FAST HEAD TAIL INSTABILITY DUE TO ELECTRON CLOUD UNDER THE PRESENCE OF THE DISPERSION

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Abstract

Electron cloud causes a transverse single bunch instability above a threshold of the cloud density. The threshold is determined by the strength of the beam-electron cloud interaction and Landau damping due to the synchrotron oscillation and/or momentum compaction. We discuss that the threshold is remarkably degraded due to the dispersion, one of the parameter of the circular accelerator optics. The single bunch instability is more serious than previous predictions without considering the dispersion, especially in the case that the horizontal beam size due to dispersion dominates compare than that due to emittance.

INTRODUCTION

A head tail instability induced by electron cloud has been studied for ILC damping ring [1]. The threshold of the instability given by coasting beam model agrees with the simulations, in which arc transformation is expressed by a simple transfer matrix with a phase advance of ν/n_{IP}, where n_{IP} is the number of the interaction point locating electron cloud. In the simulation, an averaged beta function, which characterizes coupling between the beam and electron cloud, are taken into account, but the dispersion is not taken into account. Instability simulations including the lattice information had been carried out [1]. The threshold in the simulation was degraded to a half of the analytical estimate. One of the authors (K.O.) made sure that the dispersion made worse the instability, comparing the simulated threshold for the lattice without dispersion. In this paper, we reanalyze the dispersion effect, and explain the mechanism why the dispersion degrades the instability threshold.

SIMULATION RESULTS UNDER THE PRESENCE OF DISPERSION

Figure 1 shows the beam size evolution for three cases. Parameters of the ring are as follows; Circumference, $L = 3016$ m, emittance $\varepsilon_y = 2$ pm, beta function, $\beta_x = \beta_y = 30$ m, synchrotron tune, $\nu_s = 0.011$. Plot (a) depicts for $\eta = 0$ m. The threshold is clearly seen as 1×10^{11} m^{-3}. Plot (b) depicts for $\eta = 1$ m, where the horizontal beam size increases $\sqrt{\beta_x \varepsilon_x + \eta^2 \sigma_\delta^2}$. The threshold degrades 0.4×10^{11}. Plot (c) depicts for $\varepsilon_x = 0$, $\eta = 1$ m, where the horizontal beam size is given only by the dispersion $\sigma_x = \eta\sigma_\delta$. The growth is seen even very low cloud density 0.2×10^{11} m^{-3}. We have to note that there are no big difference in the growth rate above the threshold. It seems that only threshold degrade due to the dispersion. We know the threshold is determined by the growth and the Landau damping due to synchrotron motion. Landau damping does not seem to work well for a finite dispersion.

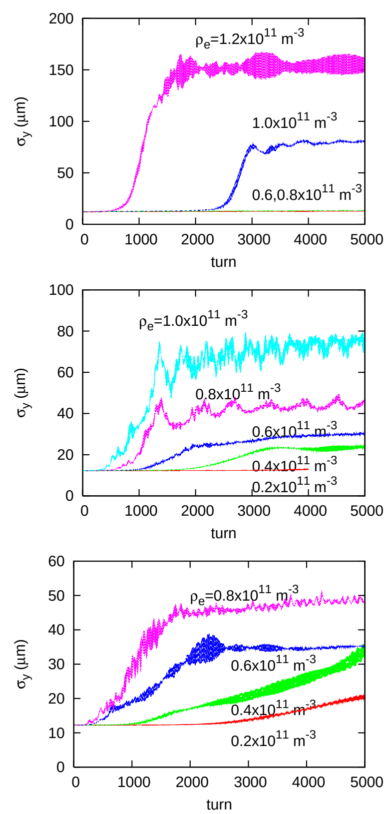

Figure 1: Growth of the vertical beam size caused by the electron cloud induced fast head-tail instability. Top is for $\eta = 0$ m, centre is for $\eta = 1$ m, and bottom is for $\varepsilon_x = 0$, $\eta = 1$ m.

Electron cloud in the bending magnets also gives the similar results: that is, the dispersion degrades the instability threshold.

WAKE FIELD DUE TO ELECTRON CLOUD UNDER THE PRESENCE OF DISPERSION

As is well-known the horizontal dispersion should not affect the head tail instability in horizontal nor vertical.

The instability theory without dispersion which is analogy of the ordinary head-tail theory, is outlined as follows. Electrons oscillate in a bunch with a frequency,

$$\omega_{e,y} = \sqrt{\frac{\lambda(s) r_e}{\sigma_y(\sigma_x+\sigma_y)}}c, \tag{1}$$

where the beam-electron force is linearized. Note this frequency actually strongly depends on x [4]. A tranverse wake field with the frequcncy component is induced by the electrons as,

$$W_y(z-z') = K\frac{\lambda_e r_e}{\lambda_+}\frac{L}{\sigma_y(\sigma_x+\sigma_y)}$$
$$\exp(\omega_e(z-z')/2Qc)\sin\left[\frac{\omega_e}{c}(z-z')\right]. \tag{2}$$

The wake force causes the fast head-tail instability. The stability criteria in the coasting beam model is expressed as follows,

$$\frac{\sqrt{3}KQr_e\beta L\rho_e}{2\gamma\nu_s\omega_e\sigma_z/c} = 1, \tag{3}$$

where $K\omega_e\sigma_z/c$, $Q = \min(\omega_e\sigma_z/c, Q_{nl})$ and $Q_{nl} \approx 10$ [1, 2]. In the mode coupling theory, the threshold is determined by solving an eigenvalue problem for a matrix [3]

$$M_{\ell\ell'} = \ell\delta_{\ell\ell'} - i\frac{Nr_eL}{16\pi^3\gamma\nu_\beta\nu_s}i^{\ell-\ell'}$$
$$\int_{-\infty}^{\infty} d\omega Z_\perp(\omega)J_\ell(\omega z/c)J_{\ell'}(\omega z/c) \tag{4}$$

where the airbag model is used.

We now discuss why the dispersion affects the head-tail instability induced by electron cloud. The wake force in Eq.(2) is not right in two point of view. One is that the frequency depends on the horizontal position, and another is electrons oscillate also in horizontal direction. One more point is that the horizontal position of the beam particles are represented by the energy deviation $x = \eta\delta$ but not horizontal betatron coordinate x_β, when the dispersion dominates for the beam size ($\eta^2\sigma_\delta^2 > \beta_x\varepsilon_x$).

The electrons in a bunch oscillate in horizontal plane with the frequency,

$$\omega_{e,x} = \sqrt{\frac{\lambda(s) r_e}{\sigma_x(\sigma_x+\sigma_y)}}c, \tag{5}$$

A vertical perturbation given at x are transferred to another horizontal position x', and $\omega_{e,y}(x)$ is a function of x. Therefore the wake field has a complex function of (x, x', z, z'),

$$F_y(x,z) = \frac{N_p r_e}{\gamma}\int_{z'}^{\infty}\int W_y(x,x',z,z')\rho_y(x',z')dx'dz' \tag{6}$$

where ρ_y is the vertical dipole moment at a position (x', z') in a bunch.

$$\rho_y(x,z) = \int y\psi(x,p_x,y,p_y,z,\delta)dydp_xdp_yd\delta \tag{7}$$

The oscillation is suppressed in a bending magnet due to very fast frequency and small radius cyclotron motion. However since the vertical oscillation frequency depend on the horizontal coordinate, the wake force has a structure like $W(x,x',z,z')\delta(x-x')$.

The wake field with these complex features is given by the numerical method [2]. The wake field is a kind of Green function, which is response for a delta function like distortion of distribution, $\rho_y(x,y) = \delta y\rho_0(x,y)\delta(x-x')\delta(z-z')$. The force $F_y(x,y)$ induce by the distortion is just the wake field. A part of bunch in (x', z'), which interact with electron cloud, are shifted in vertical, and calculate kick, which experiences following part of bunch.

$$W_y(x,x',z,z') = F_y(x,z)\delta_y\rho_0(x',z') \tag{8}$$

Figure 2 shows the wake field in the $z-x$ plane. Plot (a) depicts the ordinary wake field as a reference. It is planar wave along z and has no structure in x. Plots (b) and (c) depicts the wake field for drift space and bending magnet, respectively. It is seen that the horizontal motion of electrons with the frequency ($\omega_{e,x}$) transferred the force in $z-x$ plane in Plot (b). Plot(c) shows the characteristics in which electrons in bending magnet do not move in x direction. Since the beam displacement at (x', z') affects electrons near x', the wake does not have the form $W(x,x',z,z')\delta(x-x')$ exactly.

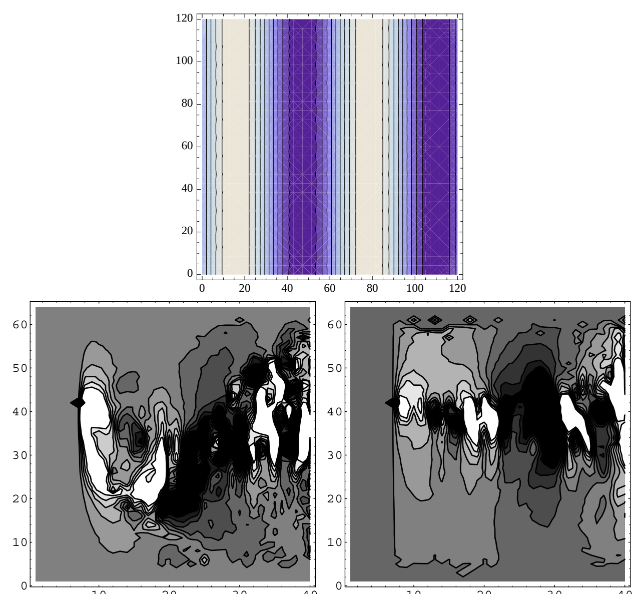

Figure 2: Wake field in $x-z$ plane. Hor. and Vert. axes are z and x, respectively. The ontour displays F_y. (top) ordinary wake, (bottom left) drift space, (bottom right) in bending magnet.

This characteristic of the wake field does not give remarkable effects, when x is mainly contributed from the betatron oscillation. Integrating the wake field over one revolution, the dependence on x is averaged and the wake force is approximated to Eq.(2), and the threshold in simulations agree well with Eq.(3).

The situation changes drastically, when the dispersion function and synchrotron motion dominates for the betatron motion in the horizontal beam distribution. The wake field is now function of z and δ, $W_y(z,z',\delta,\delta')$.

The mode coupling theory should be drastically modified as follows,

$$(\Omega-\omega_\beta-\ell\omega_s)\alpha_\ell R_\ell(r) \tag{9}$$
$$=\frac{r_e\nu_s c}{2\gamma\eta\nu_\beta L}g_0(r)\sum_{\ell=-\infty}^{\infty}\alpha_{\ell'}\int r'dr'V_{\ell\ell'}(r,r')R_{\ell'}(r')$$

where

$$V_{\ell\ell'}(r,r')=\int W_1(r,\phi,r',\phi')e^{-i\ell\phi+i\ell'\phi'}d\phi d\phi' \tag{10}$$

$z=r\cos\phi$ and $\eta L\delta/2\pi\nu_s=r\sin\phi$.

For the wake field which is only the function of $z-z'$, V is wellknown form,

$$V_{\ell\ell'}(r,r')=-\frac{i}{2\pi}\int d\omega Z(\omega)$$
$$\int e^{i\omega r\cos\phi/c-i\ell\phi}e^{-i\omega r\cos\phi/c+i\ell'\phi'}d\phi d\phi'$$
$$=\sum_p Z(\omega')J_\ell(\omega' r/c)J_{\ell'}(\omega' r'/c) \tag{11}$$

The diagonal component of $V_{\ell\ell}$ is real, if the wake field is symmetric for x or equivalently for δ and ϕ, $W_1(r,-\phi,r',-\phi')=W_1(r,\phi,r',\phi')$. The system is stable as far as considering only a mode. Conversely speaking, this means the system is unstable only when mode coupling between different ℓ for zero chromaticity. Off diagonal component $V_{\ell\ell'}$ is very small for the planar wake when $\omega_e\sigma_z/c\gg 1$.

UNSTABLE MODE IN SIMULATION

The simulation (PEHTS) gives unstable mode for the cases, drift space without dispersion, with dispersion, bending section with dispersion, as shown in Figure 3. When electron cloud in drift section without dispersion is dominant for the instability (top plot), the unstable mode depend only on z but is independent of δ. When the cloud in drift space with dispersion (centre plot) is dominant, the unstable mode depends on both of z and δ. When the cloud in bending section with dispersion is dominant, the unstable mode depends on both of z but no correlation for δ. These features of the unstable mode makes sense for the characteristics of the wake field in each case.

CONCLUSIONS

The threshold of the strong head-tail instability is remarkably degraded by the dispersion function in the beam-electron cloud interaction. The wake force induced by electron cloud depends on the horizontal position, while it is independent in the ordinary wake force.

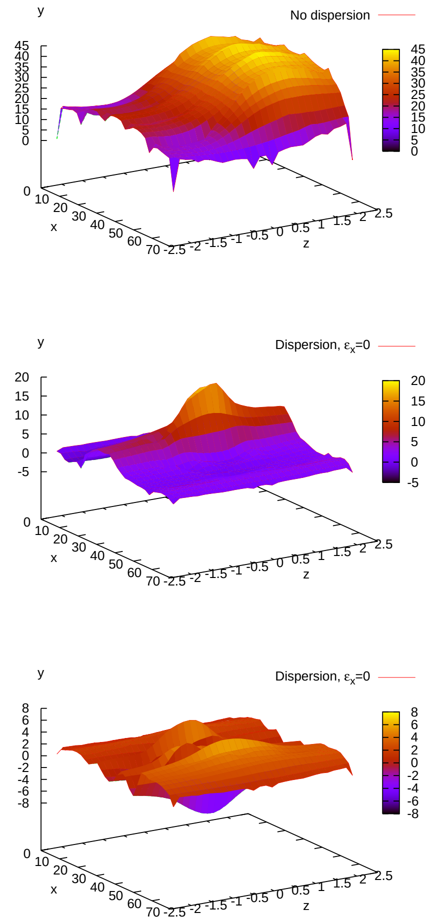

Figure 3: Vertical dipole momemnt in $z-\delta$ plane, (a) drift space without dispersion, (b) dispersion dominant drift space, (c) in bending magnet dominant dispersion.

In the dispersive section, the horizontal coordinate can be mainly due to the energy deviation (δ). Therefore the wake depends on energy deviation via dispersion. The wake force, which depends on both of z and δ, can drastically degrade the threshold of the fast head-tail instability.

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