OPTICAL INJECTOR BASED ON PARTICLE ACCELERATION BY STIMULATED EMISSION OF RADIATION IN A PENNING-TRAP*

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Abstract

We present results of an analysis demonstrating that electrons oscillating in a Penning trap may drain the energy stored in an adjacent active medium. Energy imparted to the electrons allows them to leave the trap

INTRODUCTION

Motivated by Niels Bohr postulate regarding discrete energy states in an atom, Franck and Hertz (FH) were the first to demonstrate experimentally [1] in 1914 that electrons in atoms can absorb energy from a moving free electron only in discrete quanta. They have shown that a bounded electron in a mercury atom is raised from a lower to a higher quantum-state at the expense of the kinetic energy of free electron moving nearby. Later, in 1921 Klein and Rosseland [2], coined for this process the notion of "collision of the first kind". A decade later (1930), Latyscheff and Leipunsky (LL) demonstrated experimentally the inverse process [3]. Relying on the fact that stimulated absorption of radiation manifests itself as a transition of the atom's outer electron from a low to a higher energy-state, they illuminated vapors of mercury with light from a mercury lamp. When a free electron was injected into the vapors, they found that it may gain energy in quanta corresponding to that stored in the mercury atoms. In this process, the excited outer electron, has dropped to the lower energy-state delivering the energy to the free electron, enhancing its kinetic energy - the process being known as "collision of the second kind". In both FH and LL experiments the vapors' pressure was designed such that, in average, there was only *one collision* of a free electron with a mercury atom and consequently, the average electron's energy gain/loss was of the order of a few eV's.

It was only in 1958, that Schawlow and Townes demonstrated [4] that energy stored in atoms may be used for amplification of radiation by a series of *multiple collisions* of photons with excited atoms. Today this process is known as light amplification by stimulated emission of radiation (LASER). Recently [5], we have demonstrated that a train of relativistic bunches of electrons may be accelerated by an ensemble of excited atoms provided that the resonant frequency of the medium corresponds to the frequency of the bunches. In the process of particle acceleration by stimulated emission of radiation (PASER) [6-7] a macro-bunch emits a virtual photon impinging upon an excited atom, and as a result, a real photon is emitted. The two resulting photons are absorbed by the train of bunches since their phases are identical, and as a result, the electrons' kinetic energy increases. According to the 200keV gained by a significant fraction of electrons in the experiment, these encountered order of 2 million coherent collisions of the second kind. For this process to occur, the *relativistic* electrons were bunched in a wiggler by an intense CO_2 laser; the experiment was performed at the Accelerator Test Facility of Brookhaven National Laboratory

In a recent publication [8] we presented a novel paradigm which relies on the possibility that *non-relativistic* electrons confined by a Penning trap will experience collisions of the second of kind leading to bunching of the electrons at the resonant frequency of the medium. The bunched electrons drain energy from the active medium and the resulting kinetic energy allows them to escape the trap.

For envisioning the concept we need to bear in mind that essentially collisions of the second kind facilitate coupling between two independent processes: storage of charged particles in a Penning trap and storage of electromagnetic energy in active medium. In the absence of the latter, electrons oscillate in the trap virtually indefinitely whereas in the absence of interaction the active medium decays on a time scale from milliseconds to nano-seconds according to the active material characteristics, $T_{\rm eq}$ (and its geometry if self-amplified spontaneous emission is significant).

In Ref.8 an idealized model was adopted whereby the self-amplified spontaneous emission (SASE) was ignored and so was the space-charge effect associated with the electrons, explicitly the particles' trajectory was assumed to be given by $z_i(t) = (L/2)\cos\left[\Omega\left(t - iT_{\rm opt}\right)\right]$ where L is the length of the trap, Ω is the angular frequency of the oscillation in the trap and $T_{\rm opt}$ is the period of the photon stored in the medium. This bunching is a result collisions of the second kind experienced by free electrons moving in the vicinity of active medium. If bunched, the electrons stimulate the active medium (PASER) and they gain kinetic energy at the expense of that stored in the medium. Consequently, this stimulated process leads to an enhanced decay rate of the population inversion density. And vice versa, with proper choice of parameters that facilitates a draining-time significantly shorter than $T_{\rm eq}$, we may expect the ensemble of electrons to become bunched.

By estimating the *maximum power exchange* between the active medium and the electrons oscillating in a trap it was demonstrated the overall energy drainage time may be significantly faster. In fact, subject to the assumptions above, it was demonstrated analytically that this power exchange is proportional to the energy stored in the medium. Consequently, energy balance implies

$$\frac{\rm d}{\rm dt}\Delta W + \frac{1}{T_{\rm eq}}\Delta W = -P_{\rm ex}^{(\rm max)} = -\frac{2}{\tau_{\rm ex}}\Delta W \qquad (1)$$

and for Nd:YAG (population inversion of the order of $10^{22}[m^{-3}]$, 10cm long system, the exchange time was found to be

$$\tau_{ex}\left[\mu\sec\right]=10\left(\frac{10^{13}}{N_{el}}(2)\right)^{2}\sqrt{\frac{V_{an}[V]}{100}}$$

with V_{an} representing the anode voltage in the trap and N_{el} is the number of electrons in the volume.

1-D DYNAMICS

In this short communication we present a brief summary of the equations of motion of a 1D system with special emphasis on the space-charge effect and ignoring SASE. In a normalized form, these equations have the following form:

$$\begin{aligned}
&\bar{E}_z(\bar{z},\tau)=-\xi_{SC}\int_0^{\tau}d\tau' G_E(\tau,\tau')\left\langle \bar{h}\left[\bar{z}-\bar{z}_i(\tau')\right]\right\rangle_i\\
&\left(\frac{d^2}{d\tau^2}+\frac{2}{\tau_s}\frac{d}{d\tau}+1\right)\bar{z}_\nu(\tau)=-\bar{E}_z\left[\bar{z}_\nu(\tau),\tau\right]\\
&\frac{d}{d\tau}\overline{\Delta W}+\frac{1}{\tau_{eq}}\overline{\Delta W}=\\
&-\frac{\bar{\sigma}}{\xi_{SC}}\overline{\Delta W}\int_{-\infty}^{\infty}d\tau' C(\tau,\tau')\int_{-\infty}^{\infty}d\bar{z}\,\bar{E}_z(\bar{z},\tau)\bar{E}_z(\bar{z},\tau').
\end{aligned}\tag{3}$$

The first relation determines the relation between the electric field and the current tacitly assuming that the energy stored varies slowly on the scale of the optical ω_0 or trap Ω angular frequency. Particles dynamics is described by the second whereas the third determines the variations of the stored energy. It can be readily checked that the global energy conservation associated with Eqs.(3) is

$$\begin{aligned}
&\frac{d}{d\tau}\left\{\frac{1}{2\xi_{SC}}\int d\bar{z}\bar{E}_z^2(\bar{z},\tau)+\frac{1}{2}\left\langle\dot{\bar{z}}_\nu^2\right\rangle+\frac{1}{2}\left\langle\bar{z}_\nu^2\right\rangle+\overline{\Delta W}\right\}=\\
&-\frac{1}{\tau_{eq}}\overline{\Delta W}-\frac{2}{\tau_s}\left\langle\dot{\bar{z}}_\nu^2\right\rangle
\end{aligned}\tag{4}$$

In the last two equations the following definitions were used

$$\begin{aligned}
&\bar{E}_z(\bar{z},\tau)=-\xi_{SC}\int_0^{\tau}d\tau' G_E(\tau,\tau')\left\langle \bar{h}\left[\bar{z}-\bar{z}_i(\tau')\right]\right\rangle_i\\
&G_E(\tau,\tau')=\delta(\tau-\tau')+\bar{\sigma}\overline{\Delta W}S(\tau,\tau'),\ \ \bar{\omega}_0=\frac{\omega_0}{\Omega}\\
&C(\tau,\tau')\,\square\,\cos\left[\bar{\omega}_0(\tau-\tau')\right]\exp\left(-\frac{\tau-\tau'}{\tau_2}\right)h(\tau-\tau')\\
&S(\tau,\tau')\,\square\,\frac{1}{\bar{\omega}_0}\sin\left[\bar{\omega}_0(\tau-\tau')\right]\exp\left(-\frac{\tau-\tau'}{\tau_2}\right)h(\tau-\tau')\\
&\bar{E}_z(\bar{z},\tau)=\frac{e}{m}\frac{1}{L\Omega^2}E_z(\bar{z},\tau),\overline{\Delta W}=\frac{W_{pop}-W_{eq}}{mN_{el}L^2\Omega^2}\\
&\xi_{SC}=\frac{1}{\Omega^2 A/c^2}\frac{4\pi r_e N_{el}}{\varepsilon_r L},\bar{\sigma}=4\frac{mc^2}{\hbar\omega_0}\frac{\sigma_{21}L}{AcT_2}N_{el}
\end{aligned}\tag{5}$$

It should be pointed out that as it stands, this model has in addition to the natural decay factors, two parameters, the space-charge ξ_{SC} which is proportional to the number of electrons in the trap and normalized cross-section $\bar{\sigma}$ representing the stimulated emission probability – from the upper to the lower energy state in the resonant medium; $\bar{z}=z/L,\ \tau=\Omega t$.

For a numerical solution of this set of equations one needs to bear in mind that for describing the bunching process it is necessary to have a sub-micron spatial resolution (Nd:YAG) for the entire extent of the trap (10cm) whereas the temporal resolution has to be significantly shorter than a few fsec. Moreover, the duration of a simulation needs to be of the order of the time it takes the system to return to equilibrium which can be of the order of msec. Obviously, neither resolutions are reasonable and for a reasonable accuracy, it is necessary to employ a slowly varying amplitude approach.

SIMPLIFIED 1-D EQUATIONS

In this brief communication an intermediary approach is adopted. Substituting the explicit expression for G_E we realize that the first term is the space-charge effect whereas the second, is proportional to the energy stored in the medium. Moreover, *in average* over the optical oscillation, the electric force due to collisions of the second kind is expected to be proportional to velocity of the particle but acting in the opposite direction. This would be equivalent to a "negative" friction term. Consequently we introduce a phenomenological parameter ξ_{AM} that is proportional to the stimulated emission cross-section. Subject to this assumption and futher assuming that the space-charge effect has a dominant effect on the electric field, we get

$$\begin{aligned}
&\frac{d^2\bar{z}_\nu}{d\tau^2}+\frac{2}{\tau_s}\frac{d\bar{z}_\nu}{d\tau}+\bar{z}_\nu=\xi_{SC}\left\langle\bar{h}(\bar{z}_\nu-\bar{z}_i)\right\rangle_i+\xi_{SC}\xi_{AM}\overline{\Delta W}\frac{d\bar{z}_\nu}{d\tau}\\
&\left(\frac{d}{d\bar{\tau}}+\frac{1}{\tau_{eq}}\right)\overline{\Delta W}=-\xi_{SC}\xi_{AM}\left\langle\dot{\bar{z}}_\nu^2\right\rangle\overline{\Delta W}\\
&\frac{d}{d\bar{\tau}}\left[\frac{1}{8}\zeta_{sc}-\frac{1}{4}\zeta_{sc}\left\langle\left|\bar{z}_i-\bar{z}_\nu\right|\right\rangle_{i,\nu}+\frac{1}{2}\left\langle\dot{\bar{z}}_\nu^2\right\rangle+\frac{1}{2}\left\langle\bar{z}_\nu^2\right\rangle+\overline{\Delta W}\right]=\\
&-\frac{2}{\tilde{\tau}_s}\left\langle\dot{\bar{z}}_\nu^2\right\rangle-\frac{1}{\tau_{eq}}\overline{\Delta W}
\end{aligned}\tag{6}$$

the last expression representing the global energy conservation in the framework of our simplified model.

SIMULATION RESULTS

Although in this set of equations the explicit optical signature is "lost", we still can get some flavour of the interaction of electrons in the trap with the active medium. Figure 1 shows the phase-space at $\tau = 0$ compared with that at $\tau = 50$ namely after about 8 oscillations in the trap. Evidently part of the electrons may escape the trap since $|\bar{z}_\nu| > 0.5$. Figure 2 shows the percentage of particles that may escape the trap with the active medium on and off. During the entire simulation 30% more electrons escaped the trap when the active medium was on. Figure 3 reveals the enhanced decay of the stored-energy due to the presence of the electrons. Since in the framework of our model, this decay may be shown to be proportional to $\langle \dot{\bar{z}}_\nu^2 \rangle$, asymptotically in the trap this term vanishes and as a result, the decay rate behaves as if no electrons were present.

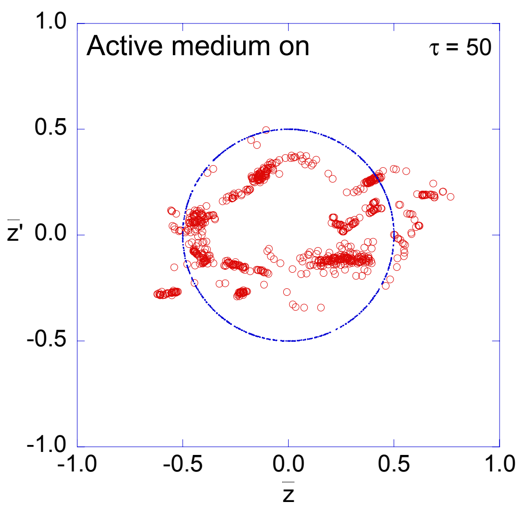

Figure 1: Phase-space at $\tau = 0$ and $\tau = 50$.

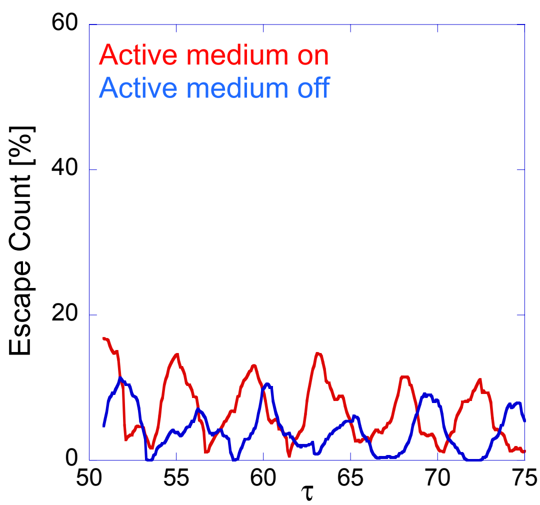

Figure 2: Percentage of electrons that may escape the trap.

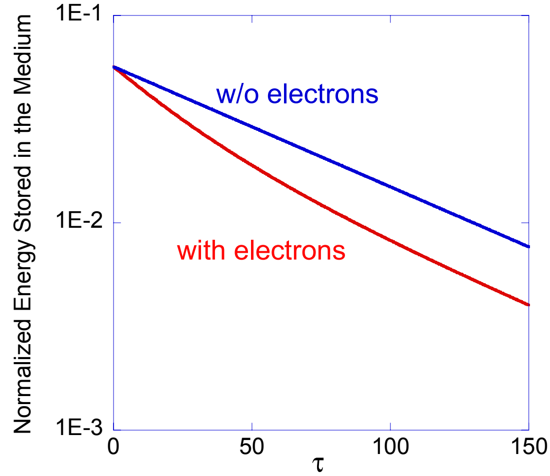

Figure 3: Energy stored as a function of time.

SUMMARY

A simplified version of the 1D dynamics of electrons in a Penning trap and in the presence of active medium has been solved numerically. Two main features were revealed: first that electrons may escape the trap due to energy absorbed from the active medium and second that this absorption process leads to enhance decay rate of the energy stored in the medium.

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REFERENCES

[1] J. Franck and G. Hertz, Dtsch. Phys. Ges. 16, 457 (1914).
[2] O. Klein and S. Rosseland, Z. Phys. 4, 46 (1921).
[3] G. Latyscheff and A. Leipunsky, Z. Phys. 65, 111 (1930).
[4] A.Schawlow and C. Townes, Phys. Rev. 112, 1940 (1958).
[5] S. Banna, V. Berezovsky, and L. Schächter, Phys. Rev. Lett. 97, 134801 (2006).
[6] L. Schächter, Phys. Lett. A 205, 355 (1995).
[7] S. Banna, V. Berezovsky, and L. Schächter, Phys. Rev. E 74, 046501 (2006).
[8]L.Schächter, Phys. Rev. Lett. 102, 034801 (2009).