

# TUNE SHIFT DUE TO CROSSING COLLISION AND CRAB COLLISION\*

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## Abstract

The use of crab cavities in the LHC may not only raise the luminosity, but it could also complicate the beam dynamics, e.g. crab cavities might not only cancel synchrotron resonances excited by the crossing angle but they could also excite new ones. In this paper, we use weak-strong beam-beam model to study the incoherent linear tune shift of the weak beam, for the crossing collision case and crab collision case with a finite crossing angle. The tune shift is also compared among the head-on collision, crossing collision and crab collision cases, both analytically and numerically.

## INTRODUCTION

The beam instability induced by beam-beam interaction is one of the most severe problems for the existing and future colliders. The head-on luminosity in a storage ring is given by

$$L = \frac{N^2 f k}{4\pi \sigma_x \sigma_y} \quad (1)$$

where  $N$  denotes the bunch population (for collision with two equal population bunch),  $f$  the revolution frequency,  $k$  the bunch number in one beam,  $\sigma_x$  the horizontal bunch size at IP, and  $\sigma_y$  the vertical bunch size at IP. From the experiments, it is well known that above some bunch density, the luminosity will not increase proportional to  $N^2$ , but proportional to  $N$  instead. This is the so-called beam-beam limit and the associated beam-beam parameter  $\xi$  will be a constant above some saturated bunch intensity. For a tune far enough away from linear resonances, and a test particle (in the weak beam) crossing a strong beam at small amplitudes, the test particle experiences a linear quadrupole field and thus has the linear tune shift (which is also the linear beam-beam parameter  $\xi$ ) as [1]

$$\xi_z = \frac{\beta_z \delta}{4\pi} = \frac{N r_0 \beta_z}{4\pi \gamma \sigma^{*2}} \quad (2)$$

where  $r_0 = e^2/4\pi\epsilon_0 m c^2$  denotes the classical particle radius,  $\gamma = 1/\sqrt{1-\beta^2}$  the Lorentz factor,  $\delta$  the inverse focal length of the quadrupole which represents the beam-beam kick,  $z$  represents  $x$  or  $y$ ,  $\beta_z$  the horizontal or vertical beta function at IP,  $\sigma^*$  the horizontal bunch size at IP.

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## CROSSING COLLISION

### Theory

To calculate the tune shift, Ruggiero and Zimmermann first applied Maxwell's equation to get the particle's equation of motion in the strong beam's coordinate frame, for head-on collision case [2]. Then for the crossing collision case with an angle  $\theta = 2\phi$ , they performed transformation for coordinates, also for the electric and magnetic fields, from strong beam coordinate frame to weak beam coordinate frame. By that they could get the beam-beam force in the weak beam coordinate frame, and integrate the derivative of the force times the beta function at IP over the longitudinal direction to get the linear tune shift of the test particle (in the weak beam). Figure 1 shows the coordinate systems used for their analysis, with a horizontal crossing angle  $\theta = 2\phi$ .

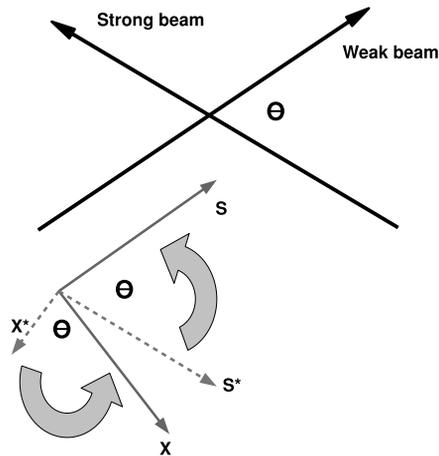


Figure 1: Schematic of coordinate systems for the two colliding beams under a horizontal crossing angle  $\theta = 2\phi$ . The  $s$ - $x$  coordinates without asterisk refers to the frame for the weak beam [2].

If the beams cross at one IP under a vertical crossing angle  $\theta$ , and cross at another IP under a horizontal angle  $\theta$ , the total tune shift  $\Delta Q_{tot}$  is the same in the two planes and given by the sum of  $\Delta Q_x$  and  $\Delta Q_y$  [2]. For a special example, if we consider the Gaussian bunches, with a bunch length which is much shorter than the IP beta function  $\beta^*$ ,

but much larger than  $\sigma^*$ , and with a small crossing angle  $\theta$ , we get a simplified expression of the total tune shift as

$$\Delta Q_{tot} \approx -\frac{N_b r_0 \beta^*}{2\pi\gamma\sigma^* \sqrt{\sigma^{*2} + \theta^2\sigma_z^2/4}} \quad (3)$$

where  $N_b$  denotes the number of particles in a bunch,  $\gamma$  the normalized beam energy,  $\sigma_z$  the bunch length.

### Simulation and Comparison

We perform simulation studies with the well known code MADX [3] and SixTrack [4], to get the tune shift for a test particle with small transverse amplitudes, and compare them with the theoretical results expressed in formulae 2. For the simulations, LHC sequence V6.5 for beam 1 is used and the beam parameters are as follows (also used for the theoretical prediction): bunch length  $\sigma_z = 0.077m$ , bunch intensity  $N_b = 1.1E + 11$ ,  $\beta^* = 0.55m$ , normalised emittance  $\epsilon = 3.75\mu m$ , proton energy  $E = 7TeV$ , full crossing angle  $\theta = 2\phi = 0 - 600\mu rad$ , RF voltage  $V_{rf} = 16MV$ . In MADX 4-D beam-beam element is used and the crossing angle at this element is changed by modifying the strength of the crossing bumps. For SixTrack the beam-beam element is available in the 6D form by applying Hirata's treatment. In both codes the beam-beam element is longitudinally 5 sliced. For the case that one vertical beam-beam element is placed at IP1, and at the same time one horizontal beam-beam element is placed at IP5, the linear tune shift versus half crossing angle  $\phi$  from the simulations and from the theory (formulae 2) are plotted together in Figure 2 where good agreement is found.

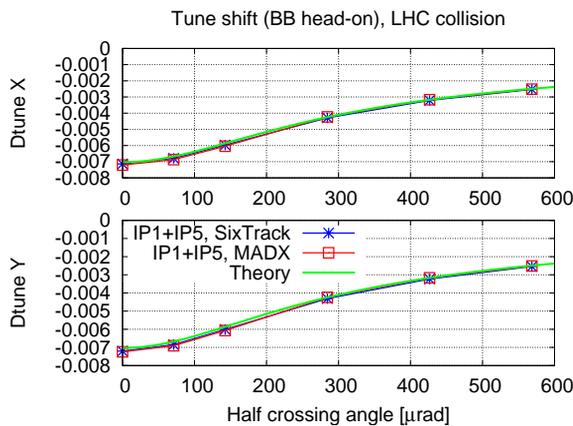


Figure 2: Comparison of tune shift between the MADX (red), SixTrack simulations (blue) and the theory (green): horizontal detune (top); vertical detune (bottom).

Also simulation is performed for another two cases: one vertical beam-beam element at IP1 alone, and one horizontal beam-beam element at IP5 alone. The tune shift versus half crossing angle  $\phi$  for these two cases are shown together with the previous case, also with the crab collision case which will be introduced in next section, in Figure 3.

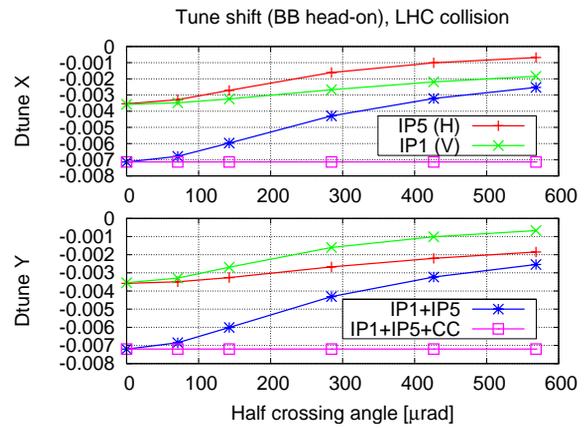


Figure 3: Horizontal detune (top) and vertical detune (bottom): one horizontal BB element at IP5(H) (red); one vertical BB element at IP1(V) (green); one vertical BB element at IP1(V) plus one horizontal BB element at IP5(H) (blue); and crab collision case (magenta).

## CRAB COLLISION

### Theory and Modified Code

The crab cavities (CC) have been proposed for both linear [5] and circular colliders [6], to restore an effective head-on collision at the IP. The crab cavity gives rise to a z-dependent transverse kick on the beam particles, as well as to a change in the longitudinal momentum. To calculate the tune shift of crab collision case, we use weak-strong model, Hirata's formalism [7] and follow the treatment implemented in SixTrack code by Leunissen et al. [8]. For the crab collision case, we use the same Lorentz boost (formulae 2.23 of [8]) which is introduced by Hirata [9] as expressed in formulae 4, which consists of a transformation from Cartesian to accelerator coordinates, and a Lorentz boost from crossing collision to head-on collision.

$$L_0 = \begin{pmatrix} 1/\cos\phi & -\sin\phi & -\tan\phi\sin\phi & 0 \\ -\tan\phi & 1 & \tan\phi & 0 \\ 0 & -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (4)$$

where  $L_0$  denotes the boost,  $\phi$  the half crossing angle.

The calculation of the beam-beam force is done by approximating the strong bunch by a number of longitudinal slices. Due to crab collision, the first and second moments of the particle distribution at the longitudinal slices are modified and expressed in formulae 5 (which is different from formulae 2.42 of [8]), and then the beam-beam force is calculated accordingly.

$$\begin{aligned} X^\dagger &= 0, & P_X^\dagger &= 0, & Y^\dagger &= 0, \\ P_Y^\dagger &= 0, & P_Z^\dagger &= 0, & \Sigma_{11}^\dagger &= \Sigma_{11}, \\ \Sigma_{22}^\dagger &= \Sigma_{22}, & \Sigma_{33}^\dagger &= \Sigma_{33}, & \Sigma_{44}^\dagger &= \Sigma_{44}, \\ \Sigma_{12}^\dagger &= \Sigma_{12}, & \Sigma_{13}^\dagger &= \Sigma_{13}, & \Sigma_{14}^\dagger &= \Sigma_{14}, \\ \Sigma_{23}^\dagger &= \Sigma_{23}, & \Sigma_{24}^\dagger &= \Sigma_{24}, & \Sigma_{34}^\dagger &= \Sigma_{34}. \end{aligned} \quad (5)$$

where  $X = (X, P_X; Y, P_Y; Z, P_Z)^T$  denotes the coordinates of the strong bunch,  $\Sigma$  the  $6 \times 6$  phase-space envelope matrix of the strong bunch.

The synchrobeam mapping (SBM) formulated only for head-on collision [7] is then applied at IP. After that the particle's coordinates are transformed back to the original accelerator frame, and transformed through arc. The Six-Track code is modified according to the above mentioned crab collision case and a new version is used to calculate the tune shift.

### Tune Shift of Crab Collision

The optics and beam parameters used are the same with the crossing collision case, and two local linear crab cavities (20-MHz) are placed at both sides of IP5 to recover head-on collision at IP5. The tracking is done for beam 1 (weak beam) as a test particle, and beam 2 is treated as the strong beam which the beam-beam element represents. For the crab collision case (two beams crabbed) that has one vertical beam-beam element at IP1 plus one horizontal beam-beam element at IP5, the linear tune shift from the simulation is found to be the same as the head-on tune shift, as shown by the magenta curve in Figure 3. A similar result is found for the crab collision case that has only one horizontal beam-beam element placed at IP5, as shown in Figure 4.

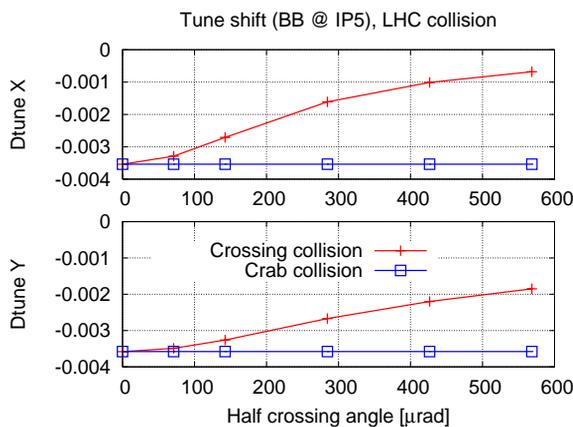


Figure 4: Horizontal detune (top) and vertical detune (bottom): crossing collision with one horizontal beam-beam element at IP5(H) (red); crab collision with the same beam-beam element at IP5(H) and two beams crabbed (blue).

With synchrotron oscillation, we compare the tune shift of crossing collision case, crab collision with only beam 2 crabbed, crab collision with two beams crabbed and head-on collision case, for particles with different longitudinal coordinate  $\Delta_z$  in a bunch (and with small transverse offsets), with the results shown in Figure 5 (with only one horizontal beam-beam element at IP5). We observe that the tune shift of the crab collision with two beams crabbed is almost recovered to be equal to the head-on collision case, especially for particles with smaller longitudinal offset. For the two beams crabbed case, as the local linear crab

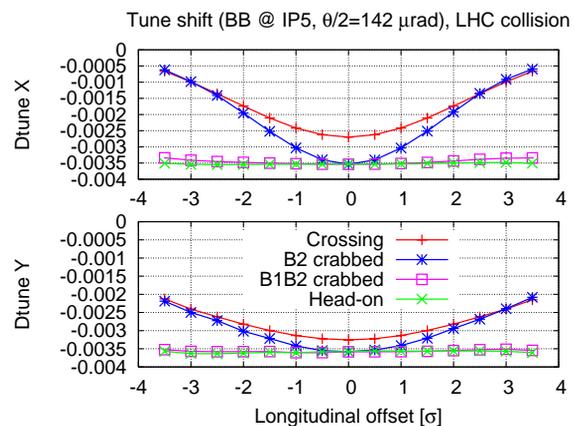


Figure 5: Horizontal detune (top) and vertical detune (bottom) versus  $\Delta_z$ : crossing collision (red); crab collision with only beam 2 crabbed (blue); crab collision with both beams crabbed (magenta); head-on collision (green).

cavities are used, a particle that has a large longitudinal offset ( $\Delta_z = \pm 3.5\sigma_z$ ) also gets a linear crab cavity kick, and thus has a detune just as the head-on collision case. Since beam 1 is treated as a test particle, for  $\Delta_z = 0$ , the beam 2 crabbed case is identical to the two beams crabbed case. For  $\Delta_z \neq 0$ , the tune shift of one beam crabbed case is between that of crossing collision case and head-on collision case; and especially for particles with large longitudinal coordinate ( $\Delta_z > 2\sigma_z$  or  $\Delta_z < -2\sigma_z$ ), the tune shift from one beam crabbed case is quite small which almost equals the tune shift of crossing collision case.

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