TRANSVERSE SINGLE BUNCH INSTABILITY IN PEP-X*

L. Wang# and G. Stupakov, SLAC, CA 94025, USA.

Abstract

A proposed high-brightness synchrotron light source (PEP-X) is under design at SLAC. The 4.5-GeV PEP-X storage ring has four theoretical minimum emittance (TME) cells to achieve the very low emittance and two double-bend achromat (DBA) cells to provide spaces for IDs. Damping wigglers will be installed in zero-dispersion straights to reduce the emittance below 0.1nm. In this paper, we present a preliminary estimation of the threshold of the transverse mode coupling instability (TMCI). Three approaches have been used in the estimation and they agree well with each other.

INTRODUCTION

In this paper, we study the transverse single bunch instability in the PEP-X storage ring [1]. The storage ring has 476 MHz RF and a revolution period of T_0=7.33 μs. The baseline design has 3154 bunches with a single bunch current of 0.44mA to mitigate the intra-beam scattering (IBS) effect. The total beam current is 1.5A. There is no transverse single bunch instability in PEPII. However, the impedance of PEP-X significantly increases due to the small aperture in IDs and the transition sections. The transverse mode coupling instability becomes one key issue in the design. As a preliminary study, we only study the effect of resistive wall impedance. A detailed investigation of the impedance with three dimensional code is under the way. Thereafter, we will include other impedance in our further study.

The resistive wall impedance is dominant in most of the light sources due to their small aperture of beam pipe. The resistive-wall instability depends on the aperture and material of the vacuum of chamber. The standard transverse impedance of a round pipe of length L is given by [2]

$$Z_{\perp}(\omega) = (\mathrm{sgn}(\omega) - i)\frac{LZ_0}{2\pi b^3}\sqrt{\frac{2c}{Z_0\sigma}}\frac{1}{\sqrt{|\omega|}}. \qquad (1)$$

Here Z_0 is the impedance of free space, b the radius of the beam pipe, σ the conductivity of the pipe material and c the speed of light. A more realistic model of the shape of the vacuum chamber will be studied in the future. The wake function corresponding to (1) is

$$W_{\perp}(s) = \frac{cL}{\pi b^3}\sqrt{\frac{Z_0}{\pi\sigma}}\frac{1}{\sqrt{s}} \quad (s>0)\,. \qquad (2)$$

Table 1 lists the beam pipe radius, material and length in different sections of the ring. The impedance in the 90 meters insertion section is dominant due to its small aperture. While the impedance from the arc and straight sections is negligible. Therefore, replacement of the beam pipe in arc and straight sections with copper doesn't effectively reduce the total impedance.

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#email address: wanglf@slac.stanford.edu

Table 1: Source of Resistive Wall Impedance in PEP-X.

	ARC	Insertion	Wiggler	Straight
Material	Al	Cu	Cu	Al
Radius[cm]	2.8	0.3	0.75	4.8
Length [m]	1522	90	90	337
L/b^3 [m^{-2}]	6.9×10^7	3.3×10^9	2.1×10^8	3.0×10^6

APPROACHES

Three methods have been used to estimate the threshold of TMCI.

Mode Coupling Theory

TMCI occurs when the frequencies of two neighboring head-tail modes approach each other due to the detuning with increasing beam current. For a Gaussian bunches, the threshold of the instability can be expressed with the transverse loss factor [3]

$$I_0^{th} = \frac{2Q_s\omega_0 E/e}{\sum_j \beta_{\perp,j}k_{y,j}}\Theta\ , \qquad (3)$$

where I_0^{th} is the threshold of the beam current, Q_s is the synchrotron tune, $\beta_{y,j}$ is the average beta function in the jth element, $\kappa_{y,j}$ is its loss factor, E is the beam energy and $\Theta \approx 0.7$.

Eigen-value Solver

The threshold of transverse mode coupling instability can be found by solving the following eigen-value problem [2, 4]:

$$\left(\frac{\Omega-\omega_\beta}{\omega_s}\right)a_{lk} = M_{lk,l'k'}a_{l'k'}\,. \qquad (4)$$

We consider azimuthal mode coupling only for the lowest radial mode (k=0)

$$M_{ll'} = l\delta_{ll'} - i\frac{I_b r_e <\beta_\perp>}{4\pi\gamma e\omega_s}i^{l-l'}\frac{1}{\sqrt{l!l'!}} \int_{-\infty}^{+\infty} d\omega Z_\perp(\omega)\left(\frac{\omega\sigma_z}{\sqrt{2}c}\right)^{l+l'}\exp\left(-\frac{\omega^2\sigma_z^2}{c^2}\right)\ , \qquad (5)$$

here $\langle\beta_\perp\rangle$ is the average beta function in the region where the transverse impedance is important. I_b is bunch current and σ_z is *rms* bunch length. The tune of each mode $(\Omega-\omega_\beta)/\omega_s$ is obtained by solving the eigen-value problem for matrix M. The frequency $\Omega=\omega_\beta \pm l\omega_s$ corresponds to the ±lth synchrotron sideband. The threshold can be found when the head-tail modes approach each other.

Simulation

Computer simulation programs have been used for more exact calculation of the threshold. The bunch is represented by a number of macro-particles. The threshold of instability can be found by tracking with different bunch currents.

RESULTS

Table 2 shows the parameters used for the calculations. The threshold of bunch current from Eq. (3) is $I_{th} \cong 0.826\,mA$. The threshold of the transverse loss factor for the nominal bunch current is

$$k_{\perp}^{th} \cong 8.7\times 10^{3}\,V/pC/m \qquad (6)$$

Figure 1 shows wake convoluted with 0.5 *mm* Gaussian bunch and figure 2 shows the frequency of head-tail modes from Eq (4). The threshold is 0.93 *mA* as shown in the Figure.

The macro-particle simulation can give both the spectrum and bunch oscillation amplitude. Figure 3 shows the FFT and oscillation of the bunch centroid at different bunch currents. It clearly shows that the bunch becomes unstable at 0.9 *mA*. The beam spectrum also shows the merge of the different modes at the same beam current. The simulation agrees very well with the calculation shown in Figure 2. It is surprising that the simple method based on Eq. (2) also gives a good approximation.

Table 2: parameters used for threshold calculations.

	Symbol	Value
Beam Energy	E (GeV)	4.5
Synchrotron Tune	Q_s	0.0074
Average Betatron Function	β_y (m)	9.7
Momentum compaction	α	4.72×10^{-5}
Revolution period	T_0 (μs)	7.33
RF Voltage	V (MV)	10
Energy Loss	U(MV)	3.273
Bunch length	σ_z (mm)	5
Nominal bunch current	I_b (mA)	0.476

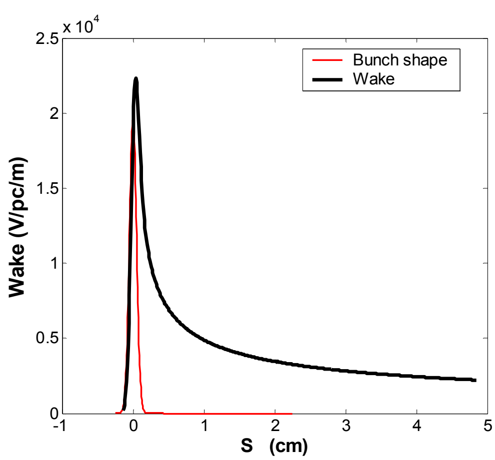


Figure 1: Transverse Wake convoluted with 0.5mm Gaussian bunch.

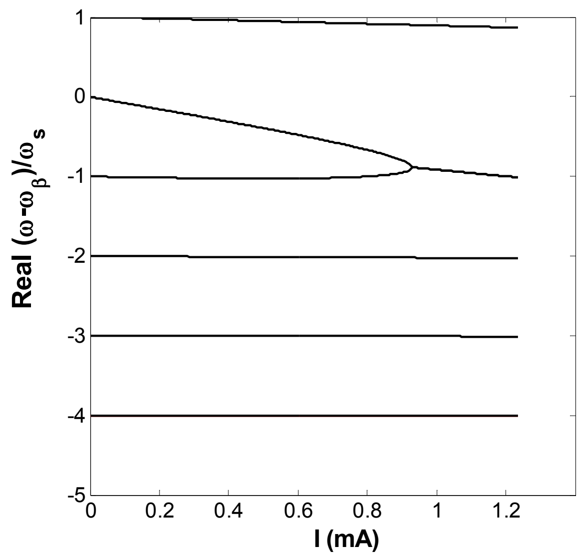
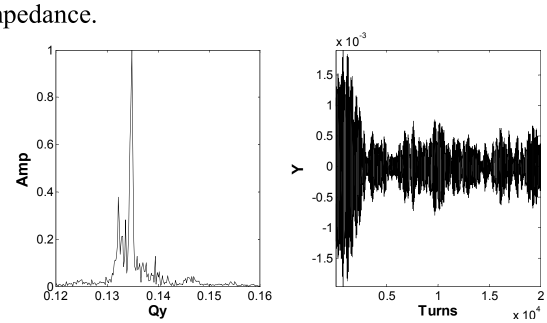
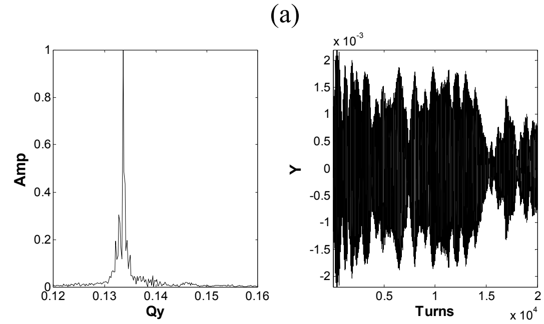


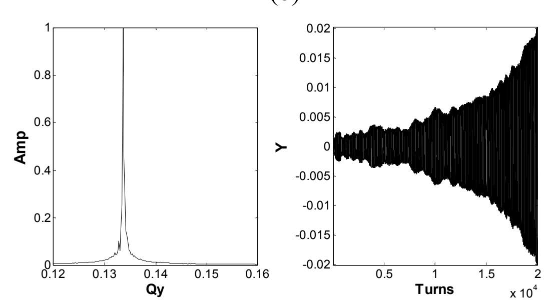
Figure 2: TMCI in PEP-X due to the resistive wall impedance.



(a)



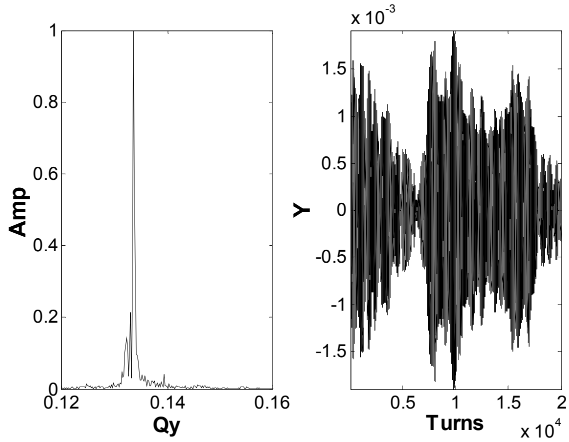
(b)



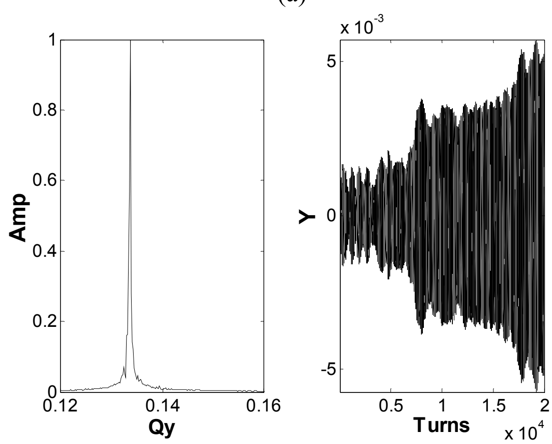
(c)

Figure 3: Simulated beam spectrum and dipole oscillation in vertical plane for different bunch current: (a) 0.80 *mA*; (b) 0.89 *mA* and (c) 0.90 *mA*.

The nominal bunch length of the recent design is 3 *mm*. The simulated threshold of the instability is 0.69 *mA* as shown in Figure 4. It is proposed to lengthen the bunch with third harmonic cavities. The bunch length can largely vary due to the third harmonic cavity. Figure 5 shows the instability threshold for various bunch length. Data fitting results in a quadratic dependence of the threshold on bunch length as shown in the Figure. The improvement of the threshold is still significant for a bunch length of 6.0 mm.



(a)



(b)

Figure 4: Simulated beam spectrum and dipole oscillation in vertical plane for different bunch current: (a) 0.69 *mA*; (b) 0.70 *mA*. Bunch length is 3mm.

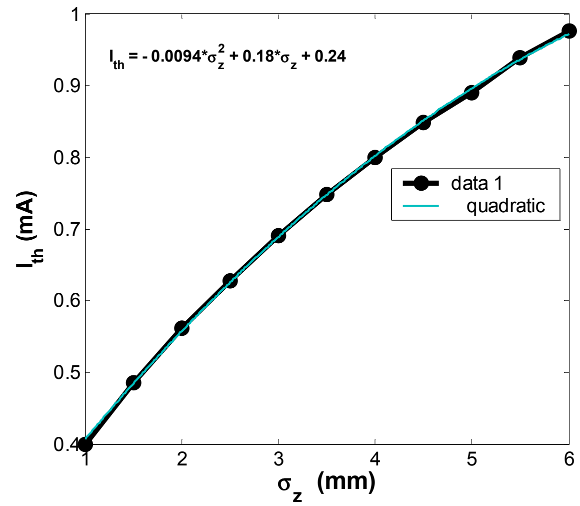


Figure 5: Dependence of the instability threshold on bunch length.

SUMMARY

Three different approaches have been used in this paper to study the threshold of the transverse mode coupling instability. The eigen-value solver and simulation method agree very well with each other, and the simple analytical model also gives a good agreement. The present study considers the resistive wall impedance only. Other impedance and the realistic geometry of the vacuum chamber will be considered late.

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