is the distance between bunch l and k, which is valid inside a bunch train (but can be generalized to any bunch position in the case of several trains), where m is the number of the preceding revolution. Note that a quadrupolar term, which arises for asymmetric geometries, could also be introduced in Eq. (1). The terms with index IB are the correction terms due to the inductive bypass at low frequency [2].
Equation 1 leads to an eigenvalue problem, which can then be solved numerically. From the imaginary part of the most critical eigenvalue the instability rise-time can be computed. Applying this approach to the numerical values of Table 1 yields the results summarised in the following Table 2 (considering only the current revolution, i.e. $m = 0$ in Eq. (1)).

Table 2: Computed rise times (turns)

	w/o ind. by-pass	w ind. by-pass
x	446	490
y	223	245

Simulation Results

By using the modified version of the HEADTAIL code, as described in the previous section, we have simulated the case of the SPS resistive wall instability for a train of LHC-type bunches with the parameters listed in Table 2. The chamber has been assumed to be flat, with the height given in the table and a much larger width. In terms of wake field, this means that both dipolar and quadrupolar components of the wakes have been included in the simulation, and their amplitudes are scaled by the Yokoya coefficients.
A quick instability appears along the bunch train, as can be seen from the plots in Figs. 1, in which the snapshots of the horizontal and vertical bunch-by-bunch centroid signals ($\Delta_{x,y}$) along the train over 5000 turns are superimposed. As expected from the model, the head of the train remains stable, whereas the tail starts oscillating with a pattern of wave gradually propagating toward the head of the train.

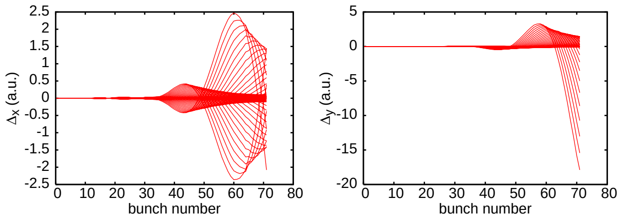

Figure 1: Evolution of the horizontal and vertical Δ signals over 5000 turns. Snapshots at different turns are superimposed. The head of the train is on the left side.

To extrapolate a global rise time of the instability, we refer to the exponentially growing motion of the centroid of the full train. Figure 2 shows the evolution of the horizontal and vertical centroids over the first 1500 turns. The vertical instability is faster than the horizontal one, because the unstable motion is mainly determined by the dipolar component of the wake, which is double in the vertical plane because of the flat chamber.

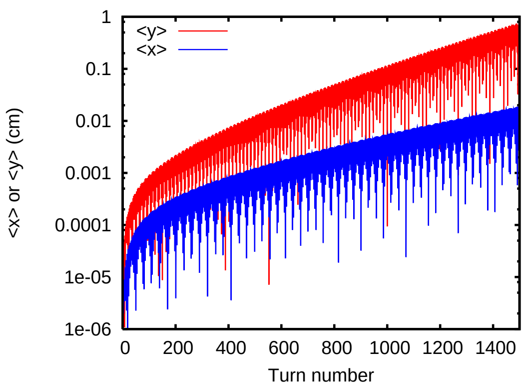

Figure 2: Horizontal and vertical evolution over 1500 turns of the global centroid of the train.

We have run simulations using both the classical thick wall resistive wake formula and with the modified formula including the inductive by-pass effect [2]. The wake field including the inductive by-pass effect exhibits a faster decay over the distance, which can become specially pronounced at the large distances or even over short distances for close walls. Figure 3 shows that the motion with the resistive wall wake with inductive by-pass is less unstable by a predictably marginal amount, given the small difference in the wakes over one train length. The summary of the rise times found from simulations is presented in Table 3. Not surprisingly, the rise times extrapolated in simulations from the evolution of the centroid of the full train are found to be about 40% longer than the rise times of the most critical coupled bunch eigenmode, which were computed in the previous section. We believe that the reason for that lies in that the rise times fitted from the simulated data intrinsically express a combination of the oscillation modes of all bunches. This results in a transient, during which all stable modes are damped, followed by an exponentially growing signal with a global rise time larger than the one corresponding to the most unstable mode alone. Furthermore, in simulations the horizontal rise times are found not to be exactly twice the vertical ones, which suggests that the quadrupolar wake field, included in simulations but neglected in the analytical approach, may also have an influence on the instability evolution.

Table 3: Simulated rise times (turns)

	w/o ind. by-pass	w ind. by-pass
x	595	625
y	330	350

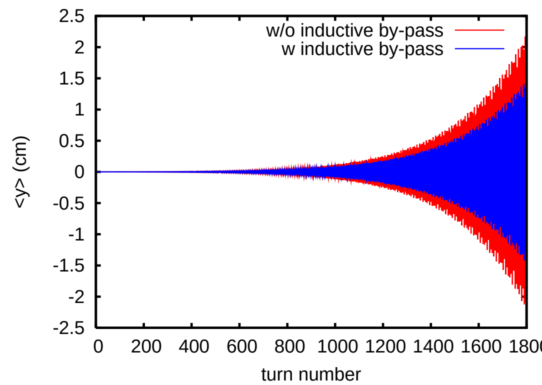

Figure 3: Vertical evolution over 1800 turns of the global centroid of the train.

CONCLUSIONS

In summary, we have explored the capability of the HEADTAIL code to simulate multi-bunch effects with minor modifications, and established a simplified model that neglects the synchrotron motion within bunches as well as multi-turn effects. As a first exercise, we have applied this model to describe coupled bunch resistive wall phenomena in the CERN-SPS, which were both observed in absence of tranverse feedback and studied through an analytical approach. The rise times computed with HEADTAIL are about 40% larger than those associated to the most critical mode, which were computed using the theory. This can be explained because of the different way the rise times were calculated in the two cases. We plan to include in HEADTAIL both synchrotron motion and multi-turn effects in the near future.

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