# DESIGN STUDY OF COMBINED MAGNET WITH COMBINED FUNCTION METHOD*

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*Abstract*

This paper describes the method of designing quadrupole and quadrupole-sextupole magnet pole profiles. Using this method to design quadrupole and quadrupole-sextupole magnets, not only is high quality magnetic field found, but also the high-order harmonic magnetic field components are eliminated. Usually when designing a quadrupole, in order to acquire good magnetic field quality, the pole face of the magnet must be shimmed repeatedly to get a desired pole profile.This method is too much dependent on the designers' experiences during the design procedure..

## INTRODUCTION

In order to compensate the natural chromaticity, during the design of new synchrotron radiation light source, generally sextupoles are put on the places where the βη is larger than that of others. Therefore, the use of a quadrupole with sextupole component can both be used effectively to compensate the natural chromaticity and to save space . At present, different quadrupole-sextupole combined magnets have been commonly used, such as quadrupole-sextupole which the quadrupole-sextupole components are generated by asymmetric excitation method [1], or by adding assistant coils to the conventional quadrupole magnets to generate a sextupole field component [2] and so on. Other ways of design quadrupole-sextupoles are still in the research stages. Designing a quadrupole-sextupole conbined magnet by the optimization of combined function of magnetic pole profile [3] is a relatively new approach. This method not only can generate relatively stronger quadrupole and sextupole components at the same time, but also can effectively eliminate main error components. In this paper, there are two parts, the first section is about how to optimize the magnetic pole profile by two-dimensional magnetic scalar potential equations, the second section is discussion and design of quadrupole magnet and quadrupole-sextupole magnet.

## MAGNETIC FIELD THEORY

Static magnetic field of Maxwell's equations:

$$\nabla \bullet \vec{B} = 0, \qquad \nabla \times \vec{H} = \vec{J} \qquad (1)$$

In the absence of current conditions, that is $\vec{J} = 0$ , the above equation become Laplace equation:

$$\nabla^2 \Phi = 0 \qquad (2)$$

Here $\Phi$ is the magnetic scalar potential, Consider the situation of two-dimensional (z is constant), In polar coordinates( $r,\theta$ ), solving for the Laplace equation [4]:

$$\Phi = (E + F\theta)(G + Hlnr)\sum_{n=1}^{\infty} \begin{matrix}(J_n r^n \cos n\theta + K_n r^n \sin n\theta + \\ L_n r^{-n} cosn\theta + M_n r^{-n} \sin n\theta)\end{matrix} \qquad (3)$$

In the application to actual magnetic field, the above expression can be changed into the following form [2]

$$\Phi = \sum_n (J_n r^n \cos n\theta + K_n r^n \sin n\theta) \qquad (4)$$

n are integers, $J_n$ and $K_n$ are decided by the geometric structures. Therefore, in polar coordinates the polar shape, for all multi-polar moments, magnetic scalar potential can be given by [3]:

$$U(r,\theta) = C_1 r \sin(\theta) + C_2 r^2 \sin(2\theta) + C_3 r^3 \sin(3\theta) + C_4 r^4 \sin(4\theta) + C_5 r^5 \sin(5\theta) + ..... \qquad (5)$$

For the quadrupole magnet, The second term is the ideal component. For the fourfold symmetric structure, The magnetic scalar potential equation can be written as (6)

$$U(r,\theta) = C_2 r^2 \sin(2\theta) + C_6 r^6 \sin(6\theta) + C_{10} r^{10} \sin(10\theta) + C_{14} r^{14} \sin(14\theta) + ..... \qquad (6)$$

here $C_6$ $C_{10}$ etc. are coefficients of error harmonics, the value of these items determine the quality of quadrupole magnet. For a sextupole, sixfold symmetric structure, $C_3$ term is the ideal sextupolar component,The magnetic scalar potential equation can be written as:

$$U(r,\theta) = C_3 r^3 \sin(3\theta) + C_9 r^9 \sin(9\theta) + C_{15} r^{15} \sin(15\theta) + C_{21} r^{21} \sin(21\theta) + ..... \qquad (7)$$

$C_9$, $C_{15}$ terms etc.are error harmonics.

## QUADRUPOLE MAGNET DESIGN

Actually, when one designs a quadrupole magnet, because of the limited width of pole profile of the magnet, It will certainly bring a lot of error harmonics, in order to reduce the effects of these error harmonics, one can shim the pole face to get a good field quality. The shimming method of the pole face has no fixed rules to follow, mainly depends on the skills of designer. By optimizing the pole profile equation, one can get a desired pole shape. $C_6$ $C_{10}$ are put to be 0 in order to reduce the error harmonics, By carefully choosing $C_{14}, C_{18}$ etc., one can get a satisfied polar equation (8),

$$U_0 = C_2 r^2 \sin(2\theta) + C_{14} r^{14} \sin(14\theta) + C_{18} r^{18} \sin(18\theta) + C_{22} r^{22} \sin(22\theta) + ..... \qquad (8)$$

While $U_0 = 1$, To get the condition, one put half of the aperture as unit, $\theta = 45^0$ , $r = 1$,

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$$C_2 = 1 + C_{14} - C_{18} + C_{22} - \cdots \qquad (9)$$

Solution of equation (8) the desired pole profile can be acquired. $C_{18}, C_{22}$ etc. are error harmonics, if r is less than 1, $r^n$ is very small and they do not effect the field quality inside the good field region; If r is more than 1, These items become great, adjusting the values of $C_{18}, C_{22}$ etc. until one gets a polar profile of fixed width, Using Poisson program [5], one can get the desired pole profile. Table 1 is a group of optimized coefficients of the pole profile equation of quadrupole magnet.

Table 1: The Coefficients of Pole Face Equation.

| Coefficients | Values |
|---|---|
| C2 | $1.0\times10^{0}$ |
| C3 | 0.0 |
| C14 | $1\times10^{-4}$ |
| C18 | $1.75\times10^{-4}$ |
| C22 | $2.0\times10^{-6}$ |
| C26 | $2.0\times10^{-8}$ |
| C30 | $4.0\times10^{-12}$ |

Using the above coefficients, we designed the quadrupole magnets [6] of HLS, This size of the magnet is shown in figure1. The field distribution of harmonic magnetic field is shown in Table 2, meeting the design requirements. The quality of the field is better conventionally designed magnet being used in HLS. The distribution of good field area of magnet is shown in Fig. 2.

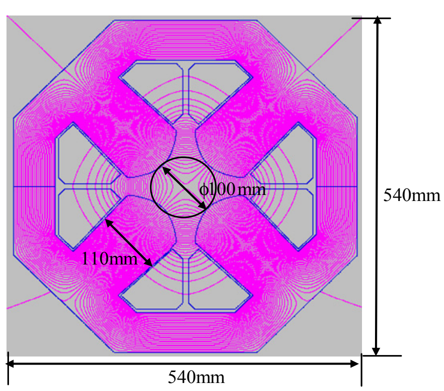


Figure1: Construction of quadrupole magnet.

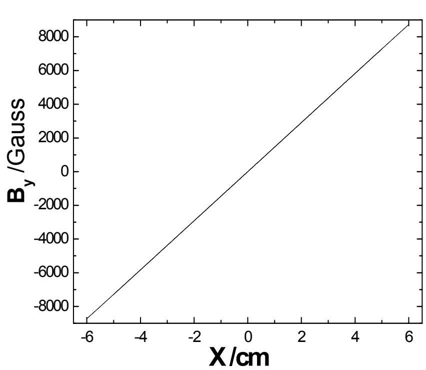


Figure 2: $B_y$ of quadrupole varied with X.

Table 2: The Ratio of Harmonic Field

| n | Value | Bn/B1 |
|---|---|---|
| B1 | 1452.15776 | 1 |
| B2 | $1.413\times10^{-3}$ | $2.89\times10^{-5}$ |
| B3 | $8.99\times10^{-3}$ | $5.57\times10^{-5}$ |
| B4 | $9.71341\times10^{-4}$ | $1.80\times10^{-5}$ |
| B5 | $9.22636\times10^{-5}$ | $5.15\times10^{-6}$ |
| B6 | $3.09227\times10^{-5}$ | $5.30\times10^{-6}$ |
| B7 | $1.68663\times10^{-6}$ | $8.48\times10^{-7}$ |
| B8 | $1.73388\times10^{-6}$ | $2.61\times10^{-6}$ |
| B9 | $1.44019\times10^{-6}$ | $6.51\times10^{-6}$ |

## QUADRUPOLE-SEXTUPOLE MAGNET

Putting equation (6) and (7) together one gets the equation of combined function magnet , Adding sextupole components to the right side of the equation and removing the higher order term, then the scalar potential equation can be written as:

$$U(r,\theta) = C_2 r^2 \sin(2\theta) + C_3 r^3 \sin(3\theta) + C_6 r^6 \sin(6\theta) + C_{10} r^{10} \sin(10\theta) + C_{14} r^{14} \sin(14\theta) + \ldots.. \qquad (10)$$

In the above equation , all of the remaining terms is error harmonic except the $C_2, C_3$ terms. Similarly according to equation (10), the coefficients of the six order terms can be found. In order to reduce the effect of higher error harmonics, $C_6, C_{10}$ are made to be zero. Adjusting the coefficients of the terms, one can get the most desired pole profile of the quadrupole-sextupole combined magnet. Table 3 is a group of optimized coefficients of the pole profile equation of quadrupole-sextupole magnet. The field distribution of quadrupole-sextupole is shown in Table 4.

Table 3: The Coefficients of the Pole Face Equation

| Coefficients | Values |
|---|---|
| C2 | $1.0\times10^{0}$ |
| C3 | $1.5\times10^{-1}$ |
| C18 | $1.75\times10^{-4}$ |
| C22 | $2.0\times10^{-10}$ |
| C26 | $1.5\times10^{-12}$ |
| C30 | $4\times10^{-14}$ |

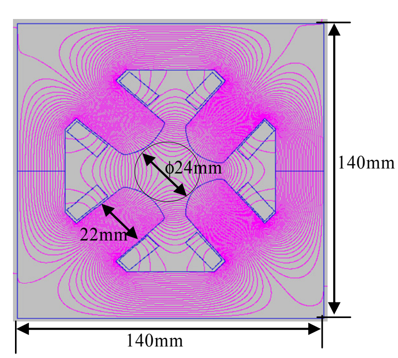


Figure 3: Construction of quadrupole-sextupole magnet.

Table 4: The Ratio of Harmonic Field

| n | Value(GS) | Bn/B1 |
|---|---|---|
| B1 | 3119.58 | 1 |
| B2 | 444.623 | 0.15 |
| B3 | 6.21455 | $2.0\times10^{-3}$ |
| B4 | 8.04518 | $2.5\times10^{-3}$ |
| B5 | 2.13703 | $6.7\times10^{-4}$ |
| B6 | 0.15826 | $5.1\times10^{-5}$ |
| B7 | 0.41425 | $1.3\times10^{-4}$ |
| B8 | 0.38458 | $1.2\times10^{-4}$ |
| B9 | 0.63617 | $2.0\times10^{-4}$ |

The field gradient of the quadrupole is K,31.2T/m, the gradient of sextupole component is $\lambda$, 444.5T/m$^2$, is in line with the design requirements. The size and apperance of the combined magnet is shown in Fig. 3. The distribution of good field area of combined magnet is shown in Fig. 4.

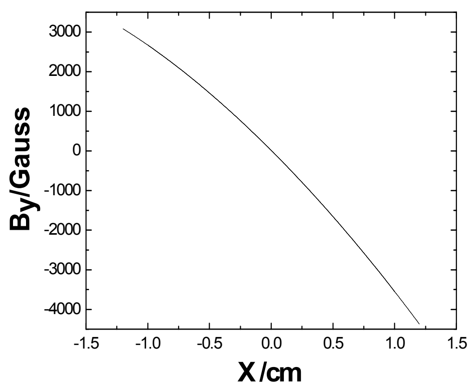


Figure 4: $B_y$ of quadrupole-sextupole varied with X.

## CONCLUSIONS

By the methods described in this paper, the conventional accelerator quadrupole magnets and quadrupole-sextupole combined magnets can be designed. The design procedure is standardized and easy to operate. The field quality of designed magnet is better than that of the magnets conventionally designed being used in HLS. The ratio of harmonic field is fewer than $10^{-5}$.

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