DYNAMIC BETA/EMITTANCE EFFECTS IN THE MEASUREMENT OF HORIZONTAL BEAM SIZES

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Abstract

It is well known that the beam-beam interaction has a focusing effect and therefore causes a dynamical beating of beta function around the rings. This effect becomes greatly enhanced when a collider, such as KEKB, is operated near half integer. The beating makes it difficult to interpret the measurement of horizontal beam size. We derived two coupled nonlinear equations and solved them analytically to obtain the beam sizes at the interaction points, taking into account of dynamical beta and emittance. It has been demonstrated its effectiveness using actual measured data at the synchrotron light monitors. It is expected that it will be implemented in the control room.

INTRODUCTION

KEKB is operated with the operating point of horizontal tune very close to a half integer. Horizontal beam size is strongly affected by the beam-beam interaction.

The beta function of a certain position (s) is determined by parameterization of eigenvector of revolution matrix (M_0). Particles in a beam experience a linear force for their amplitude due to the electro-magnetic force of the colliding beam as follows,

$$\Delta p_{x,\pm} = -q_\pm x = -\frac{2N_{\rm m} r_e}{\gamma}\frac{1}{\sigma_{x,{\rm m}}^2} x_\pm = -\frac{4\pi\xi}{\beta^*} x_\pm \qquad (1)$$

where N and s are bunch population and horizontal size of the colliding beam, respectively. ξ and β^* are the beam-beam parameter and beta function at the interaction point (IP).

The revolution matrix containing the beam-beam force is expressed by

$$M_\pm = K_\pm M_{0,\pm} K_\pm \qquad K_\pm = \begin{pmatrix} 1 & 0 \\ -q_\pm/2 & 1 \end{pmatrix} \qquad (2)$$

where the beam-beam force is divided to two parts to keep α_x=0 in the revolution matrix. The beta function is distorted by the beam-beam force all over the ring. The distorted beta function at IP ($\bar{\beta}^*$) is obtained by the revolution matrix, M [1]. The ratio for the nominal beta function β^* is given by

$$\left(\frac{\beta^*}{\bar{\beta}^*}\right)^2 = 1 + q\beta^* \cot\mu - \frac{q^2\beta^{*2}}{4}\ , \qquad (3)$$

Equilibrium beam distribution, envelope matrix ($\Sigma = \langle x_i x_j \rangle$), is given by solving the series of equations [2]

$$\Sigma_0 = (1-D^t) M_0^t \Sigma_0 M_0 (1-D) + B \qquad (4)$$

where

$$D = \oint M_0(s^*,s)^{-1} D_0(s) M_0(s^*,s) ds \qquad (5)$$

$$B = \oint M_0(s^*,s)^t B_0(s) M_0(s^*,s) ds \qquad (6)$$

where B_0 is diffusion due to photon emission at s,

$$B_0(s)_{ij} = \frac{55}{24\sqrt{3}}\frac{r_e \hbar}{mc}\frac{\gamma^5}{|\rho|^3}\delta_{i6}\delta_{j6} \qquad (7)$$

D_0 and D are matrices, which characterize the radiation damping. The structure of D matrix does not have effects on the dynamic emittance. D can be regarded as a diagonal matrix which the diagonal component is the radiation damping rate per turn, T_0/τ_i, i=x,y,z.

The equilibrium distribution with beam-beam interaction is obtained by solving similar matrix equation of Eq.(4) in which M_0 replacing by M,

$$\Sigma = (1-D^t) M^t \Sigma M (1-D) + B\,. \qquad (8)$$

The damping and diffusion matrices do not change. The transfer matrix also does not change, though the beta function is distorted by the insertion of K.

Normal coordinate of M is changed for that of M_0. Generally B can be approximated a diagonal matrix in the normal mode with the diagonal component of $2\varepsilon_x T_0/\tau_x$. In beam-beam simulations the approximation is frequently performed. When the dispersion randomly varies for the betatron phase, the diffusion is along the normal direction. Thus this approximation is reliable. Figure 1 shows the dynamic beta, emittance and inverse of the cap beam size. The inverse beam size, which is directly related to the luminosity, increases quickly for increasing current, where $\xi^2 \sim 0.01 I_+ I_-$ in KEKB. This behaviour of the luminosity is also seen in the beam-beam simulation based on the strong-strong model as shown in Figure 2.

The dynamic emittance depends on the structure of the diffusion matrix. The dynamic emittance is estimated for the diffusion rate in δx^2 and δp_x^2. The diffusion along the normal direction is $\delta x^2/\delta p_x^2 = \beta^{*2}$ and $\delta x^2 \delta p_x^2 = (2\varepsilon_x T_0/\tau)^2$ Figure 3 shows the dynamic beta and emittance for the diffusion assignment, $\delta x^2 = 2r\beta^* \varepsilon_x T_0/\tau$ $\delta p_x^2 = 2(2-r)\varepsilon_x T_0/\tau\beta^*$. with keeping $\delta x^2/\beta^* + \beta^* \delta p_x^2 = 4\varepsilon_x T_0/\tau$ The normal diffusion is r=1. The dynamic beta depends on r via the beam size of the colliding beam. The diffusion assignment is evaluated by perform the integral in Eq.(6) for the ideal lattice. SAD gave the diffusion assignment of 1.1. This value can be changed by lattice error, therefore should be determined by measurements.

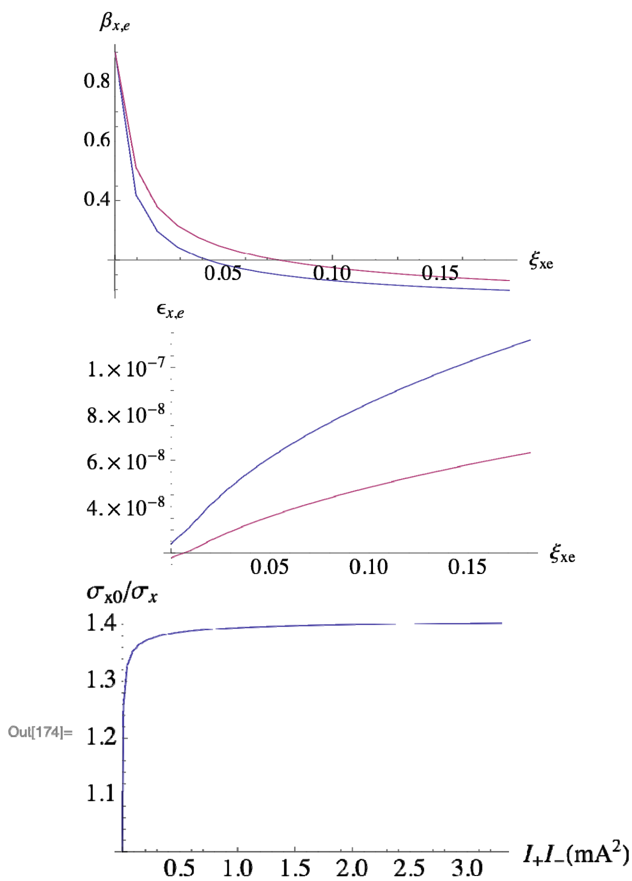

Figure 1: beta function, emittance and beam size ratio for nominal value as functions of nominal beam-beam parameter. The diffusion assignment is r=1. In the last picture, the horizontal axis is the single bunch current product of the two beams.

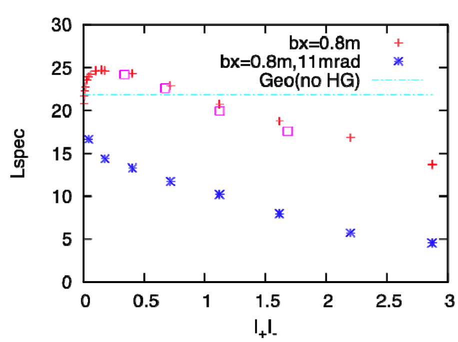

Figure 2: Specific luminosity (L/I+I-/Nbunch/1026) in KEKB given by a strong-strong simulation. The horizontal axis is the single bunch current product of the two beams.

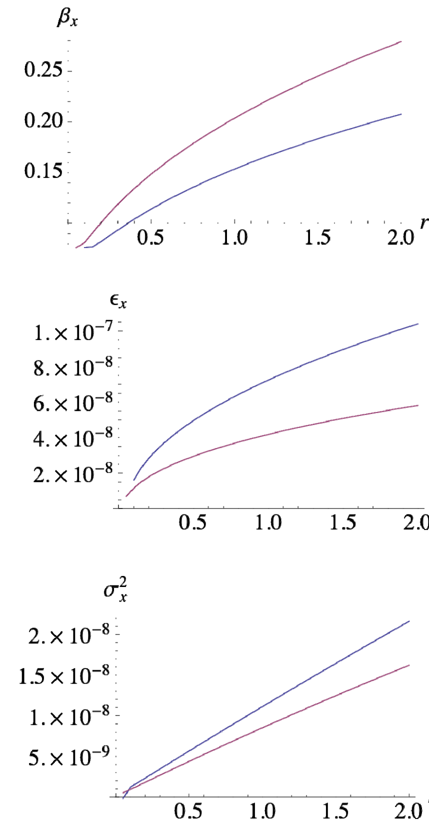

Figure 3: Dynamic beta and emittance for various diffusion rate with keeping nominal emittance.

EVALUATION OF DYNAMIC BETA AND EMITTACE FROM MEASURED BEAM SIZE

Beam size is measured by an interferometer of synchrotron light (SRM) in KEKB. The measurement apparatus is installed at a location with the nominal beta function of 14 m and 21 m in LER (positron) and HER (electron) rings respectively. We show that the beam size at IP can be estimated from the beam size measured by the SRM with taking into account of the beam-beam interaction of the two beams.

The transfer matrix from IP to SR monitor is given as

$$M_0(s_m,s^*)=\begin{pmatrix}\sqrt{\frac{\beta_m}{\beta^*}}\cos\Delta\mu & \sqrt{\beta_m\beta^*}\sin\Delta\mu \\ -\sqrt{\frac{1}{\beta^*\beta_m}}(\sin\Delta\mu+\alpha_m\cos\Delta\mu) & \sqrt{\frac{\beta^*}{\beta_m}}(\cos\Delta\mu-\alpha_m\sin\Delta\mu)\end{pmatrix} \quad (9)$$

where $\Delta\mu$ is betatron phase difference between IP and SRM and a*=0. Twiss parameters are transferred as

$$\begin{pmatrix}\alpha^* & \beta^* \\ -\gamma^* & -\alpha^*\end{pmatrix}=M_0(s_m,s^*)\begin{pmatrix}\alpha_m & \beta_m \\ -\gamma_m & -\alpha_m\end{pmatrix}M_0(s_m,s^*)^{-1} \quad (10)$$

Under the presence of the beam-beam interaction, the transfer matrix is expressed by

$$M(s_m,s^*)=M_0(s_m,s^*)K \quad (11)$$

Thus Twiss parameters are transferred as

$$\begin{pmatrix}\bar{\alpha}^* & \bar{\beta}^* \\ -\bar{\gamma}^* & -\bar{\alpha}^*\end{pmatrix}=M(s_m,s^*)\begin{pmatrix}\bar{\alpha}_m & \bar{\beta}_m \\ -\bar{\gamma}_m & -\bar{\alpha}_m\end{pmatrix}M(s_m,s^*)^{-1} \quad (12)$$

The relation of beta functions of IP and SRM is given by

$$\frac{\bar{\beta}_m}{\bar{\beta}^*} = \frac{\beta_m}{\beta^*}\left[\left(\cos\Delta\mu - \frac{q\beta^*}{2}\sin\Delta\mu\right)^2 + \left(\frac{\beta^*}{\bar{\beta}^*}\right)^2 \sin^2\Delta\mu\right] \quad (13)$$

The relation of the beam sizes at SRM and IP is obtained as

$$\frac{\bar{\sigma}^2_{m\pm}}{\bar{\sigma}^{*2}_{\pm}} = \frac{\beta_{m\pm}}{\beta^*_{\pm}} \times \quad (14)$$

$$\left[1 + \frac{2N_{\mp} r_e \beta^*_{\pm}}{\gamma}\frac{1}{\bar{\sigma}^{*2}_{\mp}}(\cot\mu_{\pm}\sin^2\Delta\mu_{\pm} - \sin\Delta\mu_{\pm}\cos\Delta\mu_{\pm})\right]$$

This equation contains the beam size of the colliding beam, thus it is coupled nonlinear equations. These equations are solved as follows,

$$\bar{\sigma}^{*2}_{\pm} = \frac{-A_{\pm}A_{\mp} + B_{\pm}B_{\mp}}{A_{\pm} + B_{\mp}} \quad (15)$$

where

$$A_{\pm} = \frac{2N_{\mp} r_e \beta^*_{\pm}}{\gamma_{\pm}}(\cot\mu_{\pm}\sin^2\Delta\mu_{\pm} - \sin\Delta\mu_{\pm}\cos\Delta\mu_{\pm})$$

$$B_{\pm} = \frac{\bar{\sigma}^2_{m\pm}\beta^*_{\pm}}{\beta_{m\pm}} \quad (16).$$

MEASUREMENT OF HORIZONTAL BEAM SIZE

The beam sizes of the two colliding beams are measured every seconds and monitored at the control room in KEKB. The vertical beam size is very delicate for the collision condition, thus the measurement is powerful tool for tuning and maintain the luminosity with a high value. Synchrotron light, which passes through two slits, interferes and describes a stripe image on a screen located back. The beam size is estimated from the interference image. Figure 4 shows a snapshot of the beam size measurement. The measured beam sizes at the monitors are 2611 and 1041 μm for HER (e^-) and LER (e^+), respectively. The beta function and phase at the monitors are ($\beta_{x,m}$,$\Delta\nu_x$)=(21.89 m, 35.3577) and (14.23m, 20.9509) for HER and LER, respectively. Other machine parameters are as follows, β_{x+-}=0.9m, (ν_{x-}, ν_{x+})=(0.5108, 0.5054), (N_-, N_+)=(3.66, 6.20)x10^{10}, (E_-, E_+)=(8, 3.5) GeV.

The vertical beam size is well calibrated, because it is used for the beam-beam optimization everyday. The horizontal size is not perfectly calibrated, thus the beam size values are preliminary.

The beam sizes at IP are estimated to be 100 μm and 132 μm for LER and HER, respectively. This results mean the diffusion assignment, r=1.2-1.5. The diffusion assignment is estimated to be r=0.89 by SAD code, in which the integral of Eq.(6) is performed for an ideal lattice. The strong-strong simulation for r=1 and r=0.89 showed smaller $\sigma_{x+/-}$=152/126 μm and 100/100 μm, respectively, for β_x=1.5 m, and 15% higher luminosity for r=0.89.

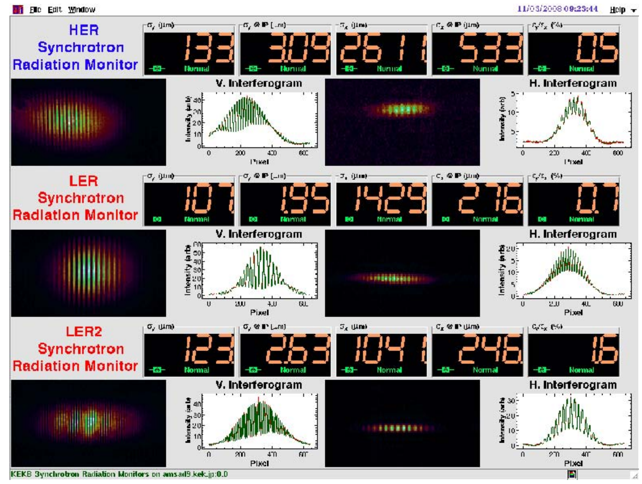

Figure 4: Beam size measurement using an interferometer of synchrotron light in KEKB.

CONCLUSIONS

Dynamic beta and emittance effects are discussed for the operating point very closed to half integer in the horizontal tune. The dynamic emittance depends on the diffusion assignment (r), which is integration of the local effect of the synchrotron radiation excitation.

The beam size at the interaction point is determined by both of the dynamic beta and emittance of the both beams: i.e., measurement of the horizontal beam size gives information of the dynamic beta and emittance. The measured diffusion assignment is somewhat lager than 1, while it is slightly less than 1 in the radiation integral. Since the SR monitors are not perfectly optimized for the horizontal beam size measurement, this result is preliminary.

When the diffusion assignment is larger than 1, the strong-strong simulation of the beam-beam interaction should give smaller luminosity, though it does not seem to be dominant to explain the measured luminosity in KEKB.

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