RF BARRIER COMPRESSION WITH SPACE CHARGE FOR THE FAIR SYNCHROTRONS

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Abstract

The conservation of the longitudinal beam quality through the SIS-18/100 synchrotron chain is of major importance for the FAIR accelerator project as well as for the SIS-18 upgrade. The generation of a short, intense heavy ion bunch at the end of the machine cycle defines a tight budget for the tolerable longitudinal emittance growth. Potential sources of bunch quality degradation are intensity effects and non-adiabatic rf ramps during the rf capture in SIS-18 and during the barrier bucket pre-compression in SIS-100. The time spend on rf manipulations has to be as small as possible in order to maximize the repetition rate. In the present simulations studies we show that longitudinal space charge can improve the efficiency of rf manipulations. As an example we present an optimized barrier bucket pre-compression scheme for SIS-100.

INTRODUCTION

The pre-compression of an intense U^{28+} bunch at 1 GeV/u using two rf barrier pulses for the subsequent fast (0.1 ms) bunch rotation just before extraction is one of the central and most demanding rf manipulations in the SIS-100 synchrotron [1]. The pre-compression should be completed in about $T = 100$ ms in order to meet the desired average intensities on the production target. After acceleration to 1 GeV/u the eight bunches will be debunched between two barrier rf half-waves into a single long bunch of length $l_0 \approx 0.8L$ (ring circumference $L = 2\pi R = 1080$ m). Afterwards one of the barriers is moved inside the bunch for pre-compression up to $l_1 \approx 0.4L$. Because of the low momentum spread of initially $(\Delta p/p)_{\mathrm{rms}} = 10^{-4}$ and the corresponding small synchrotron frequency of $T_s \approx 300$ ms the compression cannot be truly adiabatic. However, for low momentum spreads and high intensities ($N = 5 \cdot 10^{11}$ ions) space charge effects play an important role. It is the aim of this contribution to demonstrate that space charge can significantly improve the performance of the barrier compression. A novel 'shock' compression scheme for will be described.

POTENTIAL DISTORTION DUE TO SPACE CHARGE

The stationary line density profile for a bunch that is confined between two barrier rf half-waves is obtained assuming a local elliptic distribution. Let z be the deviation in position from the synchronous particle. The resulting line density is [2]

$$\lambda(z) = \lambda_0 \left(1 - \frac{Y(z)}{Y(z_m)}\right) \tag{1}$$

with the potential function at the bunch ends $z = z_m$

$$Y(z_m) = \frac{Lm^* v_m^2}{2qV_0}, \tag{2}$$

the effective mass $m^* = -\gamma_0 m/\eta$, the relativistic parameter γ_0, the slip factor $\eta_0 = 1/\gamma_t^2 - 1/\gamma_0^2$, the transition parameter γ_t, the ring circumference L, the charge q and the voltage amplitude V_0. $v_m = -\eta_0\beta_0 c\Delta p/p$ is the maximum velocity and $\Delta p/p$ the momentum spread between the barriers. The potential function

$$Y(z) = \frac{1}{V_0}\int_0^z V\,dz, \tag{3}$$

is obtained from the voltage profile $V(z)$. The voltage profile can be divided into the external (rf) voltage part and the space charge part $V = V_{\mathrm{rf}} + V_{\mathrm{sc}}$. Between the barriers the force is zero ($Y = 0$) and the line density is constant $\lambda(z) = \lambda_0$, if we only account for space charge effects. The space charge voltage is given through (see e.g. [2])

$$V_{\mathrm{sc}}(z) = -q\beta_0 c X_{sc} R \frac{\partial\lambda}{\partial z} \tag{4}$$

with the space charge reactance

$$X_{\mathrm{sc}} = \frac{g}{2\epsilon_0\beta_0 c\gamma_0^2} \tag{5}$$

and the g-factor [4]. The space charge factor Σ for a matched bunch distribution can be derived from [3]

$$\Sigma = 2\frac{c_{s0}^2}{v_m^2} \tag{6}$$

with the coherent phase velocity of space charge waves [3]

$$c_{s0} = \sqrt{\frac{qX_s I_0}{2\pi m^*}} \tag{7}$$

and the current I_0 between the barriers. The matched bunch distribution at SIS-100 extraction energy (1 GeV/u) is shown in Fig. 1.

NON-ADIABATIC BARRIER COMPRESSION

In order to pre-compress the bunch for the subsequent fast bunch rotation the phase of one barrier rf wave is varied and the barrier moves with the velocity $u_b = \dot{l}$ into

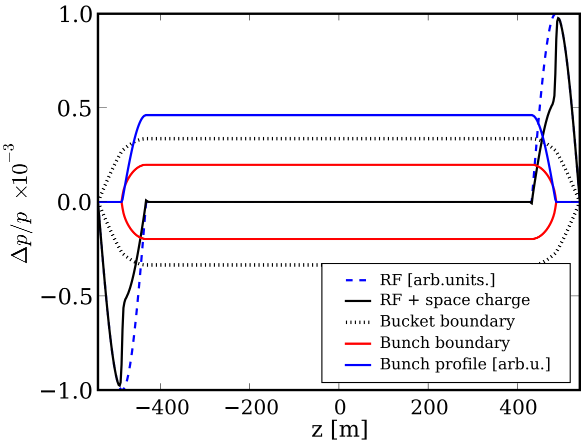

Figure 1: Matched bunch and voltage profiles between two barrier rf half-waves for $\Sigma = 3$.

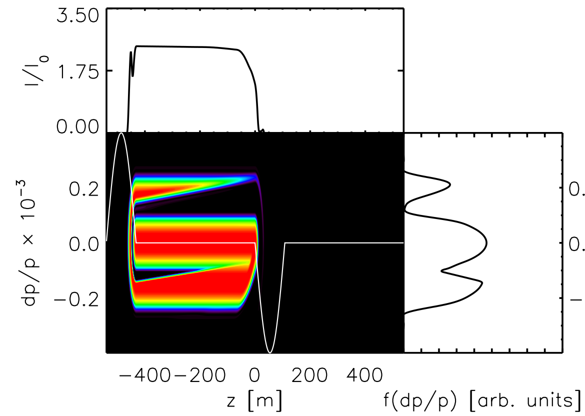

Figure 2: Compressed bunch distribution at $T = 100$ ms and for $\Sigma = 0$.

the beam. l is the distance between the barriers. The synchrotron period for particles with the velocity v_m is $T_s \approx 2l/v_m$. For SIS-100 parameters we obtain $T_{s0} \approx 300$ ms for the synchrotron period before compression. Adiabatic compression requires $\mu = T_s/T \ll 1$, with the adiabaticity parameter μ and the compression time T (see e.g [5]). The criterium $\mu \ll 1$ is equivalent to $u_b \ll v_m$. An adiabatic compression ramp can be derived from

$$\frac{1}{l}\frac{dl}{dt} = \frac{\mu}{T_s(l)} \tag{8}$$

with the synchrotron period during compression $T_s(l) = T_{s0} l^2/l_0^2$ and the initial barrier distance l_0 Eq. (8) yields

$$l = l_0 \sqrt{1 - 2\mu \frac{t}{T_{s0}}}. \tag{9}$$

The final barrier distance in SIS-100 is $l_1 = l_0/2$ and the corresponding adiabatic compression time $T/T_{s0} = 3/(8\mu)$. For $T = T_{s0}$ the adiabaticity parameter is $\mu = 3/8$. In SIS-100 the requirement is $T = 100$ ms for $T_{s0} = 300$ ms. Therefore we deliberately attempt a non-adiabatic compression with a constant velocity $u_b \lesssim v_m$. The maximum velocity v_m changes at the moving barrier to $v_{m1} = -(v_m + 2u_b)$. In this scheme we adjust the compression time exactly in such way that the particle with initial velocity v_m and initial position at the barrier z_m reaches the moving barrier again. In this way no particle hits the moving barrier more than once. The compression time is

$$T = \frac{\Delta l}{u_b} = \frac{l_0 + l_1}{v_m + 2u_b} \tag{10}$$

with $\Delta l = l_0 - l_1$. From the above equation we obtain

$$u_b = v_m \frac{\Delta l}{3l_1 - l_0}. \tag{11}$$

and finally $T = (3l_1 - l_0)/v_m$, which yields $T \approx 100$ ms for SIS-100 parameters. Fig. 2 shows the final beam distribution obtained from a Vlasov simulation [3]. The increase of the bunch area due to the emerging voids in the occupied phase space area is clearly visible. The rms bunch area increases by a factor 1.4.

SHOCK COMPRESSION

In the limit of a space charge dominated beam with $c_{s0} \gg v_m$ the Vlasov equation for the evolution of the distribution function can be reduced to the cold fluid equations for the line density $\lambda(z)$ and the local velocity $u(z)$ (see Ref. [6])

$$\frac{\partial \lambda}{\partial t} + \frac{\partial}{\partial z}(\lambda u) = 0 \tag{12}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z} + \chi \frac{\partial \lambda}{\partial z} = 0$$

with

$$\chi = \frac{q X_s \beta_0 c}{2\pi m^*} \tag{13}$$

These equations comply with Euler's fluid equations for the equation of state $p = \chi\lambda^\gamma/2$ with $\gamma = 2$. It is well known that for a piston moving into a gas the equations above exhibit a shock wave solution [7]. In our compression scheme the barrier moving with the constant velocity u_b acts like a piston. The resulting shock velocity is c_s. In Fig. 3 the shock front propagation is depicted. λ_1 is the line density in front of the moving barrier. λ_0 is the unperturbed line density. By applying the conservation laws for the particle number and momentum across the shock discontinuity to Eqs. 12 we can obtain

$$2c_s(c_s - u_b)^2 = c_{s0}^2(2c_s - u_b) \tag{14}$$

For weak shocks with $u_b \ll c_{s0}$ we obtain

$$c_s = c_{s0}, \quad c_{s0} = \sqrt{\chi\lambda_0} \tag{15}$$

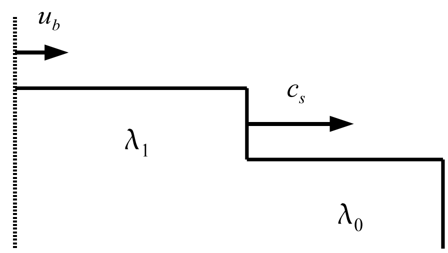

Figure 3: Shock front moving with the velocity c_s from the barrier (velocity u_b) into the undisturbed beam.

The analytic solution to Eq. 14 can be approximated as $c_s \approx c_{s0}$ for $u_b \lesssim 0.5c_{s0}$. For larger u_b Eq. 14 exhibits only complex roots. The compression time is tuned exactly in such a way that the shock front returns back to the moving barrier after being reflected at the fixed barrier. The resulting time is $T = \Delta l/u_b = (l_0 + l_1)/c_{s0}$ and the barrier velocity

$$u_b = c_{s0}\frac{\Delta l}{l_0 + l_1} \tag{16}$$

The compression time for space charge dominated beams can be much shorter, because the relevant velocity is c_s and not v_m. However, the most important aspect is that the shock compression should conserve the bunch area. For the initial bunch in SIS-100 the space charge parameter is still moderate ($c_s \gtrsim v_m$). Fig. 4 shows the result of a Vlasov simulation of the barrier compression using the expected beam parameters in SIS-100. In Fig. 4 the steep shock front launched from the moving barrier (on the right) is clearly visible. In about $T = 100$ ms the shock front returns to moving barrier and the pre-compression is completed. The compression does not lead to a noticeable growth of the occupied phase space area. From the simulation we obtain a shock velocity of $c_s \approx 2c_{s0}$ and a barrier velocity $u_b/c_s \approx 0.5$ for a compression time $T = 100$ ms. Instead, the compression time and barrier velocity obtained from Eq. 16 would be $u_b/c_s \approx 0.3$ and $T = 180$ ms. This discrepancy is due to the fact that the beam is not space charge dominated. The agreement with Eq. 16 improves with increasing beam intensity. However, the shock compression scheme works even for moderate space charge.

CONCLUSIONS

A novel barrier compression scheme for space charge dominated bunches is presented. The scheme takes advantage of the shock front that launches naturally if a barrier moves into a space charge dominated beam. The duration of the compression depends on the velocity of the shock front c_s and not on the incoherent velocity spread v_m. Therefore the compression can be much faster. In addition the bunch area is conserved in the shock compression scheme. Variation of the details of the scheme are possible. Using Vlasov simulations for SIS-100 parameters we showed that the scheme can be applied also to beams with moderate space charge. It is important to point out that for strong space charge collective effects can lead to a collapse of the shock front.

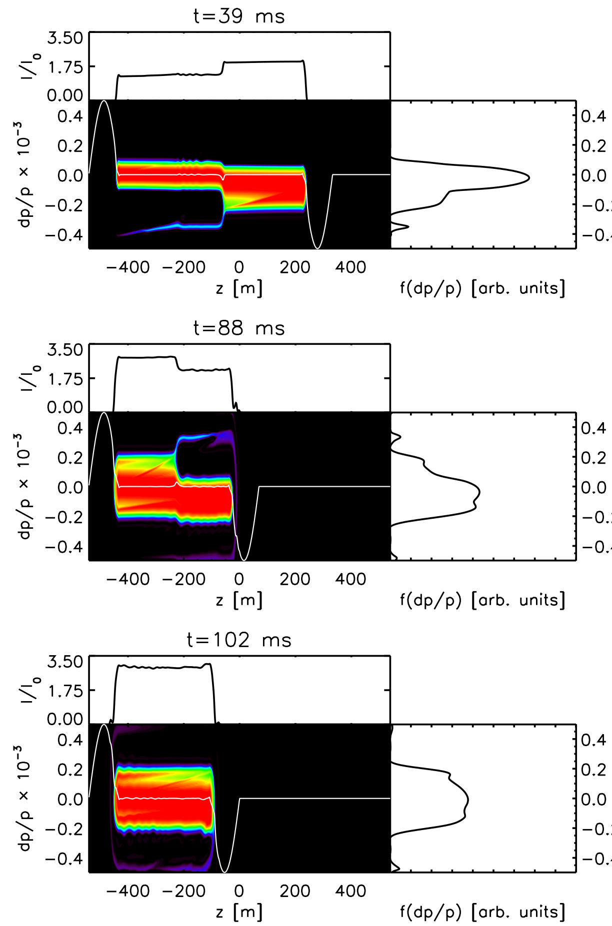

Figure 4: Evolution of the beam distribution during barrier compression obtained from a Vlasov simulation for the beam parameters expected in SIS-100.

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