RFQ PARTICLE DYNAMIC SIMULATION DEVELOPMENT

J. M. Maus*, R. A. Jameson, A. Schempp, N. Müller, U. Bartz, IAP, Frankfurt, Germany

Abstract

For the development of high energy and high duty cycle RFQs accurate particle dynamic simulation tools are important to optimize designs especially in high current applications. To describe the fields in RFQs the Poisson equation has to be solved taking the boundary conditions into account. In the newly developed subroutines this is done by using a finite difference method on a grid. The results of this improvement are shown using PteqHI and compared to the old two term and multipole expansions.

PTEQHI

PteqHI is a RFQ particle dynamic simulation program which is based on an older version of Parmteq with improvements and extensions. It has been continuously developed and kept up to date by R. A. Jameson [1]. It presently uses the same external field description as Parmteqm [2], using a multipole expansion with some geometrical data. In ParmteqM the fields are calculated using a multipole expansion with some geometry data and the image effect also uses geometry data to get coefficients for line and point charges which are assumed to represent the beam.

PARMTEQM DESCRIPTION OF THE EXTERNAL FIELD

To describe the external field in RFQs the Laplace equation

$$\Delta\varphi = 0 \tag{1}$$

has to be solved. The symmetry of the problem leads to cylindric coordinates in which the Laplace equation can be expressed:

$$\frac{1}{r}\cdot\frac{\partial\varphi}{\partial r}\left(r\cdot\frac{\partial\varphi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\varphi}{\partial\vartheta^2}+\frac{\partial^2\varphi}{\partial z^2} \tag{2}$$

This equation can be separated and solved. Taking the symmetry into account it leads to a multipole expansion for the field in a RFQ cell. In ParmteqM the expansion is truncated after the first eight terms:

$$\begin{aligned}\varphi(r,\vartheta,z) \;=\; & \frac{V_0}{2}\{A_{01}\left(\frac{r}{R_0}\right)^2+A_{03}\left(\frac{r}{R_0}\right)^6 \\ & +A_{10}I_0(kr)\cos(kz) \\ & +A_{12}I_4(kr)\cos(4\vartheta)\cos(kz) \\ & +A_{21}I_2(2kr)\cos(2\vartheta)\cos(2kz) \\ & +A_{23}I_6(2kr)\cos(6\vartheta)\cos(2kz) \\ & +A_{32}I_4(3kr)\cos(4\vartheta)\cos(3kz) \\ & +A_{30}I_0(3kr)\cos(3kz)\}\end{aligned} \tag{3}$$

* maus@iap.uni-frankfurt.de

The multipole coefficients A_{nm} have been calculated by integrating over an arc with the radius of the aperture a of the field. The field calculation have been done with Charge3D and the coefficients have been normalized to certain cell parameters and stored in a geometry data file. These files are used in ParmteqM and ptyeqHI to find the coefficients at each cell.

The advantage of the multipole method is - at least for the z-code ParmteqM - the comparatively short execution time. A major disadvantage of this method is, that the field found by the multipole expansion is only accurate in the region from the beam axis to the minimum aperture. Particles which leave that region will be exposed to an inaccurate field. Using the multipole expansion it is not possible to simulate the region between the tank wall and the beginning and end of the electrodes and to simulate the effect of a misalignment of a single electrode. Also, the image effect has to be taken into account using some approximation.

ITERATIVE POISSON SOLVER

In order to overcome the disadvantages of the multipole expansion method one can use an iterative Poisson solver which solves in case of the external fields of a RFQ the Laplace equation on a grid. The iterative solver used here is a Gauß-Seidel using the successive overrelaxation technique (GS-SOR). The first step is to discretize the Laplace equation on the grid (here for the 2D case) (See [3] and [4]):

$$0=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} \tag{4}$$

$$0=\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{(\Delta x)^2}+\frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{(\Delta y)^2} \tag{5}$$

This can now be solved for $u_{i,j}$:

$$u_{i,j} \;=\; \frac{1}{4}\left(u_{i+1,j}+u_{i-1,j}+u_{i,j+1}+u_{i,j-1}\right) \tag{6}$$

The new value for a grid point will be calculated using this combination of the neighboring grid points. Using successive overrelaxation method a combination of the old value and the new value is assigned to the grid point

$$u^{m+1} \;=\; u^m+\omega\left(u_{i,j}-u^m\right) \tag{7}$$

where ω is the overrelaxation parameter, which has to be chosen to fit the problem and m stand for the m-th iteration.

On a regular equidistant grid the smooth vane surface would be represented by small steps. This will lead to a huge demand on memory and computation time. To avoid this, a semi non equidistant grid is used. While generating the mesh and assigning initial values to the grid points the routine shifts grid points from the inside of the conducting material to the surface of the conducting part (See figure 1). With this procedure a smooth surface of the electrodes can be simulated without increasing the number of grid points. Table 2 gives some data about the design of the simulated RFQ. We have set up a set of 10 RFQ designs which we use for intensive code checking and comparing. The RFQ used here one of this set. It has the smallest aperture and is therefore the shortest RFQ of the set. It is not an optimal design, but still useful, because it shows limitations of simulation tools. Figure 2 shows the potential in the x-y plane in the middle of a cell. The beam axis is at the front right corner, where the potential is equal to zero. The flat plateaus represent the electrodes with voltage of 27 kV which is half the design voltage of 54 kV. The difference between the acceleration from the field calculation with multipoles and with the iterative Poisson solver are shown in figures 3, 4, and 5. The first figure shows the acceleration that act on the synchronous particle vs. longitudinal position of the particle for the whole structure and the second figure for the first part of the RFQ in more detail. In the low energy region the two method agree quite well for the synchronous particle. For particles further away from the beam axis the two methods give different results which is due to the fact that the MP routine does not include the tank wall while the Poisson solver does (Figure 5). The gap between the tank wall and the start of the electrodes has a length of $3 \cdot \frac{\beta\lambda}{2}$ therefore 3 oscillation can take place in part.

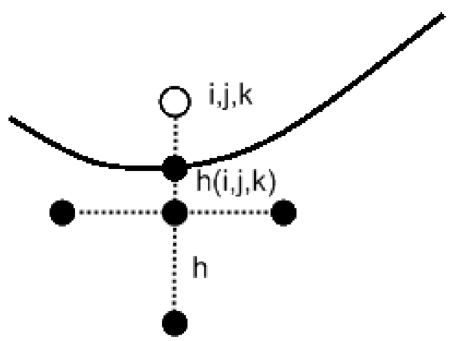

Figure 1: Shifted grid point.

Table 1: Some RFQ Parameters

Frequency	175 MHz
Input energy	95 keV
Output energy	5 MeV
Current	130 mA
Intervane Voltage	54 kV
Minimum aperture	0.247 cm
Length	5.7 m

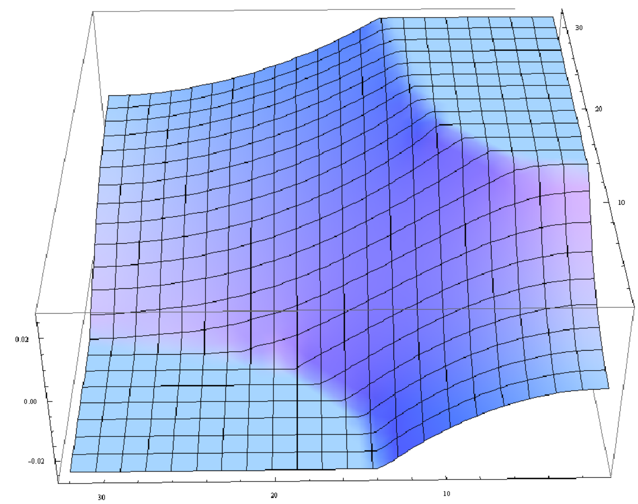

Figure 2: x-y-plane of the potential in a RFQ cell.

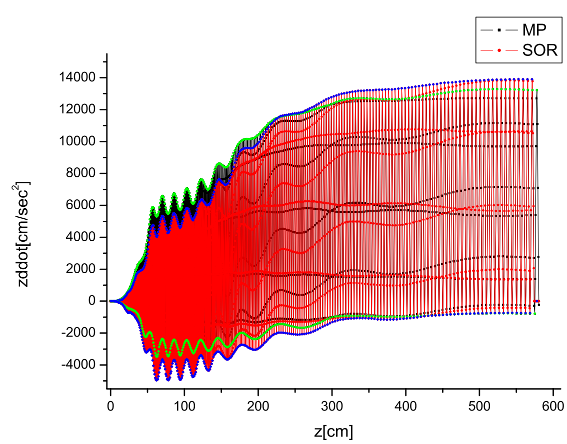

Figure 3: Acceleration based on electric field the synchronous particle sees vs. longitudinal position.

On the left side the rf-field falls to zero in the tank endplate. When the magnitude of the field starts to increase the Poisson solver gets a smaller answer and on the high energy end of the RFQ the multipole method finds a smaller acceleration.

RESULTS OF THE SIMULATION

The results of simulations are shown in table 2. For zero current the two methods give the same transmission. The

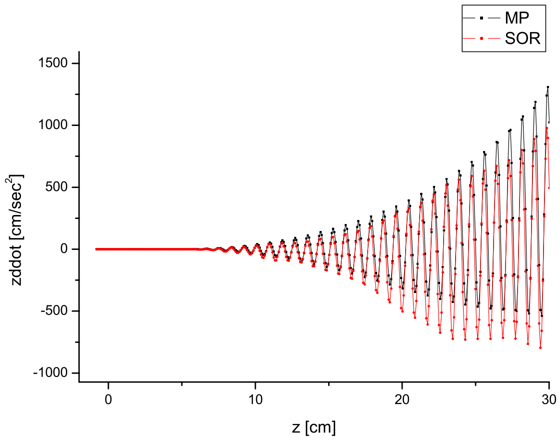

Figure 4: Acceleration based on electric field the synchronous particle sees vs. longitudinal position for the low energy part of the RFQ.

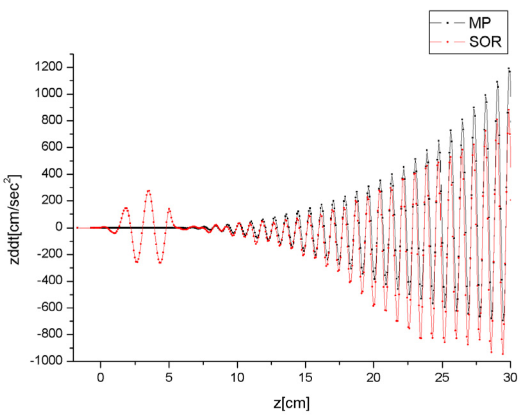

Figure 5: Acceleration based on electric field of particle with x=.1cm vs. longitudinal position for the low energy part of the RFQ.

difference between the two method become obvious in case of high current when the beam becomes bigger and particles come closer to the electrodes. In this region the multipole method gives inaccurate results due to the integration path of the coefficient calculation.

CONCLUSION

The paper shows that the limitations of the old, widely used multipole method can be reached in modern high current designs. Therefore it is necessary to develop adequate and accurate simulation tools. Basic steps have been done with the GS-SOR-solver. Further developments will lead towards multigrid solver. The described GS-SOR-solver will still be important to check more fancy solvers.

Table 2: Overview of the Results

	MP	Poisson solver
Transmission 0mA	95.7%	95.7%
α_x	0.0702	0.0336
β_x	47.0150	45.1134
$\epsilon_{x,rms}$	0.0005	0.0005
α_y	0.0067	-0.0809
β_y	33.5640	31.9465
$\epsilon_{y,rms}$	0.0005	0.0005
Transm. 130mA (scheff)	69.8%	58.2%
Transm. 130mA (Iterative SC solver)	—	52.6%

REFERENCES

[1] R.A. Jameson, "LA-UR-07-0876", Los Alamos National Laboratory

[2] K. R. Crandall and T. P. Wangler and L. M. Young and J. H. Billen and G. H. Neuschaefer and D. L. Schrage, "RFQ Design Codes", Los Alamos National Laboratorym, 2005

[3] U. Trottenberg and C. Osterlee and A. Schüller, "Multigrid", Academic Press, 2001

[4] R. W. Hockney and J.W. Eastwood, "Computer simulation using particles", IOP Publishing, 1988