

# TOUSCHEK LIFETIME MEASUREMENTS AT SMALL HORIZONTAL EMITTANCE IN THE ALS\*

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## Abstract

The Touschek lifetime in low energy or small emittance lepton storage rings strongly depends on the particle density in bunches. In the usual parameter range, this dominates other effects and the lifetime gets shorter with higher the bunch density, i.e. with smaller beam emittance. However, once one gets to extremely small horizontal emittances, this is no longer the case. Since the Touschek scattering process is an energy transfer from the transverse plane to the longitudinal one, the Touschek lifetime actually increases, once the transverse temperature (i.e. emittance) gets small enough. In the usual Touschek lifetime formulas, this is accounted for with a complicated multi-parameter function (form factor). This presentation will present to our knowledge the first direct measurements of the Touschek lifetime in this region of reversed dependence on horizontal emittance, as well as comparison with theory. The measurements were carried out at the ALS at reduced beam energy and very small horizontal emittance.

## INTRODUCTION

The beam lifetime at most modern synchrotron light sources is (mostly) dominated by the Touschek effect. In a Touschek scattering event transverse momentum (associated with the transverse emittances) can be transferred into the longitudinal plane. If the energy transfer is large enough, the particles can fall outside the momentum acceptance  $\delta_{acc}(s)$  of the ring and therefore they will be lost. If the scattering happens at a position with nonzero dispersion, it can also lead to large transverse oscillations. This effect usually reduces the (dynamic) momentum acceptance for lattice locations with dispersion [1, 2]. The Touschek lifetime can be calculated as [3]

$$\frac{1}{\tau} = \frac{r_e^2 c q}{8\pi e \gamma^3 \sigma_s} \frac{1}{C} \oint_C \frac{F\left(\left(\frac{\delta_{acc}(s)}{\gamma \sigma'_x(s)}\right)^2\right)}{\sigma_x(s) \sigma'_x(s) \sigma_y(s) (\delta_{acc}(s))^2} ds, \quad (1)$$

where  $r_e$  denotes the classical electron radius,  $q$  the bunch charge,  $\sigma_s$  the bunch length,  $C$  the circumference of the storage ring, and  $\sigma_x(s)$  and  $\sigma_y(s)$  the rms horizontal and vertical beamsizes.  $c$  is the speed of light,  $e$  the electron charge, and  $\gamma$  the relativistic Lorentz factor of the beam.

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The beam divergence  $\sigma'_x(s)$  in this case is given by

$$\sigma'_x(s) = \frac{\epsilon_x}{\sigma_x(s)} \sqrt{1 + \frac{\mathcal{H}(s) \left(\frac{\Delta p}{p}\right)^2}{\epsilon_x}}, \quad (2)$$

where  $\mathcal{H}(s)$  is the usual invariant

$$\mathcal{H}(s) = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta'_x + \beta_x \eta'^2_x. \quad (3)$$

Finally the Touschek form factor  $F(x)$  in above formula is given by

$$F(x) = \int_0^1 \left(\frac{2}{u} - \ln \frac{1}{u} - 2\right) \exp\left(-\frac{x}{u}\right) du. \quad (4)$$

Touschek lifetime measurements and calculations in existing third generation light sources nowadays agree very well [2, 4]. However, in all of those cases the conditions were on the same side of the minimum in the form factor. In that regime, smaller emittances (and therefore higher brightness) usually mean shorter lifetimes because of the higher bunch density resulting in a larger probability for collisions. However, there is an additional effect. Once the transverse emittances (particularly the larger horizontal one) gets small enough, the Touschek lifetime becomes larger again, since there is not enough energy anymore to fall outside the momentum acceptance. This regime is of some interest, since planned machines like NSLS-II [5] and MAX-IV begin to explore it and proposed ultimated storage rings would operate well beyond the minimum in Touschek lifetime. Therefore we carried out measurements beyond the minimum of the form factor to confirm that lifetime calculations in that parameter regime are accurate as well.

Figure 1 shows the calculated Touschek lifetime as a function of horizontal emittance for a 1 nC bunch in the ALS at 1.23 GeV. For the calculation, an RF-acceptance of 0.03 was used, and the dynamic momentum acceptance and lattice functions were calculated from a measured orbit response matrix - see next sections. The nominal ALS lattice has an emittance of about 2.5 nm at 1.23 GeV, i.e. one is just beyond the minimum of the form factor.

## ORBIT RESPONSE MATRIX ANALYSIS

As one can see in the Touschek lifetime formula, one needs to know beamsizes and divergences, as well as lattice functions and the momentum acceptance for every location around the ring to calculate the Touschek lifetime. One very

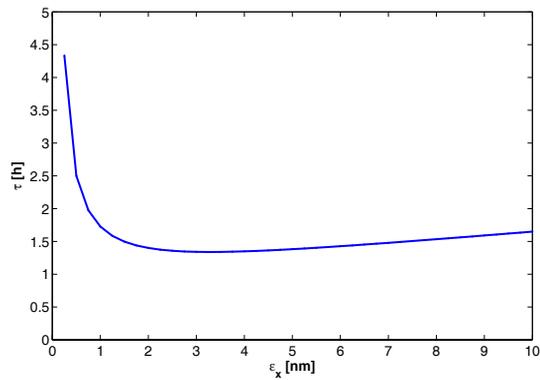


Figure 1: Calculated Touschek lifetime as a function of horizontal emittance for a 1 nC bunch in the ALS at 1.23 GeV. The nominal ALS lattice has an emittance of about 2.5 nm at 1.23 GeV.

powerful tool to provide all required input variable is orbit response matrix analysis. Orbit response matrix analysis has been used very successfully for years at modern storage rings, either to correct lattice errors and restore lattice symmetry (gradients and skew gradients), as well as to calibrate machine models. For this study it is used to generate the calibrated lattice model, which is used to calculate lattice functions, beamsizes and even the dynamic momentum aperture to compute the Touschek lifetime and compare it to measurements. An example of an orbit response matrix measurement for the ALS is shown in Figure 2. It includes more than 100 beam position monitors as well as almost 100 horizontal and about 70 vertical corrector magnets.

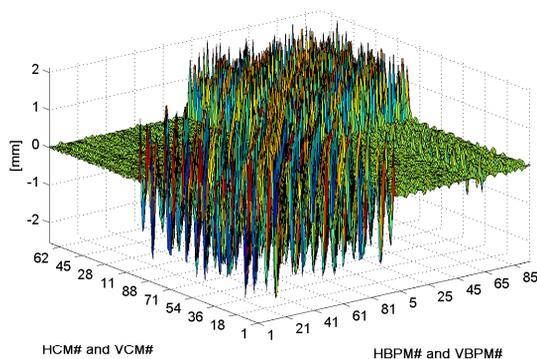


Figure 2: Measured orbit response matrix of the ALS.

A complete response matrix measurement only takes about 10 minutes for the ALS and the analysis can be completed in about one minute on modern personal computers. Fit parameters for this analysis include about 70 quadrupole as well as 70 skew quadrupole gradients, as well as gains and coupling/tilt for all corrector magnets and BPMs.

## MOMENTUM APERTURE

To calculate the dynamic momentum acceptance (which is then used in the lifetime calculation), the calibrated machine model from the orbit response matrix analysis is used. The tracking is carried out using AT. For each lattice location around the ring particles are launched with various energy offsets (but otherwise on the closed orbit) and tracked for about one damping time with damping switched on. Figure 3 shows the result of the tracking for the calibrated machine model of the ALS. As a side remark, the dynamic momentum acceptance of the ALS can be fully described by nominal sextupole fields, as well as measured gradient and skew gradient errors. No higher multipoles or fringe effects needed to be included.

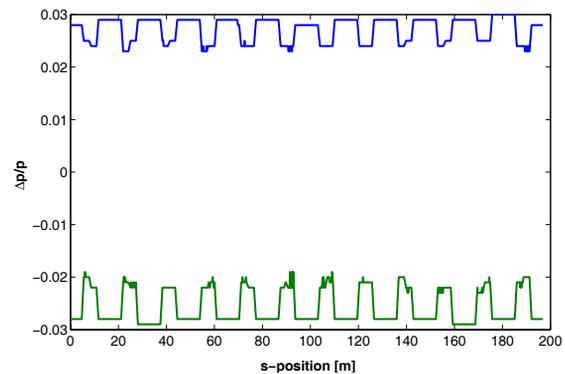


Figure 3: Position dependent dynamic momentum acceptance of the full ALS ring calculated from calibrated LOCO model.

The tracking was used to find the smallest energy deviation for each point of the lattice, at which particles were lost in tracking. The largest energy deviation below this, for which particles still survived in tracking was then used as the (dynamic) momentum acceptance. The dynamic momentum acceptance is almost 3% for particles lost in the straight sections and between 2.0 and 2.5% for particles lost in the dispersive arcs.

## MEASUREMENTS AND COMPARISON WITH CALCULATIONS

For the lifetime measurement, the RF amplitude was changed and the lifetime was measured for each value of the RF bucket size (and therefore RF momentum acceptance). The results are shown in Figure 4.

In addition to the measured beam lifetimes with errorbars, the figure also shows the calculated beam lifetime using the calibrated machine model and dynamic momentum acceptance as explained above. The blue curve is the case if the momentum acceptance would only be defined by the RF bucket size, the red one includes the effects of the dynamic momentum acceptance. As one can see, using the calibrated machine model, excellent agreement is reached between the measured Touschek lifetime (as a function of

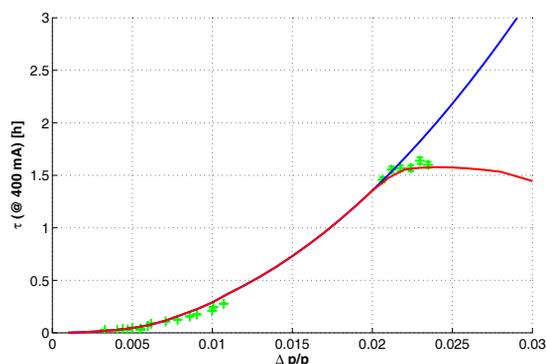


Figure 4: Measured Touschek lifetime for 1 nC per bunch at 1.23 GeV (horiz. emittance is about 2.5 nm) as a function of RF acceptance compared to calculation based on LOCO model (blue without dynamic momentum acceptance, red with dynamic momentum acceptance again calculated from calibrated LOCO model).

the acceptance of the RF bucket by changing the RF voltage) and the calculated one. One should reiterate, that the calculation does not use any fit parameters but uses all calculated values (for beamsizes, lattice functions, dynamic momentum acceptance) from the calibrated LOCO model which was derived from an orbit response matrix measurement.

As a conclusion, for this set of parameters, which are just beyond the minimum of the form factor, these measurements confirm that the lifetime formulas are still correct.

## MEASUREMENTS AT LOWER ENERGIES

To probe deeper into the region beyond the lifetime minimum measurements were also started at 1.0 GeV. In this case, the emittance is small enough, that one is well on the other side of the lifetime minimum. Of course, the reduction of  $\gamma$  contributes as well. Figure 5 again shows the calculated Touschek lifetime as a function of horizontal emittance for a 1 nC bunch, but this time for 1.0 GeV. As an RF-acceptance we again used 0.03, and the dynamic momentum acceptance and lattice functions were calculated from a calibrated machine model base on orbit response matrix analysis. The nominal ALS lattice has an emittance of about 1.7 nm at 1.0 GeV.

Initial measurements were carried out and agree reasonably with the calculations. However, at this energy, collective effects, including potential well distortion, intra beam scattering and the effects of the coherent synchrotron radiation wake, cannot be neglected anymore and a precise quantitative analysis requires simultaneous measurements of lifetime, beamsizes and bunchlength. Another fundamental limitation however might be the fact, that the collective effects result in non-gaussian beam distributions, the exact shape of which are challenging to measure, particularly in the vertical plane because of resolution limits of

**Beam Dynamics and Electromagnetic Fields**

**D03 - High Intensity - Incoherent Instabilities, Space Charge, Halos, Cooling**

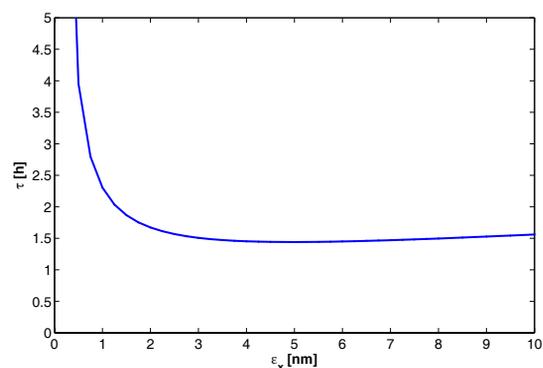


Figure 5: Calculated Touschek lifetime as a function of horizontal emittance for a 1 nC bunch in the ALS at 1.0 GeV. The nominal ALS lattice has an emittance of about 1.7 nm at 1.0 GeV.

the synchrotron radiation monitors. Furthermore, the Touschek lifetime formulas we used so far assume gaussian distributions. Therefore a full analysis of the results might be difficult and we will also attempt to perform the measurements at smaller per bunch currents.

## SUMMARY

Touschek lifetime measurements have been carried out at low energy and very small emittances in the Advanced Light Source. The parameter regime is near the minimum of the Touschek lifetime (in fact just beyond the minimum). The lifetime measurements as a function of momentum acceptance of the RF bucket agree very well with calculations based on calibrated lattice models (using orbit response matrix analysis). This is even true in the case where the lifetime rolls over due to the dynamic momentum acceptance. The measurements are ongoing at even smaller energies, where one is well beyond the minimum of the Touschek lifetime form factor.

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