THE EFFECT OF AN OXIDE LAYER ON RESISTIVE-WALL WAKE FIELDS*

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Abstract

Shorter and shorter electron bunches are now used in the FEL designs. The fine structure of the wall of a beam vacuum pipe plays more noticeable role in the wake field generation. Additionally to the resistance and roughness, the wall may have an oxide layer, which is usually a dielectric. It is important for aluminium pipe, which have Al2O3 layer. The thickness of this layer may vary in a large range: 1-100 nm. We study this effect for the very short (20-1000 nm) ultra relativistic bunches in an infinite round pipe. We solved numerically the Maxwell equations for the fields in the metal and ceramics. Results showed that the oxide layer may considerably increase the wavelength and the decay time of the resistive-wall wake fields, however the loss factor of the very short bunches does not change much.

INTRODUCTION

The effect of the wake fields in an undulator becomes noticeable if variation in energy due to wake fields exceeds the FEL bandwidth. There are three major sources of the wake fields: resistive walls, geometric discontinuities and rough surfaces. Surface roughness wake fields for beam pipes with smooth shallow corrugations have been calculated in [1]. It is found that the roughness of the surface contributes similarly to the wake as an oxide layer. However any aluminum pipe has a dielectric layer, therefore a short bunch excites fields due to resistance of the surface and dielectric material. We will try to analyze how these parts work together.

Resistive Wall Wake Fields

There is an analytical formula [2] for the wake field Green's function for a pipe of radius a

$$W(\chi) = \frac{Z_0 c q}{\pi a^2}\left\{\frac{4}{3} e^{-\chi}\cos(\sqrt{3}\chi) - \frac{4\sqrt{2}}{\pi}\int_0^{\infty}\frac{e^{-u^2\chi}}{u^6+8}x^2 du\right\}$$

distance $\chi = \frac{s}{s_0^r}$ is measured in characteristic distance

$$s_o^r = a\left(\frac{2}{Z_0 \sigma a}\right)^{1/3}$$

that defines the effective length of the fields.

Z_0 is impedance of free space, c is speed of light, q is electron charge, σ is conductivity.

At smaller distances, $s \ll s_0$ Green's function takes constant value

$$W(0) = \frac{Z_0 c q}{\pi a^2}$$

For following parameters a = 2.5 mm and σ = 3.510^7 1/Ohm/m it takes value of 5.8 kV/pC/m, when parameter s_0 =9.8 micron.

Thick Dielectric Layer

Formulas for a short bunch traveling in a dielectric pipe of radius a and dielectric constant ε, were derived in reference [3]

$$W(\chi) = \frac{Z_0 c q}{\pi a^2}\left\{\left(1+\frac{1}{4\varepsilon}\right)e^{-\chi} - \frac{2\varepsilon}{8\varepsilon^2 + \chi^2\left(1+\frac{1}{4\varepsilon}\right)^2}\right\}$$

Characteristic distance in this case is

$$s_0^d = a\frac{\sqrt{\varepsilon-1}}{2\varepsilon}\left(1+\frac{1}{4\varepsilon}\right)$$

At smaller distances, $s \ll s_0$ Green's function for dielectric pipe takes same constant value as Green's function for resistive wake field. Green's functions may have even more similarity if dielectric constant ε=4/3.

Thin Dielectric Layer

We take formulas for Green's function for a shot bunch in a pipe with thin dielectric layer of thickness δ from reference [4] and [1]

$$W(\chi) = \frac{Z_0 c q}{\pi a^2}\cos\chi$$

and characteristic length is

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$$s_0^t = \frac{1}{k_o} = a\sqrt{\frac{\varepsilon - 1}{2\varepsilon}\frac{\delta}{a}}$$

We have again same value for Green's function at smaller distances.

We can estimate the thickness of a dielectric layer when oscillations of Green's function for a thin dielectric layer will be comparable with oscillations of the resistive wall Green's function

$$s_o^t = \frac{1}{\sqrt{3}} s_o^r$$

$$\frac{\delta}{a} = \frac{1}{3}\frac{2\varepsilon}{\varepsilon - 1}\left(\frac{2}{Z_0 \sigma a}\right)^{2/3}$$

For dielectric constant ε=2 and mention above values for pipe radius and conductivity we get dielectric thickness of order of 50 nm. This means that oxide layer of this thickness on aluminium surface will be noticeable.

OXIDE LAYER IN ALUMINIUM PIPE

We will calculate wake potentials of a short bunch using numerical methods for solving Maxwell equations. For steady-state solution in infinite pipe, we have one variable for z and t

$$s = ct - z$$

For azimuthally symmetric fields in a round pipe Maxwell equations say

$$\mu \frac{\partial Z_0 H_\varphi}{\partial s} = \frac{\partial E_r}{\partial s} + \frac{\partial E_z}{\partial r}$$

$$\varepsilon \frac{\partial E_z}{\partial s} = \frac{1}{r}\frac{\partial (r Z_0 H_\varphi)}{\partial r} - Z_0 \sigma E_z - Z_0 c \rho_b(s)$$

$$\varepsilon \frac{\partial E_r}{\partial s} = \frac{\partial (Z_0 H_\varphi)}{\partial s} - Z_0 \sigma E_r$$

$$-\varepsilon \frac{\partial E_z}{\partial s} + \frac{1}{r}\frac{\partial (\varepsilon r E_r)}{\partial r} = Z_0 c \rho_b(s)$$

Using this we derived second order equations for longitudinal electric field component.

In ceramics:

$$\frac{\partial^2 E_z}{\partial s^2} = \frac{1}{\varepsilon r}\frac{\partial}{\partial r}\left(\frac{\varepsilon r}{\varepsilon\mu - 1}\frac{\partial E_z}{\partial r}\right)$$

and in metal

$$Z_0 \sigma \frac{\partial E_z}{\partial s} = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial E_z}{\partial r}\right)$$

We use continuity of the azimuthal magnetic field to get values for the electric field at the boundary between ceramic and metal at radius b

$$\frac{\partial}{\partial s}\frac{\partial E_z}{\partial r}(r = b_{+0}) = -Z_0 \sigma \frac{\partial E_z}{\partial r}(r = b_{+0}) + \frac{\varepsilon Z_0 \sigma}{\varepsilon\mu - 1}\frac{\partial E_z}{\partial r}(r = b_{-0})$$

Same condition for continuity of the magnetic field is used to match field in the free space and ceramics at radius a:

$$\frac{\partial^2 E_a}{\partial s^2} = \frac{2}{a}\frac{\varepsilon}{\varepsilon\mu - 1}\frac{\partial E_z}{\partial r}(a_{+0}) - \frac{Z_0 c}{\pi a^2}\frac{\partial}{\partial s} q(s)$$

For numerical solving these equations, we use finite-difference implicit scheme.

We checked the numerical code for the cases of only metallic or ceramic tubes. Fig. 1 shows the wake potential of a 100 nm bunch in a pure aluminium pipe of radius 2.5 mm. As the bunch length is much smaller that the characteristic distance, this wake potential may serve as a Green's function and compared very well with an analytical solution.

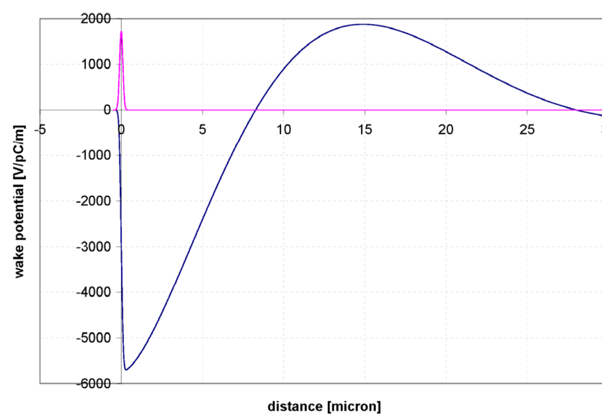

Figure 1: Wake potential of a 100 nm bunch in aluminium pipe (numerical solution).

Comparison of the wake potentials of a 1 micron bunch in a pure aluminium pipe and a pipe with 50 nm oxide layer (when oxide layer is noticeable according to above formulas) is shown in Fig. 2. Comparison in a wider range of distances for 100 nm bunch and oxide layer of 100 nm is shown in Fig. 3. We can notice that amplitudes of the wake potentials do not differ too much for shot bunches, but they may differ strongly for longer bunches. Loss factor as a function the oxide layer thickness is shown in Fig. 4.

In addition, the damping distance of the wake potentials increases with dielectric layer thickness and may become important for longer bunches and transverse wake fields.

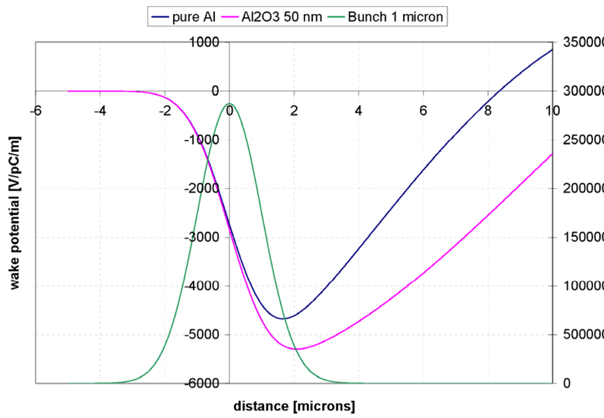

Figure 2: One micron bunch in a pure aluminium pipe (blue line) and in a pipe with 50 nm oxide layer (pink line)

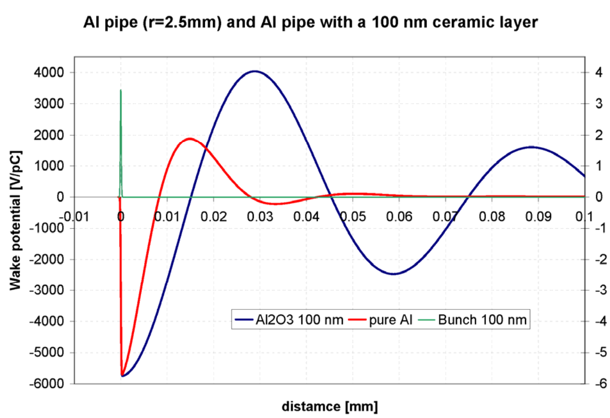

Figure 3: 100 nm bunch and 100 nm oxide layer (blue line), red line shows pure aluminium pipe.

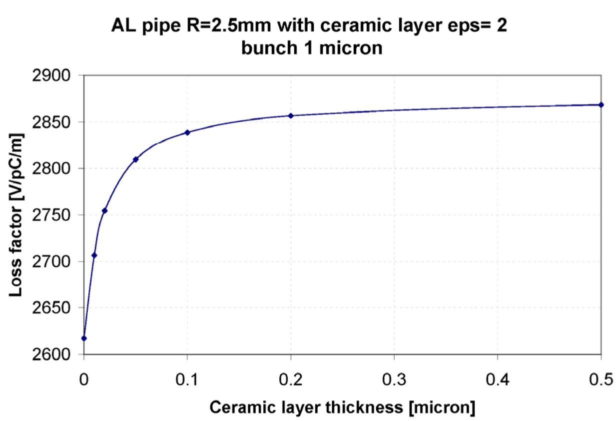

Figure 4: Loss factor of a 1 mm bunch as a function the oxide layer thickness.

TRANSVERSE WAKE POTENTIAL (TRANSVERSE FORCE)

To calculate dipole and higher modes of the wake potentials we need to solve two equations of the second order, for example for longitudinal components of electric and magnetic fields

$$\frac{\partial^2 E_z}{\partial s^2} - \frac{1}{\varepsilon r}\frac{\partial}{\partial r}\left(\frac{\varepsilon r}{(\varepsilon\mu-1)}\frac{\partial E_z}{\partial r}\right) + \frac{m^2}{(\varepsilon\mu-1)r^2}E_z = 0$$

$$\frac{\partial^2 Z_0 H_z}{\partial s^2} - \frac{1}{\mu r}\frac{\partial}{\partial r}\left(\frac{\mu r}{(\varepsilon\mu-1)}\frac{\partial Z_0 H_z}{\partial r}\right) + \frac{m^2}{(\varepsilon\mu-1)r^2}Z_0 H_z = 0$$

Boundary conditions are complicated in this case:

$$\frac{\partial^2}{\partial s^2}E_z(a) + \frac{2m(1+m)}{(\varepsilon\mu-1)a^2}E_z(a) =$$

$$= \frac{\varepsilon(1+m)}{(\varepsilon\mu-1)a}\frac{\partial E_z}{\partial r} - \frac{\mu(1+m)}{(\varepsilon\mu-1)a}\frac{\partial Z_0 H_z}{\partial r} - (1+m)\frac{Z_0 c}{\pi a^2}\left(\frac{r_0}{a}\right)^m \frac{\partial Q}{\partial s}$$

$$Z_0 H_z(a) = -E_z(a)$$

$$\frac{\partial}{\partial s}\frac{\partial Z_0 H_z}{\partial r} - \frac{\mu Z_0 \sigma}{(\varepsilon\mu-1)}\frac{\partial Z_0 H_z}{\partial r} = \frac{m}{b}\left(\frac{Z_0\sigma}{(\varepsilon\mu-1)}E_z - \frac{\partial}{\partial s}E_z\right)$$

$$Z_0\sigma\frac{\partial E_z}{\partial r} + \frac{\partial}{\partial s}\frac{\partial E_z}{\partial r} - \frac{\varepsilon Z_0\sigma}{(\varepsilon\mu-1)}\frac{\partial E_z}{\partial r} = \frac{m}{b}\left(\frac{Z_0\sigma}{(\varepsilon\mu-1)}Z_0 H_z - \frac{\partial}{\partial s}Z_0 H_z\right)$$

We present one result for a dipole wake fields (transverse force) of a 100 nm bunch in a pure aluminium pipe and in a pipe with 50 nm oxide layer (Fig. 5). This oxide layer almost doubles the transverse force.

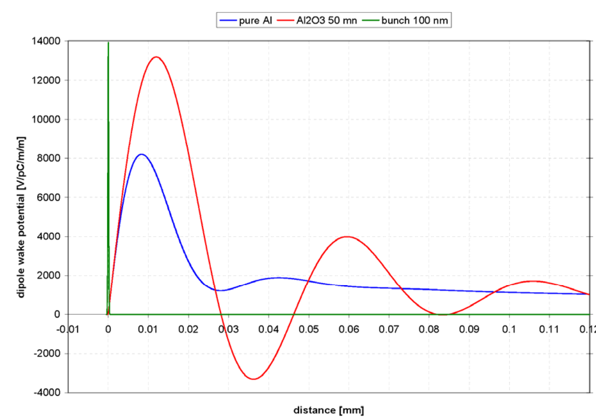

Figure 5: Dipole wake potential (transverse force) of a 100 nm bunch in a pure Al pipe (blue line) and in a pipe with 50 nm oxide layer (pink line), green line shows bunch shape.

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