

NUMERICAL CALCULATION OF WAKE FIELDS IN STRUCTURES WITH CONDUCTIVE WALLS

A. Tsakanian, Hamburg Univ., Hamburg, Germany,
M. Dohlus, I. Zagorodnov, DESY, Hamburg, Germany

Abstract

Based on TE/TM splitting algorithm a new (longitudinally) dispersion-free numerical scheme is developed to evaluate the wake fields in structures with finite wall conductivity. The impedance boundary condition in this scheme is modelled by one dimensional wire connected to boundary cells. A good agreement of the numerical simulations with the analytical results is obtained. The developed code allows to calculate multipole wake potentials of arbitrary shaped rotationally symmetric geometries with walls of finite high conductivity.

INTRODUCTION

The achievement and preservation of the very small electron beam emittance with high peak current is one of the most actual challenges in modern accelerator for fundamental and applied sciences to reach the design goals of the projected facilities [1,2]. The physics of high energy small emittance electron beams is basically dominated by the interaction of the beam with surrounding structure through the excited electromagnetic fields [3]. These fields, known as wake fields, have in general the transverse and longitudinal components, which produce the transverse kick and extra voltage for the trailing charges in the beam. The analytical solutions for the wake fields are available for the structure with relatively simple geometry [4,5].

The real structure, that can include cavities, transitions, collimators, bellows etc, in general has a complicated geometry and is composed of resistive material. Various Maxwell grid equation (MGE) based numerical codes have been developed to solve the 2D and 3D wake field problems in frequency and time domains [6-7] but usually without resistive wall losses. From existing numerical codes only CST Microwave Studio [8] can model structures with finite resistivity but the algorithm suffers from the numerical dispersion. To prevent the numerical dispersion in longitudinal direction, the dispersion-free numerical scheme is proposed, for example, in [9, 10]. Based on it a new (longitudinally) dispersion-free algorithm is developed to evaluate the monopole wake fields in structures with finite wall conductivity which was presented in EPAC08 conference [11]. In this paper we present the extended dispersion free algorithm for multipole wake field calculation. The impedance boundary condition in this scheme is modelled by one dimensional wires connected to boundary cells, as in monopole algorithm. A good agreement of the numerical simulations with the analytical results is obtained. The developed code allows to calculate wake fields of arbitrary shaped rotationally symmetric geometries with walls of finite high conductivity.

FORMULATION OF THE PROBLEM

Consider the ultrarelativistic charged particle bunch with longitudinal distribution ρ , moving along the azimuthally symmetric structure with the speed of light c (Fig.1). The internal region of the structure Ω is bounded by the resistive infinite wall with conductivity κ . The problem is to calculate the electromagnetic fields \vec{E}, \vec{H} induced by the bunch and reads as

$$\begin{aligned} \nabla \times \vec{H} &= \frac{\partial}{\partial t} \vec{D} + \vec{j}, & \nabla \times \vec{E} &= -\frac{\partial}{\partial t} \vec{B}, \\ \nabla \cdot \vec{D} &= \rho, \quad \nabla \cdot \vec{B} = 0, & \vec{H} &= \mu^{-1} \nabla \times \vec{A}, \quad \vec{D} = \epsilon \vec{E}, \\ \vec{E}(t=0) &= \vec{E}_0, \quad \vec{H}(t=0) = \vec{H}_0. \end{aligned} \quad (1)$$

The current density is $\vec{j} = \vec{j}_b + \vec{j}_c$, where $\vec{j}_b = \vec{c}\rho$ is the charge current and $\vec{j}_c = \kappa \vec{E}$ the current induced in the wall. The boundary conditions for electromagnetic fields are given by the continuity of tangential components of electric and magnetic fields on the boundary between the vacuum and the wall. In accelerator applications, the studied structure is usually supplied by ingoing pipe and the well known analytical solution for ultra-relativistic beam in a perfectly conducting cylindrical pipe [4] can be used as initial field.

For conductor the physical model described in paper [11] is used. Hence, in media with high conductivity only tangential components of the fields should be taken into account and they have to be coupled to the full three dimensional field in vacuum. In vacuum the full three dimensional grid space is used. Propagation of the tangential component of the field in the conducting media on each boundary cell with conductor is described by one dimensional (1D) set of Maxwell's equations. The numerical algorithm based on this model is given in the next section.

A SCHEME WITH CONDUCTIVITY

In this section we describe an implicit algorithm for multipole electromagnetic fields computation that

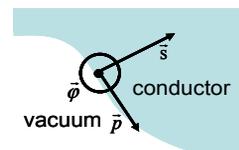


Figure 1: Boundary local coordinate system.

includes the boundaries with finite conductivity. We introduce a local coordinate system connected to the

boundary $\{p, \varphi, s\}$ Fig.1. The propagation of the tangential field in the conductor is described by two decoupled 1D electromagnetic problems: one for the $\{\widehat{h}_\varphi, \widehat{e}_p\}$ components and the second for $\{\widehat{h}_p, \widehat{e}_\varphi\}$. The excitation source of EM field in conducting media is the tangential magnetic field in boundary cell. Following the matrix notation of the finite integration technique [12] the implicit 1D scheme for each pair of the electromagnetic field components $\{\widehat{h}_\varphi, \widehat{e}_p\}$ and $\{\widehat{h}_p, \widehat{e}_\varphi\}$ will be read as

$$\begin{aligned}\widehat{e}^{n+1} &= A e^n + B P_s \frac{\widehat{h}^{n+1} + \widehat{h}^n}{2} \\ \widehat{h}^{n+1} &= A \widehat{h}^n + \Delta\tau P_s^* \frac{\widehat{e}^{n+1} + \widehat{e}^n}{2}\end{aligned}\quad (2)$$

The two-banded matrix P_s plays the role of discrete differential operator. The matrices A and B are diagonal with entries

$$a_{00} = e^{-0.5\kappa Z_0 \Delta\tau}, \quad a_{ii} = e^{-\kappa Z_0 \Delta\tau}, \quad b_{ii} = \frac{1 - a_{ii}}{\kappa Z_0} \quad (3)$$

In local coordinate system the tangential component of magnetic field in vacuum will read

$$\vec{\widehat{h}}_{\tau, vacuum} = \widehat{h}_{\varphi, vacuum} \vec{i}_\varphi + \widehat{h}_{p, vacuum} \vec{i}_p \quad (4)$$

The tangential magnetic field $\widehat{h}_{p, vacuum}$ can be found via interpolation of radial and longitudinal components of magnetic field into the centre of boundary cell and its projection on tangential vector \vec{i}_p . Hence the boundary conditions at the interface for 1D electromagnetic problems will reads $\widehat{h}_{\varphi,0} = \widehat{h}_{\varphi, vacuum}$ and $\widehat{h}_{p,0} = \widehat{h}_{p, vacuum}$.

Explicit and implicit dispersion free schemes with perfect boundary conditions are introduced in paper [10]. Using the same notations the implicit hybrid scheme with finite conductive boundaries will be read

$$\begin{aligned}\widehat{h}_\varphi^\# &= \widehat{h}_\varphi^{n-0.5} + \frac{\Delta\tau}{2} M_{\mu_\varphi} \left[P_z \widehat{e}_r^n - P_r \widehat{e}_z^{n-0.5} + \widehat{e}_{pc}^{n-0.5} \right] \\ \widehat{h}_r^\# &= \widehat{h}_r^{n-0.5} + \frac{\Delta\tau}{2} M_{\mu_r} \left[-P_z \widehat{e}_\varphi^n + m \widehat{e}_z^{n-0.5} + \widehat{e}_{\varphi c}^{n-0.5} \right] \\ \widehat{e}_z^{n+0.5} &= \widehat{e}_z^{n-0.5} + F_1 \Delta\tau M_{\varepsilon_z} \left[P_z \widehat{h}_\varphi^\# - m \widehat{h}_r^\# + \widehat{j}_z^n \right] \\ \widehat{h}_\varphi^{n+0.5} &= \widehat{h}_\varphi^\# + \frac{\Delta\tau}{2} M_{\mu_\varphi} \left[P_z \widehat{e}_r^n - P_r \widehat{e}_z^{n+0.5} + \widehat{e}_{pc}^{n+0.5} \right] \\ \widehat{h}_r^{n+0.5} &= \widehat{h}_r^\# + \frac{\Delta\tau}{2} M_{\mu_r} \left[-P_z \widehat{e}_\varphi^n + m \widehat{e}_z^{n+0.5} + \widehat{e}_{\varphi c}^{n+0.5} \right] \\ \widehat{e}_\varphi^\# &= \widehat{e}_\varphi^n + \frac{\Delta\tau}{2} M_{\varepsilon_\varphi} \left[P_z \widehat{h}_r^{n+0.5} - P_r \widehat{h}_z^n + \widehat{j}_\varphi^{n+0.5} \right] \\ \widehat{e}_r^\# &= \widehat{e}_r^n + \frac{\Delta\tau}{2} M_{\varepsilon_r} \left[-P_z \widehat{h}_\varphi^{n+0.5} + m \widehat{h}_z^n + \widehat{j}_r^{n+0.5} \right] \\ \widehat{h}_z^{n+1} &= \widehat{h}_z^n + F_2 \Delta\tau M_{\mu_z} \left[P_r \widehat{e}_\varphi^\# - m \widehat{e}_r^\# + \widehat{e}_{\varphi c}^{n+0.5} \right] \\ \widehat{e}_\varphi^{n+1} &= \widehat{e}_\varphi^\# + \frac{\Delta\tau}{2} M_{\varepsilon_\varphi} \left[P_z \widehat{h}_r^{n+0.5} - P_r \widehat{h}_z^{n+1} + \widehat{j}_\varphi^{n+0.5} \right] \\ \widehat{e}_r^{n+1} &= \widehat{e}_r^\# + \frac{\Delta\tau}{2} M_{\varepsilon_r} \left[-P_z \widehat{h}_\varphi^{n+0.5} + m \widehat{h}_z^{n+1} + \widehat{j}_r^{n+0.5} \right]\end{aligned}\quad (5)$$

with

Beam Dynamics and Electromagnetic Fields

D05 - Code Developments and Simulation Techniques

$$\begin{aligned}F_1 &= \left(I + \frac{\Delta\tau^2}{4} M_{\varepsilon_z^{-1}} P_r^* M_{\mu_\varphi^{-1}} P_r + \frac{\Delta\tau^2}{4} m^2 M_{\varepsilon_z^{-1}} M_{\mu_r^{-1}} \right)^{-1} \\ F_2 &= \left(I + \frac{\Delta\tau^2}{4} M_{\mu_z^{-1}} P_r M_{\varepsilon_\varphi^{-1}} P_r^* + \frac{\Delta\tau^2}{4} m^2 M_{\mu_z^{-1}} M_{\varepsilon_r^{-1}} \right)^{-1}\end{aligned}$$

Here m is multipole number and $\widehat{e}_{pc}, \widehat{e}_{\varphi c}$ are the electric field tangential components at the conductive boundary and in the scheme they appear in magnetic field updates. In the first time step magnetic fields in boundary cells are used to update EM fields eq. (2) in conductor and in the second time step we update the magnetic field in vacuum using updated electric fields on conductive boundaries $\widehat{e}_{pc, \varphi c} \equiv \widehat{e}_{p0, \varphi 0}$. In the case of monopole wake fields only azimuthal component of magnetic field on boundary is nonzero and remains only one field update in conductor.

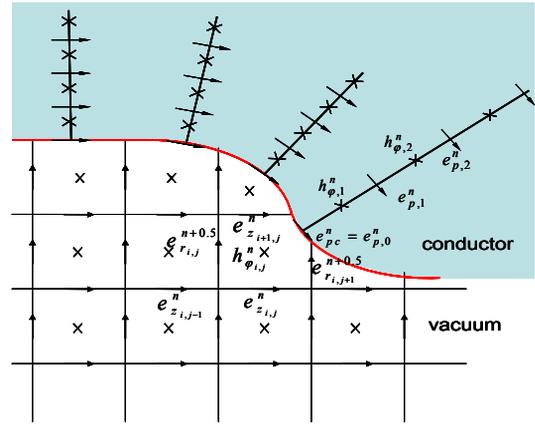


Figure 2: Vacuum grid with 1D conducting lines at the boundary for monopole case.

As in monopole case (Fig.2) [11] the stability condition of this scheme is also $\Delta\tau \leq \Delta z$. With the time step $\Delta\tau$ equal to longitudinal mesh step Δz , the scheme does not have dispersion in longitudinal direction. The transverse mesh and mesh step in conductor can be chosen independently from stability considerations in vacuum region. The above introduced scheme is for any order harmonics and doesn't require additional computational time for higher order harmonics than for dipole one. This scheme can be easily generalized to 3D case [13].

NUMERICAL EXAMPLES

As the first test we calculate the steady state wake of the

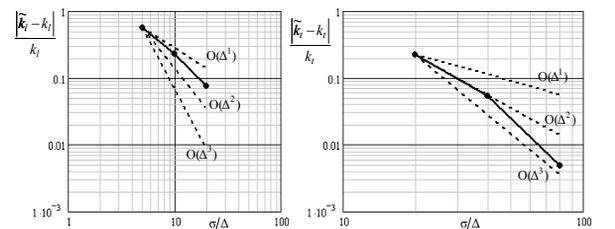


Figure 3: Convergence of the loss and kick factors.

Gaussian bunch with rms length $\sigma=1\text{mm}$ in round pipe of radius $a=1\text{ cm}$ and of conductivity $\kappa=1e5\text{ S/m}$. To obtain the steady state solution we have calculated $2m$ of the pipe and subtracted the wake of the first meter. Fig. 3 shows convergence of the loss and kick factors to the analytical values 1.31 V/pC and 75.5 V/pC/m correspondingly.

Fig. 4 compares the analytical and the numerical dipole wakes for mesh resolution of 10 points on σ . In this case the error in loss and kick factors is about 3%.

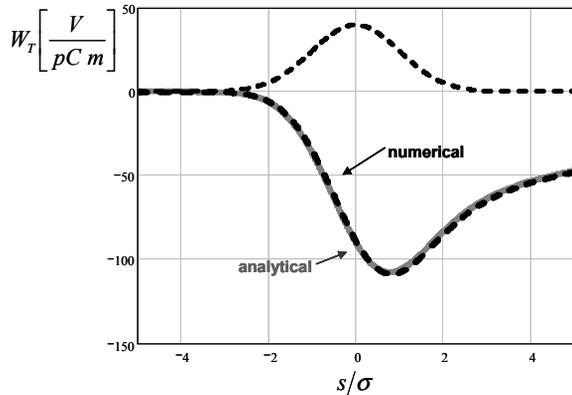


Figure 4: Comparison of numerical and analytical wakes.

As the second test we calculate a wake of finite length resistive cylinder. It has radius $a=1\text{cm}$, length $b=10\text{cm}$ and conductivity $\kappa=1e4\text{ S/m}$. For the Gaussian bunch with $\sigma=0.025\text{ mm}$ the analytical results of the paper [13] could be used. The kick factor reads

$$k_t = c \frac{cZ_0 \sqrt{g s_g} \Gamma(1/4)}{2\pi^2 a^3} K_t \left(\frac{\sigma}{s_g} \right), \quad s_g = \sqrt{\frac{g}{2Z_0 \kappa}} \quad (6)$$

where function K_t is given by Eq. (6.6) from [14].

Fig. 5 shows the numerically obtained dipole wake (black solid line) and the analytical steady state wake [3, 4] (dashed gray line). The numerically obtained kick factor is equal to 42.6 V/pC/m and coincides with that given by Eq.(6) (41.5 V/pC/m). The steady state kick factor is equal to 9.6 V/pC/m and underestimates the kick.

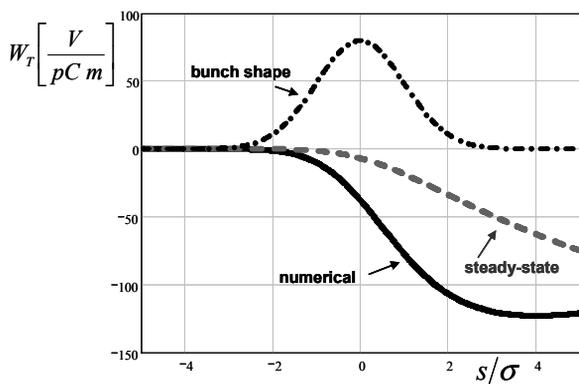


Figure 5: Comparison of transient and steady-state wakes.

Third test is monopole wake potential calculation of tapered collimator (Fig.6) with parameters $a_1=17\text{mm}$, $L_1=200\text{ mm}$, $a_2=10\text{mm}$, $L_2=100\text{mm}$, $a_3=6\text{mm}$ and conductivity $\kappa=1e4\text{ S/m}$. For a Gaussian bunch $\sigma=50\text{ }\mu\text{m}$ numerically obtained loss factor for conductive walls (270 V/pC) is twice larger than for perfectly conducting walls (133 V/pC) and cannot be obtained as direct sum of the geometrical and the steady-state solution.

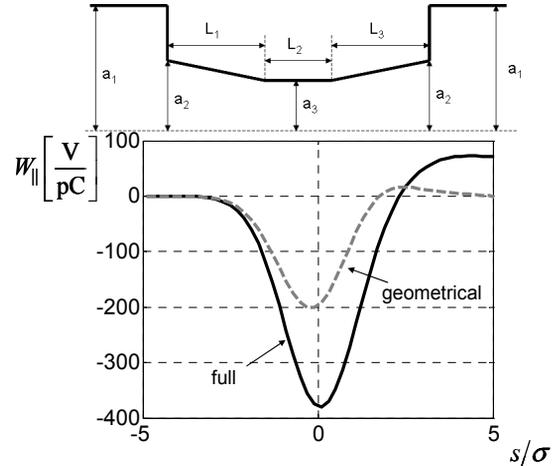


Figure 6: Comparison of wakes “with” and “without” resistivity.

Similar test on convergence and accuracy of the scheme has been done for beam pipe also for higher order harmonics and the scheme behaves in the same manner as dipole. The multipole wake field calculation with the code is not fully tested yet for more complicated geometries like collimators and currently is in improvement stage.

REFERENCES

- [1] The European XFEL, DESY 2006-097, July 2006.
- [2] LCLS CDR, SLAC Report No. SLAC-R-593, 2002.
- [3] A.W. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators (Wiley, New York, 1993).
- [4] B.W.Zotter and S.A. Kheifetz, Impedances and Wakes in High-Energy Particle Accelerators (1997).
- [5] H. Henke and O. Napolj, EPAC-1990, pp.1046 1048.
- [6] MAFIA Collaboration, MAFIA manual, CST GmbH, Darmstadt, 1997.
- [7] T. Weiland, IEEE Trans. Nucl. Sci. 30 (1983) 2489.
- [8] CST Microwave Studio is a trademark of CST GmbH
- [9] I. Zagorodnov, T. Weiland, PR-STAB, 8, 042001 (2005)
- [10] M. Dohlus, I. Zagorodnov, Journal of Comp. Phys., Vol. 228, 2822-2833, 2009
- [11] A. Tsakanian, M. Dohlus, I. Zagorodnov, EPAC-2008, pp. TUPP075, Genoa, Italy, 2008.
- [12] T. Weiland, Int. J. Num. Mod., V. 9, 295-319 (1996)
- [13] A. Tsakanian, M. Dohlus, I. Zagorodnov. TE/TM Scheme for Conductive Wall Structures (in preparation).
- [14] S.Krinsky et al, PR-STAB, 7, 114401 (2004).