

# EMITTANCE DEGRADATION OF SOURCES DUE TO UTILIZATION OF TRANSVERSE RF DEFLECTORS IN TAIWAN PHOTON SOURCE\*

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## Abstract

A pair of transverse deflecting RF cavities in the quadruple-bend achromat (QBA) lattice of 3 GeV Taiwan Photon Source (TPS) has been studied for generating ultra short X-ray pulses. The electrons after passing the first deflecting structure receive a sinusoidal vertical kick which leads to growth in their vertical amplitude and slope substantially. Therefore the non-zero momentum compaction factor and nonlinearity, and coupling of the interior sextupoles causes the second deflector cannot cancel the first kick perfectly even in perfect machine and transverse emittance increase as well. In this article, we have studied simulation and detail analysis of the effects of the non-zero momentum compaction factor and the sextupoles in between the deflecting structures for both 2<sup>nd</sup> and 3<sup>rd</sup> configurations, as sources of emittance degradation in TPS and evaluate how much emittance growths due to the effects.

## INTRODUCTION

The electrons in a segment at a longitudinal distance,  $z$ , from the bunch center after passing the first deflector receive a sinusoidal vertical kick [1-3] which is given by

$$y' = \frac{eV}{E} \sin(\omega_c t) \approx \frac{eV\omega_c z}{Ec} \quad (1)$$

where  $E$  is the nominal energy of the electrons,  $V$  is the peak deflecting voltage,  $\omega_c = h\omega_{RF}$  is the angular frequency of deflecting cavities,  $h$  is the harmonic number,  $\omega_{RF}$  is the main angular RF frequency and  $c$  is the speed of light. Although the vertical kick was almost reversed by the second deflector, but the emittance seems to grow from its nominal value of around 3 nm-rad in TPS. Since the QBA lattice functions at the deflectors for both 2<sup>nd</sup> and 3<sup>rd</sup> configurations [4-5] were the same, the main sources of emittance degradation and imperfect vertical kick cancellation process were associated with the electrons energy spread and nonlinear elements in between the deflectors. The non-zero momentum compaction factor and interior sextupoles affect the amplitude and slope of tilted electrons between the deflectors and generate errors in these parameters at the second deflector.

## EMITTANCE DEGRADATION

In this section, we present detailed analysis and simulation studies of sources of emittance degradation in a perfect TPS machine with the deflecting structures of the 2<sup>nd</sup> and 3<sup>rd</sup> configurations. For the extreme case of  $h = 8$  with deflecting voltages of 3 and 6 MV for the 2<sup>nd</sup> and 3<sup>rd</sup> configurations respectively, a Gaussian distribution of 10000 electrons per bunch was tracked for a single turn where the synchrotron radiation effects containing radiation damping and quantum excitation were excluded during the tracking. As anticipated, the effects of the emittance degradation sources would be diminished if the voltages were reduced.

## NON-ZERO MOMENTUM COMPACTION FACTOR AND ENERGY SPREAD

The non-zero momentum compaction factor effect is present even in an ideal machine where there are no errors and nonlinearities [6]. The energy spread and momentum compaction factor at TPS ring are fixed at  $8.319 \times 10^{-4}$  and  $2.712 \times 10^{-4}$ , respectively. We can get a sense of why this might matter by computing the differential time of flight of the electrons between the deflectors for a fixed energy deviation. The non-zero momentum compaction factor generates various time of flight of the electrons. Thus they have an additional phase term in the second cavity. This can be expressed as following

$$\psi_{y_2} = n\pi + \psi_{y_1} + \omega_c \Delta t \quad (2)$$

where  $\Delta t$  represents the electrons time of flight. The first two terms on the right are the nominal phase advances for an ideal cancellation and the last term is associated with the non-zero momentum compaction factor. For the fraction of the ring in between the deflectors, the differential time of flight is given by

$$\Delta t = \frac{n \alpha_c \delta}{N f_{rev}} \quad (3)$$

where  $f_{rev}$  is the revolution frequency of electrons,  $\alpha_c$  is the momentum compaction factor,  $\delta$  is the energy spread, and  $\frac{n}{N}$  is the fraction of the ring in between the deflectors.

The  $\frac{n}{N}$  is approximately 1/6 and 1/12 for the second and third configurations, respectively and results to

$$\sigma_{\Delta t-2nd} \approx 2\sigma_{\Delta t-3rd} = 6 \times 10^{-14} \text{ s.}$$

Considering the divergence of the electrons at the second deflector, the error effect in the rms electrons time of flight on the emittance blow up becomes clearer. The electrons differential time of flight generates a differential

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slope error which is evaluated by

$$\sigma_{\Delta y'} = \frac{eV\omega_c}{E} \sigma_{\Delta t} \quad (4)$$

Since the deflecting voltage for the 2<sup>nd</sup> configuration is half of the third, the error in rms vertical slope becomes equal for both configurations as follows

$$\sigma_{\Delta y'-2nd} = \sigma_{\Delta y'-3rd} = 1.5\mu\text{rad}.$$

The 1.5 $\mu\text{rad}$  is not negligible while the divergences of the un-tilted electrons at the deflectors are around 1.8 $\mu\text{rad}$  and 4.9 $\mu\text{rad}$  for the 2<sup>nd</sup> and 3<sup>rd</sup> configurations respectively, as given in Table I.

Table I. Approximate values of the vertical beta function, beam size and beam divergence at locations of the deflecting structures for the second and third configurations.

Parameter	2 <sup>nd</sup> config.	3 <sup>rd</sup> config.
$\beta_y$ ( $\mu\text{m}$ )	8.93	1.447
$\sigma_y$ ( $\mu\text{m}$ )	16.4	6.6
$\sigma_{y'}$ ( $\mu\text{rad}$ )	1.8	4.9

Therefore, using the equation

$$\sigma_{y_2} = \sqrt{\sigma_{y_1}^2 + \sigma_{\Delta y'}^2} \quad (5)$$

the non-zero momentum compaction factor increases the vertical divergence of the bunched electrons by a factor of 1.3 and 1.044 for the 2<sup>nd</sup> and 3<sup>rd</sup> configurations, respectively. If the nominal values of the beam size are presumed invariant, as given in Table I, the ratio of vertical emittance degradation at the second deflector for the 2<sup>nd</sup> and 3<sup>rd</sup> configurations are calculated as

$$\frac{\Delta\epsilon_y \Big|_{2nd}}{\Delta\epsilon_y \Big|_{3rd}} = 6.8.$$

This large factor makes the 3<sup>rd</sup> configuration seem as an optimum case.

## INTERIOR SEXTUPOLES

Sextupoles are typically required in the storage rings to correct chromatic focusing aberrations and defeat beam instabilities, but they have undesirable effects in the presence of the deflectors. The horizontal and vertical magnetic fields of a sextupole are as following

$$B_x = Sxy \quad \text{and} \quad B_y = \frac{S}{2}(x^2 - y^2) \quad (6)$$

where S is the strength of the sextupole. The electrons which have passed through the first cavity receive a large vertical kick such that their vertical amplitudes increase substantially in between the deflectors. As shown in Eq. (6), the sextupole magnetic field is nonlinear with respect to the transverse amplitude. Due to this nonlinearity, the large vertical amplitude of the electrons is significantly affected by the interior sextupoles in between the deflectors. Since the phase advance and the elimination process of the first vertical kick vary with the transverse

amplitude of electrons, nonlinearities of the interior sextupoles lead to an increase in the vertical emittance.

Furthermore, because of the interior sextupoles coupling, the horizontal emittance growth occurs as well. These undesired effects can be worse than the partial chromaticity and in such a case, switching off the interior sextupoles is favoured. Thus, both on/off operation modes of the interior sextupoles had to be investigated. In the off-mode, the residual vertical oscillation amplitude of the electrons at the second cavity is given by

$$\langle y_2^2 \rangle^{1/2} = 2\pi\Delta Q \sqrt{\beta_{y_1c}\beta_{y_2c}} y_1' \quad (7)$$

Where  $\beta_{y_1c}$  and  $\beta_{y_2c}$  are the vertical beta functions at the first and second deflectors,  $y_1'$  is the vertical kick of the first cavity,  $\Delta Q = -\frac{n}{N}\zeta_y\delta$  is the fractional vertical

betatron phase error, and  $\zeta_y$  is the vertical natural chromaticity of the ring being -30 at TPS. The betatron phase errors for the 2<sup>nd</sup> and 3<sup>rd</sup> configurations calculate as

$$\Delta Q_{2nd} \approx 2\Delta Q_{3rd} = 41.5 \times 10^{-4}.$$

The discrepancy factor of 2 in the betatron phase errors is completely compensated by the difference in their vertical kick voltages of 3 and 6 MV of the deflectors and therefore the dependence of the vertical oscillation amplitude on the vertical beta function becomes the dominant parameter as expressed in Eq. (7). Regarding the beam parameters in Table I, the residual vertical amplitude for the second configuration calculates to be 6.17 times of the third, ( $\langle y_2^2 \rangle^{1/2} \Big|_{2nd} = 6.17 \langle y_2^2 \rangle^{1/2} \Big|_{3rd}$ ).

Therefore, both the non-zero momentum compaction factor and the chromatic effect increase the vertical emittance nominal value of 30 pm-rad to 260 and 92 pm-rad for the 2<sup>nd</sup> and 3<sup>rd</sup> configurations, respectively. This indicates that the vertical emittance in the 2<sup>nd</sup> configuration is larger by a factor of 2.8 over the 3<sup>rd</sup> configuration. The simulation results of the effects are presented in Fig. 1. Both On/Off modes of the interior sextupoles for the 2<sup>nd</sup> and 3<sup>rd</sup> configurations are included in Fig. 1.

In the Off-mode, for the highest deflecting voltages (3 and 6 MV) the vertical emittance blow up almost agrees with the theoretical results obtained above. The observed discrepancy is due to the slightly smaller  $\frac{n}{N}$  values (1/6

and 1/12), and the non-approximated deflecting sinusoidal voltage that were used in the simulation. The linearly approximated deflecting voltage used in the analytical estimation was oversimplified for higher harmonics ( $h=8$ ). As a result, the vertical emittance blow up obtained in the simulation is smaller than that of the theoretical. In the On-mode, as can be seen in Fig. 1, for the highest deflecting voltages (3 and 6 MV) the vertical emittance increases to 112 and 79 pm-rad for the 2<sup>nd</sup> and 3<sup>rd</sup> configurations, respectively. As presented in Fig. 1, with deflecting voltage of 6 MV in the 3<sup>rd</sup> configuration, the nonlinearity of the interior sextupoles (On-mode) blows

up the vertical emittance almost just as much as the partial chromaticity (Off-mode).

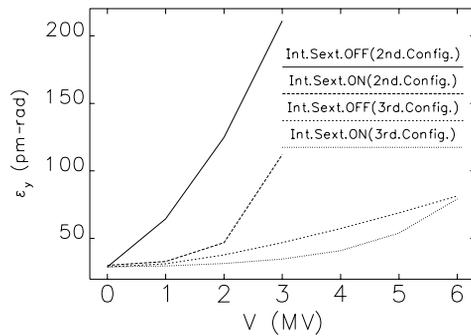


Figure 1: The vertical emittance for the 2<sup>nd</sup> and 3<sup>rd</sup> configurations after a single pass through the system as a function of deflecting voltage for h=8. With interior sextupoles off, tracking with momentum spread illustrates the impact of natural chromaticity especially for the second configuration.

However, with deflecting voltage of 3 MV in the 2<sup>nd</sup> configuration, the nonlinearity of the interior sextupoles (On-mode) blows up the vertical emittance almost half as much as the partial chromaticity (Off-mode). Consequently, for both configurations, working in the On-mode case, a lower vertical emittance blow up is produced especially for lower deflecting voltages. Overall, the 3<sup>rd</sup> configuration with the interior sextupoles in the On-mode case seems to be more favourable to the 2<sup>nd</sup> configuration. Furthermore, at higher deflecting voltages of 3 and 6 MV for both configurations, it is anticipated that the On- and Off-mode vertical emittance blow up curves would cross each other where the Off-mode case would generate a smaller vertical emittance blow up thereafter.

As far as the horizontal emittance degradation in the third configuration is concerned, Fig. 2 shows that as the deflecting voltage is increased the horizontal emittance is unaffected in the Off-mode and is blown up to 4.3 from 3.0 nm-rad for the On-mode case where the interior sextupoles coupling effect is present.

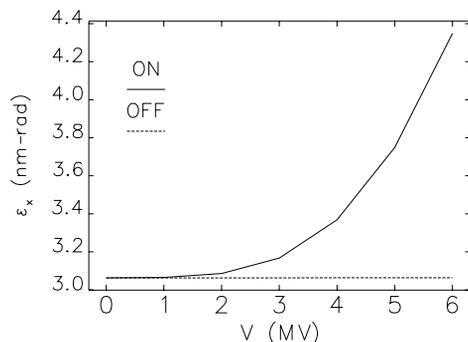


Figure 2: The horizontal emittance for third configuration after a single pass through the system as a function of deflecting voltage for h=8. No increasing is seen in the emittance when the sextupoles are off.

## CONCLUSION

Regardless of errors and synchrotron radiation effects, degradation of vertical emittance in presence of deflecting structures is mainly due to the vertical kick of cavities, nonlinear elements between the cavities, energy deviation of particles. The results show that switching the interior sextupoles on is more beneficial especially for low voltages in view of a vertical emittance blow up although the horizontal emittance blows up as well. In addition, it was found that after a single turn electron tracking, the vertical emittance of electron beam for operating the deflectors in the third configuration is almost half of second configuration.

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