

# INTRABEAM SCATTERING EFFECT CALCULATED FOR A NON-GAUSSIAN-DISTRIBUTED LINAC BEAM\*

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## Abstract

A high-brightness electron beam used for linac-based fourth-generation light sources such as x-ray free-electron lasers (FELs) and energy recovery linacs (ERLs) is often non-Gaussian distributed, especially in the longitudinal direction. In order to study the intra-beam scattering effect (IBS) in such a beam, we added a slice analysis method to `elegant`[1]. This paper explains this method and an application result to a possible ERL upgrade of the Advanced Photon Source.

## INTRODUCTION

The intra-beam scattering (IBS) effect may become an issue for the linac-based fourth-generation light sources, which use high-brightness electron beams with extremely small transverse and longitudinal emittance. Any change of beam quality could have significant impact on other issues such as bunch compression, coherent synchrotron radiation (CSR) effects, and the quality of x-rays delivered to users. An electron bunch from the gun is often non-Gaussian distributed, especially in the longitudinal direction, as shown in Figure 1.

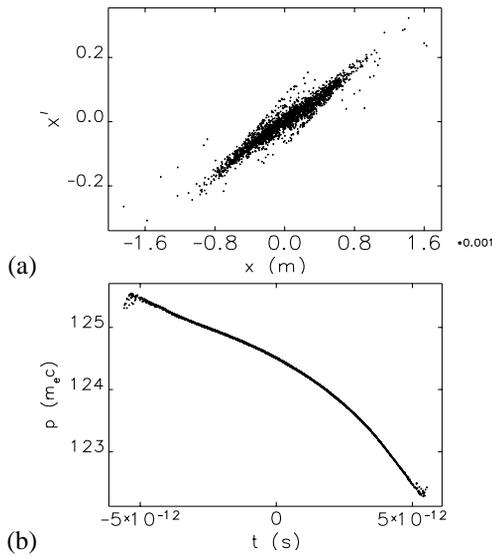


Figure 1: Particle distribution of LCLS beam: (a) horizontal. (b) longitudinal.

The particle distribution plotted here is for the Linac Coherent Light Source (LCLS) and was provided by P. Emma.

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It illustrates general bunch features for a linac beam. In the transverse plane, the distribution is more or less close to a Gaussian distribution. But in the longitudinal plane, the distribution has a strong  $t - p$  correlation which also varies along the linac. The energy spread for the entire bunch and the “intrinsic” energy spread are often different by more than an order of magnitude. The IBS calculation based on the assumption of Gaussian beam is therefore invalid.

To solve this difficulty, we tried two methods. Method 1: treat the linac beam as a whole, but remove the correlation between  $t$  and  $p$  ( $\langle tp \rangle = 0$ ). We hoped that this would result in correct IBS calculation. Method 2: modify the IBS calculation inside the code `elegant` so that the user can slice the beam longitudinally and compute IBS effects slice by slice. This paper will discuss both methods in detail. Applications to a proposed APS ERL [2, 3] upgrade lattice are also presented.

## IBS GROWTH RATE

Bjorken and Mtingwa’s [4] formula is used in `elegant` for calculating the emittance growth rate  $\tau_d$  in the direction  $d$  ( $x$ ,  $y$ , or  $z$ ) due to the IBS effect for a Gaussian-distributed beam in all directions:

$$\frac{1}{\tau_d} = \frac{\pi^2 c r_0^2 m^3 N \ln \Lambda}{\gamma \Gamma} f, \quad (1)$$

$$f = \left\langle \int_0^\infty \frac{\lambda^{1/2} d\lambda}{\sqrt{|\lambda|}} \{Tr L^d Tr [A^{-1}] - 3 Tr [L^d (A)^{-1}]\} \right\rangle,$$

where  $c$  is the speed of light,  $r_0$  is the classical particle radius,  $m$  is the particle mass,  $N$  is the number of particles per bunch (or in the beam for the unbunched case),  $\ln \Lambda$  is a Coulomb logarithm,  $\gamma$  is the Lorentz factor,  $\Gamma$  is the 6-dimensional invariant phase-space volume of the beam,

$$\Gamma = (2\pi)^3 (\beta\gamma)^3 m^3 \varepsilon_x \varepsilon_y \sigma_p \sigma_z, \quad (2)$$

$A = (L + \lambda I)$  and  $L = L^x + L^y + L^z$ , with

$$L^x = \frac{\beta_x}{\varepsilon_x} \begin{pmatrix} 1 & -\gamma\phi_x & 0 \\ -\gamma\phi_x & \gamma^2 \left( \frac{D_x^2}{\beta_x^2} + \phi_x^2 \right) & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$L^y = \frac{\beta_y}{\varepsilon_y} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \gamma^2 \left( \frac{D_y^2}{\beta_y^2} + \phi_y^2 \right) & -\gamma\phi_y \\ 0 & -\gamma\phi_y & 1 \end{pmatrix}, \quad (3)$$

$$L^z = \frac{\gamma^2}{\sigma_p^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Here,  $\phi_{x,y} = D'_{x,y} + \frac{\alpha_{x,y} D_{x,y}}{\beta_{x,y}}$ ;  $\varepsilon_{x,y}$  and  $\sigma_{p,z}$  are beam-distribution-related quantities; and  $\beta_{x,y}$ ,  $\alpha_{x,y}$ ,  $D_{x,y}$ ,  $D'_{x,y}$  are local optical functions.

Equation 1 shows a clear dependency of  $\tau_d$  on the beam distribution through the functions  $\Gamma$  and  $f$ . Since IBS only happens when two particles collide in physical space, for a non-Gaussian-distributed beam, all beam-distribution quantities should be taken locally.

For a bunch traveling through a linac with acceleration, some modifications are made in the calculation [5]. Because there are no synchrotron oscillations for a linac bunch, the longitudinal growth rate is increased by a factor of 2 based on Piwinski's [6] formula

$$\frac{1}{\tau_z} [\text{linac} - \text{bunch}] = 2 \frac{1}{\tau_z} [\text{circulating} - \text{bunch}], \quad (4)$$

and the effective bunch length is  $\sigma_z = \frac{1}{\sqrt{2\pi}} c \Delta t$ , where  $\Delta t$  is the bunch duration. In *elegant*, IBS is treated in a lumped fashion: segments of the beamline are separated by IBSCATTER elements; for each segment, the IBS growth rate is calculated locally with the incoming normalized emittance and local beam energy, then integrated over this segment. At each IBSCATTER element, particle coordinates are changed by either scaling them to a new value ("smooth" method) or by adding random values. In both cases, the resulting new beam size is consistent with the integrated IBS growth rate. The new particle distribution and beam size values are then used as input for the next segment.

## METHOD 1

One simple way of using Equation 1 to calculate the IBS effect for a linac beam with a distribution property similar to the one shown in Figure 1 is to remove the longitudinal  $t-p$  correlations of the input beam ( $\langle tp \rangle = 0$ ). After doing this (Figure 2), the particle's distribution is "close" to Gaussian with long tails, and Equation 1 is valid. To examine this method, we tracked particles with  $\langle tp \rangle \neq 0$  or  $\langle tp \rangle = 0$  through part of the LCLS lattice [7] with a bunch charge of 1 nC. Figure 3 compares the results.

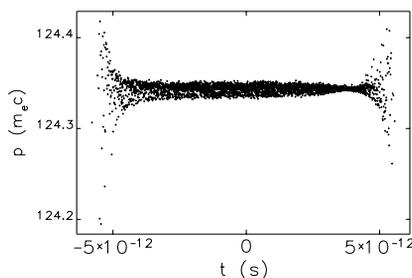


Figure 2: Particle distribution for  $\langle tp \rangle = 0$ .

We see a clear horizontal emittance growth for  $\langle tp \rangle = 0$ , while other beam parameters have no noticeable change in this simulation. This result seems reasonable, but if we stopped here, we would be very wrong. Figure 4 shows

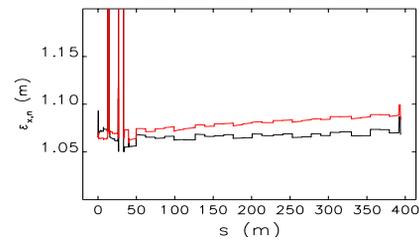


Figure 3: Normalized horizontal emittance evaluation for particles  $\langle tp \rangle \neq 0$  (black) and  $\langle tp \rangle = 0$  (red).

tracking results for  $\langle tp \rangle = 0$  with bunch charge equal to 0, 1, and 10 nC respectively. From this plot we conclude that horizontal emittance doesn't change due to the IBS effect. What, then, is the source of emittance change in Figure 3? We looked into the lattice carefully and found out that horizontal emittance is increased when the bunch passes through the bunch compressor. The increase depends on the value of  $\langle tp \rangle$  and is not related to the IBS effects. We are then forced to give up this method.

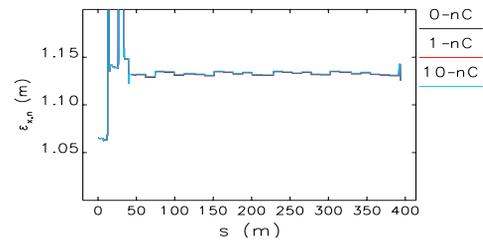


Figure 4: Normalized horizontal emittance evaluation for  $\langle tp \rangle = 0$  bunches. Bunch charges are 0, 1, and 10 nC respectively.

## METHOD 2

To study the IBS effect for a non-Gaussian-distributed beam, we modified the IBS calculation in *elegant* so that it can use slice analysis. The procedure is very straightforward: The simulated electron bunch is sliced into  $N$  pieces longitudinally ( $z$  coordinate) with equal length  $\delta z$  when it meets an IBSCATTER element, where  $N$  is input parameter provided by the user. Within each slice, we assume that particles are Gaussian distributed in the transverse phase space and  $\frac{\delta p}{p}$  direction, while in the  $z$  direction they are distributed uniformly with  $\sigma_z = \delta z$  in Equation 1. The normalized transverse emittance, energy spread, and optical functions ( $\beta_{x,y}$ ,  $\alpha_{x,y}$ ) are calculated for each slice. Here, user can select the design optical functions or the optical functions from the particle distribution. As described in the previous section, the integrated IBS growth rate is calculated for each sliced beam for the part of beamline that the IBSCATTER element represents. The beam's dimensions are updated by giving a kick to the particles' coordinates, while the kick strength is based on the IBS growth rate for the slice to which the particle belongs.

When we apply this method to the LCLS lattice used in the previous section, we see a negligible change in the

beam’s dimensions. This is because the IBS effect has insufficient time to accumulate. For a linac with length  $\sim 1$  km, the time duration is  $\sim 3$  ms. For any beam parameter to obtain a 1% change, the average IBS growth rate would have to be  $\sim 3000$  1/s. Figure 5 shows the difference of the IBS growth rate ( $\frac{1}{\tau_z}$  only,  $\tau_{x,y} \gg \tau_z$ ) with and without slicing the bunch. From this plot, we see that the IBS growth rate drops quickly as the beam is accelerated.

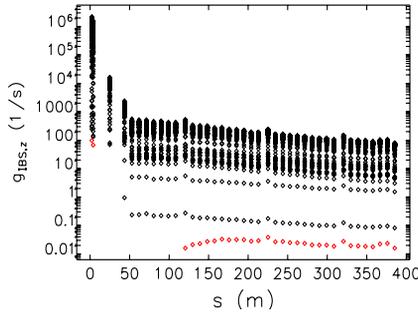


Figure 5: IBS growth rate (longitudinal) for sliced bunch (black, each dot represents a slice) and unsliced bunch (red) (LCLS lattice, beam energy from 63.5 MeV to 4.4 GeV).

Figure 5 also suggests that the IBS effects might be visible if we start our simulation from lower beam energy instead of 63.5 MeV. To explore this, we scaled the bunch energy to 10 MeV and matched it to the starting point of our APS ERL lattice, keeping the charge at 1 nC. Figures 6-8 show simulation results for the APS-ERL lattice (beam energy from 10 MeV to 7 GeV). Figure 6 indicates that we should be able to see a strong energy spread increase for the beam, and Figure 7 confirms it. The energy spread at the center of the bunch increased significantly, while the transverse emittance shows negligible changes. Figure 8 shows the bunch energy spread evolution vs.  $s$ . It indicates that the total energy spread doesn’t change significantly due to IBS.

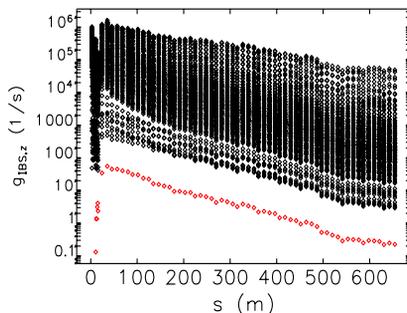


Figure 6: IBS growth rate (longitudinal) for sliced bunch (black, each dot represents a slice) and unsliced bunch (red) (APS ERL lattice, beam energy from 10 MeV to 7 GeV).

## CONCLUSION

To study the IBS effect for a non-Gaussian-distributed beam, we first tried a simple method for estimating the

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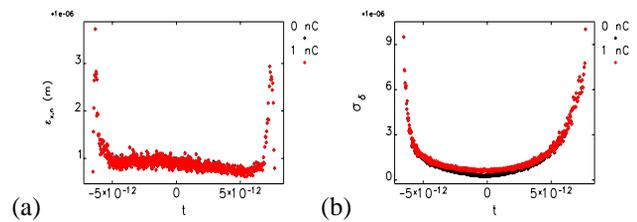


Figure 7: Particle distribution vs. longitudinal position ( $t$ ) at the end of beamline with/without IBS: (a) normalized emittance; (b) energy spread.

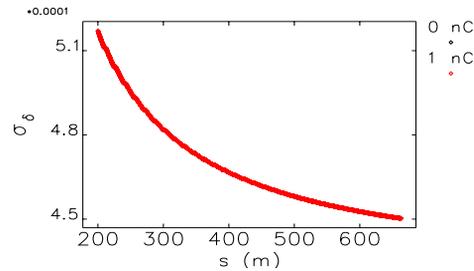


Figure 8: Bunch energy spread evolution vs.  $s$ .

IBS growth rate by removing the  $t - p$  correlation. Our results show that such simplification is not suitable, especially when there is a chicane in the beamline. Thus, we developed a slice analysis method in *elegant*. An application of this method to the APS ERL lattice was also presented. We found a noticeable local energy spread increase but little change in the full bunch properties. These results may be important for studying other issues, for example, CSR effects or FEL gain, which depend on the slice energy spread.

## REFERENCES

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