

# STABILITY OF FLAT BUNCHES IN THE RECYCLER BARRIER BUCKET \*

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## Abstract

We examine the stability of intense flat bunches in barrier buckets used in the Fermilab Recycler. We consider some common stationary distributions and show that they would be unstable against rigid dipole oscillations. We discuss the measurements which identify stable distributions. We also report on experimental studies on the impact of creating a local extremum of the incoherent frequency within the rf bucket.

## INTRODUCTION

The creation of long flat bunches is under study for the LHC upgrade as a way of increasing the luminosity [1]. The stability of such bunches is one of the key issues of interest. At the Fermilab Recycler, long flat bunches are created using a barrier bucket technique. At present intensities these bunches are observed to be stable with lifetimes around 20-50 hours (depending on intensity) without electron cooling and  $\sim 500$  hours with electron cooling at bunch intensities below  $4.5 \times 10^{12}$ . We explore the longitudinal stability of these bunches at higher intensities.

Bunches are confined within a barrier bucket by two voltage pulses of equal magnitude and opposite polarities, equal duration  $T_1$  separated by a duration  $T_2$  with no applied voltage. The body of the bunch is contained within the interval  $T_2$  but the head and tail of the bunch penetrate into the barrier on either side. Changing  $T_2$  adiabatically changes the bunch length while preserving the bunch area. The main longitudinal parameters of the Recycler bunches are shown in Table 1. Single particle dynamics within such a bucket was studied in [2]. There are some factors that may have a potential impact on the stability. The synchrotron period is rather long, hence even slowly growing instabilities can drive the beam unstable. Furthermore, since the bunches are long, they can be excited by perturbations with wavelengths of the order of the bunch length or equivalently relatively low frequency excitations. In this paper we report on measurements of proton bunches in the Recycler and theoretical studies of the rigid dipole mode.

## STATIONARY DISTRIBUTIONS

In the absence of radiation damping, the equilibrium longitudinal distribution of a proton bunch is not uniquely determined. Any phase space distribution which is a function of the Hamiltonian only is a stationary solution of the Liouville equation. Consider first a binomial distribution of the form  $\rho(\Delta E, \tau) \propto [H_b - H]^p$  where  $H_b$  is the value of

Table 1: Recycler bunch parameters

| Parameter        | Value                 | Units            |
|------------------|-----------------------|------------------|
| Circumference    | 3331.0                | m                |
| Energy           | 8.93                  | GeV              |
| Bunch intensity  | $4.5 \times 10^{12}$  |                  |
| $\gamma_t$       | 19.97                 |                  |
| Rf voltage $V_0$ | 1.8                   | kV               |
| $T_1$            | 48.0                  | 0.0189 $\mu$ sec |
| $T_2$            | 324.0                 | 0.0189 $\mu$ sec |
| Bunch area       | 100.0                 | eV-sec           |
| $\nu_s^{max}$    | $9.97 \times 10^{-6}$ |                  |

the Hamiltonian  $H$  on the bunch boundary and  $p$  is a real number. The Hamiltonian for the longitudinal variables is  $H = -\eta(\Delta E)^2/(2\beta^2 E_0) - eU(\tau)/T_0$ . Here  $E_0, T_0$  are the energy and revolution period of the synchronous particle, and  $U(\tau)$  is the rf potential function. The line density obtained by projecting the phase space density on the time axis is  $\lambda = \lambda_0[U(\tau) - U_b]^{p+1/2}$  where the constant  $\lambda_0$  is found from the normalization condition and  $U_b$  is the potential function on the bunch boundary. For the special case  $p = 1/2$ , the line density is proportional to the rf potential and the potential well distortion due to space charge or external impedance simply changes the scale of the potential without changing its form. This is a unique feature of this elliptic distribution first pointed out by Hofmann and Pedersen [3]. Assuming that the rf pulses are rectangular with constant voltage  $\pm V_0$ , the normalized density for this distribution is

$$\lambda(\tau) = \frac{N_b}{\tau_b^2 - (T_2/2)^2} \{(\tau + \tau_b), (\tau_b - \frac{1}{2}T_2), -(\tau - \tau_b)\} \quad (1)$$

in the regions  $-\tau_b \leq \tau \leq -T_2/2$ ,  $-T_2/2 \leq \tau \leq T_2/2$ , and  $T_2/2 \leq \tau \leq \tau_b$  respectively. Here  $\pm\tau_b$  are the bunch boundaries and  $W_b = \tau_b - T_2/2$  is the extent of the penetration into the barrier. The rf voltage profile and the line density are sketched in Fig 1(a). The expression for the energy density  $\mu(\Delta E) = \int \rho(\Delta E, \tau) d\tau$  can be found in [4], its form is plotted in Fig 1(b).

The frequency of coherent rigid dipole oscillations for any distribution can be found from [4]

$$\omega_c^2 = \frac{e|\eta|}{\beta^2 E_0 T_0 N_b} \int_{-\tau_b}^{\tau_b} \lambda'(\tau) \frac{\partial U}{\partial \tau} d\tau \quad (2)$$

In general this will not coincide with the small or large amplitude frequency of incoherent oscillations. In the barrier bucket, the bare incoherent frequency (without intensity effects) rises from zero at the origin to a maximum value

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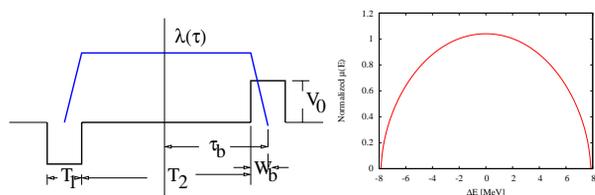


Figure 1: Sketch of the rf voltage and line density (left) and a plot of the energy distribution (right) in an elliptic distribution.

$\omega_{s0,max} = 2\pi[|\eta|eV_0/(32\beta^2 E_0 T_0 T_2)]^{1/2}$ . The ratio of coherent frequency to the maximum of the bare incoherent frequency is

$$\frac{\omega_c}{\omega_{s0,max}} = \frac{4}{\pi} \left[ \frac{T_2}{T_2 + W_b} \right]^{1/2} \quad (3)$$

Space charge shifts the incoherent frequencies downwards, thus the above ratio will be greater when space charge forces are included. For the Recycler parameters, this ratio is  $> 1$  suggesting that Landau damping would be lost even at zero intensity since the coherent frequency is outside the incoherent spread. We conclude that this is not an appropriate distribution for the Recycler since the bunches are observed to be stable; and furthermore the energy distribution seen in Figure 1 does not match the measured energy distribution.

Another simple choice of stationary distribution is the exponential distribution  $\rho \propto \exp[-H/H_0]$  where  $H_0$  is a scale constant. The line density is exponential  $\lambda(\tau) = \lambda_0 \exp[-eU(\tau)/(H_0 T_0)]$ . This line density is also constant between the pulses from  $-T_2/2$  to  $T_2/2$  and falls exponentially in the barriers. The energy distribution in this case is a Gaussian,  $\mu(\Delta E) = \mu_0 \exp[-(\Delta E)^2/(2\sigma_E^2)]$  where  $\sigma_E^2 = \beta^2 E_0 H_0 / |\eta|$ . The measured energy distribution is approximately Gaussian, as seen in Figure 3. The ratio of the coherent to the maximum of the bare incoherent frequency for this distribution is

$$\frac{\omega_c}{\omega_{s0,max}} = \frac{4}{\pi} \left[ \frac{1 - \exp(-\chi W_b)}{1 + (2/(\chi T_2))[1 - \exp(-\chi W_b)]} \right]^{1/2} \quad (4)$$

where  $\chi = \beta^2 E_0 e V_0 / (|\eta| \sigma_E^2 T_0)$ . This ratio for the Recycler parameters is greater than 1 suggesting that this distribution is also not appropriate for the Recycler bunch. The coherent frequency depends critically on the line density within the barriers, thus it is important to have a good description of the bunch tails.

## MEASUREMENTS

During normal operations, the Recycler electron cooled anti-protons for injection into the Main Injector and Tevatron. In a recent study protons were injected into the Recycler with a increasing number of Booster batches. The bunch profiles were measured with a wall current monitor at three different intensities. Longitudinal Schottky spectra

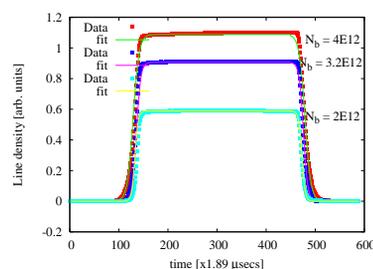


Figure 2: Comparison of the measured line density and fit to the data with a tanh profile at three different intensities.

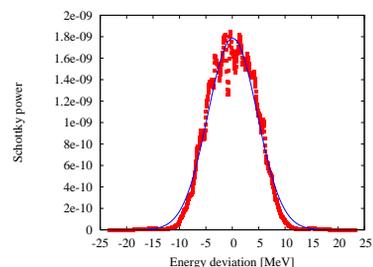


Figure 3: Measured Schottky power of a proton beam in the Recycler compared with a Gaussian fit.

were also recorded. The measured bunch length profiles are well described by the function

$$\lambda_{fit}(\tau) = a \{ [1 + \tanh(b\tau + c)] \theta(-\tau) + [1 - \tanh(b\tau - c)] \theta(\tau) \} \quad (5)$$

Here  $\theta$  is the step function. The measured profiles and the fit function are shown for all three sets in Figure 2.

For this line density, the ratio of the coherent to maximum of the bare incoherent frequency is

$$\frac{\omega_c}{\omega_{s0,max}} = \frac{4}{\pi} \sqrt{a T_2} \sqrt{\tanh[b\tau_b - c] - \tanh[\frac{1}{2}bT_2 - c]} \quad (6)$$

This ratio is 0.93 for the Recycler, i.e. at zero intensity the coherent frequency is within the incoherent spread. The coherent frequency is outside the spread and Landau damping is lost when the intensity exceeds  $8.5 \times 10^{14}$  [4], about 400 times higher than present intensities.

It has been observed in the SPS that a region where the incoherent frequency as a function of amplitude has an extremum is associated with the appearance of a local instability [5]. In a barrier bucket, the maximum of the incoherent tune occurs when the ratio of the peak energy on the orbit to the bucket height is  $\sqrt{T_2/(4T_1)}$ . If this ratio is greater than unity, as is the case for the nominal Recycler parameters  $T_2/(4T_1) = 324/(4 \times 48)$ , the maximum lies outside the bucket. The effects of this local instability are not present under normal operation but can be studied by changing the separation  $T_2$ .

These studies were carried out in the Recycler using protons. The Recycler is equipped with a broad band RF system [6] capable of providing a variety of rectangular barrier buckets at a time. For the experiment we chose  $T_1 = 48$

bkts (a 'bkt' =  $0.01893\mu\text{sec}$ ) and pulse height of 1.8kV. At the start protons were injected into the barrier bucket with  $T_2 = 84$  bkts. Once equilibrium was reached, beam intensity along with data from the wall current and Schottky monitors were recorded to establish the initial parameters. Subsequently the beam was expanded or compressed adiabatically by decreasing  $T_2$  to different values of interests without changing  $T_1$ . The beam cooling systems as well as the transverse dampers were turned off during the experiment.

Fig. 4 shows wall current monitor and Schottky data on the beam in the barrier bucket with  $T_2 = 14, 44, 84, 130, 180, 228$  and  $276$  buckets with initial beam intensity of about  $29.7 \times 10^{10}$  protons. As the separation  $T_2$  and hence bunch length is decreased, the energy spread increased. This is seen in the Schottky data which shows the spectrum broadening as  $T_2$  was decreased. At the shortest separation  $T_2 = 14$  bkts the measured energy spread exceeds the rf bucket height. For values of  $T_2 \geq 84$ bkts, the energy spread was well within the rf bucket.

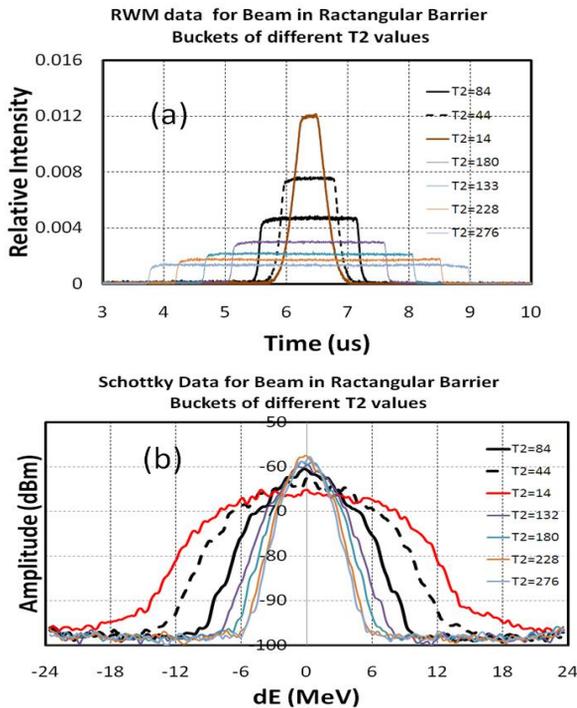


Figure 4: Recycler measurement data on beam in a rectangular barrier bucket (a) wall current monitor (b) Schottky spectrum for various  $T_2$  values with a fixed value of  $T_1=0.908\mu\text{s}$ . The barrier pulse heights were  $1.813\text{kV}$  in all these cases.

Fig. 5 shows the surviving beam as a function of time. Various  $T_2$  to  $T_1$  ratios along with the beam lifetime are also indicated. There are three distinct regions in this plot. The first section had  $T_2 < 4T_1$  with  $T_2$  assuming values of 14, 44, 84, 130, 180 bkts. The average lifetime of the beam at 8 hours was significantly lower than in the second section where it was about 50 hours with  $T_2 = 228, 276$  and

$> 4T_1$  in both cases. In the third region the beam was compressed to its original value of  $T_2 = 84$  bkts. The lifetime again decreased by about a factor of two. While some of the beam loss may have been due to the larger energy spread, we observe a shorter lifetime even when this was not expected to contribute. The data is therefore not inconsistent with the hypothesis that a local instability at the extremum of the incoherent synchrotron tune can lead to beam loss. The effects of this instability are easily avoided in the barrier bucket by choosing appropriate values of  $T_1, T_2$

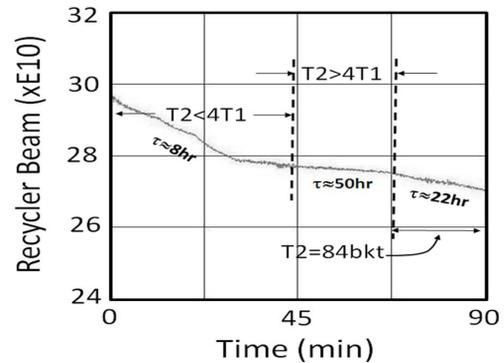


Figure 5: Recycler beam and its lifetime at various times with different  $T_2$  to  $T_1$  ratios.

## SUMMARY

We considered two typical stationary distributions and found they were not adequate descriptions of the Recycler bunches. From the measured line density distribution we find (a) the tanh function is a good fit to the line density, and (b) the coherent frequency of the rigid dipole mode for this distribution is within the incoherent spread at nominal intensities. Stability diagrams when the beam couples to space charge and external impedances will be discussed elsewhere. Our initial experimental investigations indicate that longitudinal stability in the Recycler is, consistent with expectations, influenced by the ratio  $T_2/(4T_1)$  which determines the location of the extremum of the incoherent tune. The coherent tune depends strongly on the distribution in the bunch tails which is difficult to measure. Numerical studies using both a conventional tracking code and a Vlasov solver are in progress and should provide more insight into conditions that may lead to unstable behavior.

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