TRANSVERSE PERIODIC BEAM LOADING EFFECTS IN A STORAGE RING*

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Abstract

Uneven beam fill patterns in storage rings, such as gaps in the fill patterns, leads to periodic, or transient loading of the modes of the RF cavities. We show that an analogous effect can occur in the loading of a dipole cavity mode when the beam passes off the electrical center of the cavity mode. Although this effect is small, it results in a variation of the transverse offset of the beam along the bunch train. For ultralow emittance beams, such as optimized third generation light sources and damping rings, this effect results in a larger projected emittance of the beam compared with the single bunch emittance. The effect is particularly strong for the case when a strong dipole mode has been purposely added to the ring, such as a deflecting, or "crab" cavity. We derive an approximate analytic solution for the variation of the beam-induced deflecting voltage along the bunch train.

INTRODUCTION

Periodic, or "transient", beam loading of the accelerating mode of a storage ring RF system has presented one of the challenges to operating high current electron storage rings, particularly in the case of the "flavor"–factories (B and Φ factories) and for third generation light sources with harmonic RF systems. These rings are typically operated with a gap in the fill pattern ranging from a few to about 20%. During the gap, the accelerating voltage and phase evolve such that there is a significant difference in the synchronous phase of the beam from the head to tail of the bunch train. This synchronous phase transient can create a number of problems for ring operation and most rings are limited to operating with a total transient of 20 degrees of RF phase or less.

There has been a number of studies of periodic beam loading over the past decade[1, 2, 3, 4, 5, 6]. It has been shown that the effect is proportional to the average beam current, the size of the gap, and the total R/Q of the RF system. The effect does not explicitly depend on the quality factor, or Q of the RF system and occurs for both normal and superconducting cavities. Although the effect is usually referred to as transient beam loading, it is important to note that effect is steady-state and only slowly evolves as the beam current changes.

In this paper, we introduce the analogous effect of periodic beam loading in the deflecting mode of an RF cavity. In this situation, a beam passes through a cavity with a transverse offset from the electrical center of the dipole mode, exciting the mode. Subsequent bunches are deflected, perturbing the beam orbit. If the beam is uniform, all bunches are deflected equally and therefore have the same orbit. However, if the storage ring beam fill pattern contains a gap, the beam-induced deflecting fields will evolve during the gap and the head and tail of the bunch train will receive unequal deflections and follow different orbits. As we show later, this effect is negligibly small except for a few, but important, cases. It is important to note that this effect is not an instability but a steady-state situation that slowly evolves with the beam current. The deflecting modes also give rise to a number of coherent instabilities, particularly multibunch instabilities. For the purposes of this study, we ignore these effects and assume that they are damped by an active feedback system. This paper provides an analytical calculation of the scale of this effect.

One of situations where transverse periodic beam loading is nontrivial is the case of ultralow vertical emittance rings such as damping rings for linear colliders and the next wave of ring-based synchrotron light sources such as NSLS-II, PEPX, and PETRA3. In this case, the small difference in vertical position along the bunch train can be a substantial fraction of the small vertical beam size. Another case where the effect is non-negligible is when a large cavity deflecting mode has been intentionally added to the ring for the purpose of exciting head-tail transverse oscillations of the bunch. For example, in two-ring colliders, there is a reduction of the luminosity from the geometrical overlap of beams colliding with a crossing angle. Several schemes have been proposed to use deflecting cavities to tilt the beams at the interaction point[7, 8, 9]. Another example is a scheme to use deflecting cavities to excite a head-tail vertical oscillation larger than the beam divergence[10] and use the vertical head-tail correlation of the emitted photon pulse to shorten its time duration.

TRANSVERSE TRANSIENTS

When a bunch traverses a cavity excited in the TM110 (i.e. deflecting) mode parallel to the cavity axis but with some transverse offset from the electrical center of the mode, xoff, it is excited and deflects subsequent bunches. The wake voltage excited by the bunch passage can be represented by a complex voltage phasor and is given by

$$\tilde{V} = -j2k_{\perp}qx_{off} \tag{1}$$

where $k_{\perp} = \pi\omega_r R/2Q$ is the transverse loss parameter, Q is the mode quality factor, R/Q is the geometric transverse shunt impedance, and q is the charge of the bunch. As each subsequent bunch passes it will experience a deflection based on the voltage excited by each of the previous

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bunch passes and the phase of the oscillation of the field in the cavity. In general we can express the wake voltage after the passage of the i_{th} bunch as the complex voltage given by the difference equation

$$\tilde{V}_{i,w+1} = \tilde{V}_{i,w} e^{(jw_r - \frac{w_r}{2Q})\Delta t} - j2k_\perp q x_{disp} \quad (2)$$

where Δt is the time between bunches, w_r is the angular resonant frequency, Q is the quality factor and i is the turn-by-turn index with bunch index w. If we assume that all of the bunches in the bunch train start with the same offset, x_{off}, we can rewrite the equation as

$$\tilde{V}_{i,w+1} = \tilde{V}_{i,w} e^{(jw_r - \frac{w_r}{2Q})\Delta t} - j2k_t q(x_{i,w+1} + x_{off}) \quad (3)$$

where $x_{i,w+1}$ is the relative transverse position. The beam-induced voltage will add to the external generator voltage in the cavity and impose stability criteria similar to the Robinson stability criteria[12]. For our study, we ignore the external generator voltage because it does not directly contribute to the variation of the deflecting voltage along the bunch train. However, the value of the generator voltage does affect the cavity tuning and thus the transient.

To find an approximate analytic solution for position variation, we begin with the difference equations for the transverse betatron motion. Note that we use x for the horizontal or vertical position of the beam relative to the offset in the deflecting cavity.

$$\begin{aligned} x_{i+1,w} = \; & cos(2\pi Q_x) x_{i,w} + \\ & \beta_x sin(2\pi Q_x) x'_{i,w} (1 - 2\lambda_x) \end{aligned} \quad (4)$$

$$\begin{aligned} x'_{i+1,w} = \; & \frac{-sin(2\pi Q_x) x_{i,w}}{\beta_x} + cos(2\pi Q_x) x'_{i,w} (1 - 2\lambda_x) \\ & + \Re\left(\frac{\tilde{V}_{i+1,w}}{(E/e)}\right) \end{aligned} \quad (5)$$

where β_x is the beta function, Q_x is the betatron tune, and λ_x is radiation damping decrement. To find an analytical solution of the steady state bunch-to-bunch transverse position, we first decouple the difference equations used for tracking (Eqns. 2, 3, and 4) by assuming that the transverse motion is much smaller than the offset in the cavity ($x_{i,w} << x_{off}$.) We also assume that the bunch to bunch spacing, Δt, is constant.

Suppose that the bunch train consists of N bunches followed by a gap. We assume the gap is much larger than the bunch spacing and model this as a square pulse driving function periodic with the revolution frequency, f_{rev}. We therefore expect the voltage response of a deflecting cavity driven by an uneven fill pattern in a storage ring to be the superposition of decaying sinusoids, seperated by $1/f_{rev}$ in the time domain. If the deflecting cavity happens to be tuned such that $f_{cavity} = nf_{rev}$, where n is an integer, then the decaying sinusoids will add constructively, leading to a resonance. Defining $h_{rest} = f_{cavity}/f_{rev}$, we expect particularly large deflecting mode voltage oscillations when $h_{rest} = integer$.

Table 1: Example parameters.

Ring Circumference	1 km
Beam Current	1 A
RF Frequency	500 MHz
Harmonic number	1667
Deflecting mode frequency	800-1500 MHz
Deflecting mode quality factor	1000-1e8
Beam transverse offset at cavity	1 mm
Bunch Train Gap	variable
Beta Function at cavity	10 m
Fractional betatron tune	0.2
Beam Energy	3 GeV

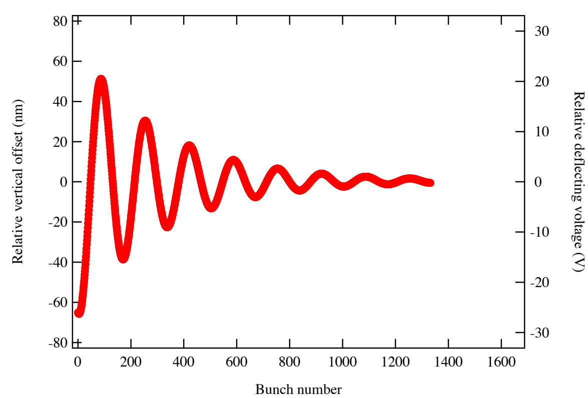

Figure 1: Calculation of the relative transverse voltage and offset along the bunch train for the parameters shown in Table 1. The dipole mode is tuned to the one revolution frequency away from the second harmonic of the RF frequency. The value of R/Q=1 Ohm.

In complex phasor notation we can write the cavity voltage as

$$\tilde{V}(w) = \tilde{A} e^{(jU+K)w} + \tilde{B} \quad (6)$$

where w is the bunch index. Note that this trial solution satisfies Eq. 6. The real part of the voltage is given by

$$V(w) = e^{\frac{-wF}{2Q}} [A_r cos(wF) - A_i sin(wF)] + B_r \quad (7)$$

$$F = \frac{2\pi b h_{rest}}{h} \quad (8)$$

With an expression for the deflecting voltage, we can solve the coupled difference equations (Eqs. 1 and 2) by requiring a steady state condition. This solution yields an expression for the transverse position along the bunch train which follows the voltage transient and is given by

$$x(w) = KV(i) + x_{off} \quad (9)$$

Detailed expressions for $\tilde{A}$, $\tilde{B}$, and K are given in another reference[13].

To give an example of the scale of the transverse periodic loading effect, we use parameters of a ring resembling a third generation light source shown in Table 1. The

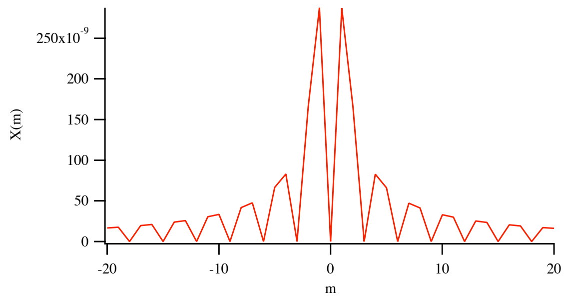

Figure 2: Shows the size variation of the transverse transient as a function of the deflecting cavity tuning, m. Here m is assumed to be an integer and the quality factor of the cavity, Q, is assumed to be very large. Notice that the largest peak occurs at $m = 1$.

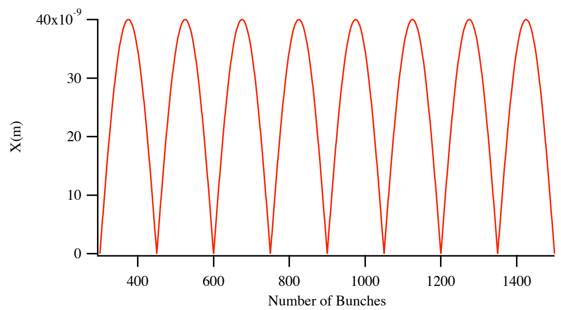

Figure 3: This shows the size of the transverse transient as a function of the number of bunches in the bunch train.

relative transverse offset and deflecting voltage along the bunch train is shown in Fig. 1. For this calculation, we used a frequency for the deflecting mode of 1002 MHz, and R/Q=1 Ohm, and $Q = 2000$. The resulting transient deflecting voltage has a rather modest amplitude of less than 20 Volts resulting in a variation in the transverse position along the bunch train of less than 0.12 microns. This value can be scaled by the actual value of R/Q. For example, an R/Q=100 Ohm would give a position variation of 10 microns. Note that we plot the transverse position at the same location as the deflecting mode. The value must be scaled appropriately if observed at another location with different beta function.

The amplitude of the transverse transient can be expressed as the peak-peak amplitude of the transverse position. That is,

$$\chi \equiv x_{max} - x_{min} \tag{10}$$

where x_{max} is the transverse position of the bunch with the largest transverse offset in steady state. As the transverse position as a function of the bunch index is just a sinusoid, we can determine the maximum and minimum transverse positions by determining the extremum values as long as the sinusoid passes through both its maximum and minimum values over the length of the bunch train[13].

Figure 2 shows χ plotted versus m, where m is an integer such that $h_{rest} = nh + m$. The peak appears when the dipole mode is tuned to the first revolution harmonic from an RF harmonic. This pattern follows directly from the spectrum of beam current harmonics for this gap size. Fig. 3 shows χ plotted versus the number of bunches in the train. Notice that χ oscillates between some maximum value and zero as the number of bunches is increased.

SUMMARY

We have derived an analytic expression for the transient voltage developed in the deflecting mode of an RF cavity from a nonuniform filling pattern. For typical parameters, the effect is usually negligible except for rings with very low vertical emittance. The effect can also be large for rings with crab cavities. The effect could be removed with correcting the beam orbit to the electrical center of the cavity dipole mode. This becomes more difficult for multiple dipole modes with different electrical centers. It is also possible that a feedback system could apply a variable voltage along the bunch train. However, this system would require high position sensitivity to detect the extremely small bunch-bunch vertical position variation.

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