A DISPERSION FREE THREE-DIMENSIONAL SPACE-CHARGE MODELING METHOD*

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Abstract

We present a summary of the theoretical and numerical results of a dispersion-free time-dependent Green's function method which can be utilized for calculating electromagnetic space-charge fields due to arbitrary current in a conducting pipe. Since the Green's function can be expanded in terms of solutions to the wave equation, the numerical solutions to the fields also satisfy the wave equation yielding a completely dispersion-free numerical method. The technique is adequately suited for modeling bunched space-charge dominated beams, such as those found in high-power microwave sources, for which the effects of numerical grid dispersion and numerical Cherenkov radiation are typically found when using FDTD type methods.

INTRODUCTION

When modeling the physics of space-charge in high-current electron beams in high-power microwave sources, such as klystrons [1-5], it is necessary to have accurate representations of the space-charge electromagnetic fields. Although the space-charge fields are generated by these high-current beams, often times the effects of image charges and image currents on the boundaries of nearby conducting surfaces, such as a cylindrical drift tube, can make important contributions to these fields, as well.

The most common method for numerically modeling space-charge fields in the presence of conductor boundaries utilizes the Yee algorithm [6]. In this method, the electric and magnetic fields are numerically computed on a grid through finite difference approximations of Faraday's Law and the Ampere-Maxwell Law. The grid also serves the purpose of tracking the charge and current densities of the beam. While the Yee algorithm is self-consistent, it is known to generate unphysical phenomena, such as numerical grid dispersion [7]. Therefore, it is helpful to develop alternative techniques, which include analytic/semi-analytic methods, for computing space-charge fields that does include the effects of numerical grid dispersion.

Recently, a number of methods have been developed explicitly for this purpose and have included the effects of circular conducting pipe boundary conditions and a flat cathode [8-10]. However, in these previous works it was assumed that the beam currents were only propagating in the longitudinal direction that is the beam currents transverse to the pipe axis were zero. While this assumption is adequate for modeling space-charge dominated accelerator systems, such as photocathode sources, it is not a good approximation for modeling high-power microwave systems, such as klystrons, which may have non-negligible transverse currents that generate important electromagnetic space-charge fields.

In this paper, we summarize the results of a significant extension to the space-charge field methodology which is presented in Ref. 11. by including the calculation of space-charge fields for cylindrically symmetric transverse beam currents. The paper is organized as follows. In Sec. II, we show how the space-charge field equations are formulated for the case of cylindrically symmetric charge and current densities in a circular conductor pipe. In Sec. III, we perform numerical calculations for the space-charge fields of a bunched annular electron beam undergoing transverse radial oscillations. In Sec. IV, we include a summary of our results.

SPACE-CHARGE FIELDS

In this section, we briefly summarize the method of Ref. 11 for computing the space-charge fields for circularly symmetric charged sources in a circular conducting pipe. We assume that the radius of the pipe is denoted by $r = a$ and the circularly symmetric charge and current densities are given by, $\rho(r,z,t)$ and $\mathbf{J}(r,z,t)$, where z denotes the longitudinal coordinate parallel to the axis of the pipe. In addition, we impose the self-consistent assumption of charge conservation, i.e. $\partial\rho/\partial t + \nabla\cdot\mathbf{J}(r,z,t) = 0$.

In order to compute the fields, we first need to find appropriate spatial expansion formulae for the beam charge and current density. The expansion formula utilizes Bessel functions to represent the sources in the radial direction, and the expansion coefficients for the source densities are functions of z and t. These coefficients are then used to compute coefficients for the space-charge fields which are also expanded in terms of Bessel functions. We use the following Bessel function expansions for the charge and current densities,

$$\rho = \sum_{m=1}^{\infty} \rho_m(z,t) J_0\left(\frac{j_{0m} r}{a}\right) \tag{1a}$$

$$J_r = \sum_{m=1}^{\infty} J_{rm}(z,t) J_1\left(\frac{j_{0m} r}{a}\right) \tag{1b}$$

$$J_\theta = \sum_{m=1}^{\infty} J_{\theta m}(z,t) J_1\left(\frac{j_{1m} r}{a}\right) \tag{1c}$$

$$J_z = \sum_{m=1}^{\infty} J_{zm}(z,t) J_0\left(\frac{j_{0m} r}{a}\right), \tag{1d}$$

where $J_l(x)$ is the l^{th} order Bessel functions of the first kind and j_{lm} is the m^{th} zero of $J_l(x)$. The space-charge fields are then expanded in the following manner, i.e.

$$E_r = \sum_{m=1}^{\infty} E_{rm}(z,t) J_1\left(\frac{j_{0m} r}{a}\right), \tag{2a}$$

$$E_\theta = \sum_{m=1}^{\infty} E_{\theta m}(z,t) J_1\left(\frac{j_{1m} r}{a}\right), \tag{2b}$$

$$E_z = \sum_{m=1}^{\infty} E_{zm}(z,t) J_0\left(\frac{j_{0m} r}{a}\right), \tag{2c}$$

$$B_r = \sum_{m=1}^{\infty} B_{rm}(z,t) J_1\left(\frac{j_{1m} r}{a}\right), \tag{2d}$$

$$B_\theta = \sum_{m=1}^{\infty} B_{\theta m}(z,t) J_1\left(\frac{j_{0m} r}{a}\right), \tag{2e}$$

and

$$B_z = \sum_{m=1}^{\infty} B_{zm}(z,t) J_0\left(\frac{j_{1m} r}{a}\right). \tag{2f}$$

We note that the Bessel expansion functions in Eq. [2] have been appropriately chosen to correctly enforce the conductor boundary conditions at $r=a$. One can relate the mth space-charge field coefficients in Eq. [2] to the mth source coefficients in Eq. [1] using the field wave equations which are readily derived from Maxwell's equations, i.e.

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\varepsilon_0}\nabla\rho + \mu_0 \frac{\partial \mathbf{J}}{\partial t} \tag{3a}$$

and

$$\nabla^2 \mathbf{B} - \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} = -\mu_0 \nabla\times\mathbf{J}\ . \tag{3b}$$

As demonstrated in Ref. 11, the solutions to the wave equations, and hence the solutions to the mth field coefficients can be expressed in terms of time-dependent Green's functions which satisfy the wave equation for delta function sources in space and time. The expressions for the mth space-charge field terms contain both spatial and temporal integrals over these Green's functions and the mth source terms. Since the mth radial expansion term in each space-charge field equation independently satisfies the wave-equation for the mth source term, then it is dispersion-free. Likewise, since each space-charge field component is a sum of dispersion-free terms, then it is also dispersion-free. Hence, the scheme has no difficulties with numerical grid dispersion.

In Ref. 10, we outlined the numerical constraints for implementing the space-charge solver scheme, such as the minimum number of radial modes and the maximum integration time step, which is needed to achieve high field accuracy, <1% field errors. We should emphasize that while the numerical solutions to the fields have truncation errors this does not impose any unphysical numerical dispersion in the calculation since the truncated field expansions are also dispersion-free.

NUMERICAL STUDIES

We demonstrate the space-charge solution method for an annular bunched beam undergoing radial (breathing mode) oscillations. This type of beam is featured in high-power klystron designs and hence calculations of its space-charge fields are important.

The annular beam is chosen to have a total charge Q, length L, and a uniform radial distribution function for time-dependent inner and outer radii. Specifically,

$$\rho(r,z,t) = \frac{Q}{\pi L\left(r_o^2(t) - r_o^2(t)\right)} \times \left[H(r_o(t)-r) - H(r_i(t)-r)\right]\left[H(L/2+z) - H(L/2-z)\right] \tag{4}$$

where the inner and outer beam radii are given by

$$r_i(t) = \begin{cases} r_i(0)\ , & t \le 0 \\ r_i(0) - \delta r' \sin(\omega t), & t > 0 \end{cases} \tag{5a}$$

$$r_o(t) = \begin{cases} r_o(0)\ , & t \le 0 \\ r_o(0) + \delta r' \sin(\omega t), & t > 0 \end{cases} \tag{5b}$$

where $H(x)$ is the Heaviside step function and the beam radii are constant for time $t<0$ and then undergo radial sinusoidal oscillations with amplitude $\delta r'$ and angular frequency ω for $t>0$. We choose the following parameters for this example, a=0.0908 m, $r_0 = 0.9a$, $\delta r' = 0.025a$, $\omega = j_{01}c/a = 7.94\times10^9\ rad/s$, $L = 0.001m$, $r_i = 0.8a$, $r_o = 0.9a$, and $\delta r' = 0.025a$. This parameter set would model the space-charge fields for a highly bunched electron beam undergoing breathing mode oscillations in a klystron drift tube with a characteristic frequency in the range of 1 GHz. We note that our choice of radii also include the effect of transient startup physics at time t=0.

From these parameters, one can first calculate the source expansion coefficients and then relate them to the field coefficients. Since the beam currents are in the radial direction, then only E_r, E_z, and B_θ are nonzero. Figures 1 and 2 show plots of normalized E_r vs. r/a for different times which are related to the characteristic beam oscillation period $T = 2\pi/\omega$. Specifically, Fig. 1 has plots of the normalized radial electric field at z=0.0 across the beam where the oscillation is occurring for times $t = 0$ (red), $t = 0.1T$ (green), and $t = 0.25T$ (black). It is immediately obvious that the electromagnetic contribution to the space-charge fields which occur after the t=0 is important compared to the electrostatic component which is the curve at t=0 despite having only a modest radial perturbation. Figure 2 shows plots of the normalized E_r for earlier times near the outer bunch edge, namely t=0.0 (red) and t=0.0022 (blue). At a time $t = L/2c \cong 0.0021T$, the space-charge field develops the first important electromagnetic perturbation within the beam midplane, which is when the beam edges begin to affect the middle of the bunch. Physically, this perturbation is due to light pulses being emitted by the longitudinal beam edges, at

$z = \pm L/2$ and $t = 0$, when the radial beam oscillations begin. To complete the study, we also show plots of normalized E_z vs. z/a at the outer beam edge $r = 0.9a$ for the times $t = 0.0$(red), t = 0.1T (green), and $t = 0.25T$ (black) in Figure 3. Plots of the normalized E_z at the inner beam edge $r = 0.8a$ look nearly identical to Figure 3. We do not show plots of the normalized B_θ vs. r/a at the bunch midplane because by symmetry $B_\theta = 0$ in the midplane.

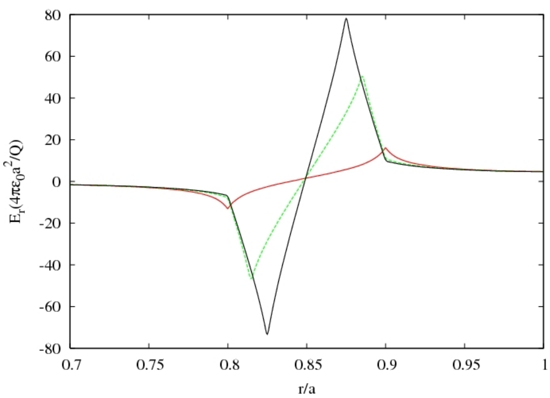

Figure 1: Normalized E_r vs. r/a for different times.

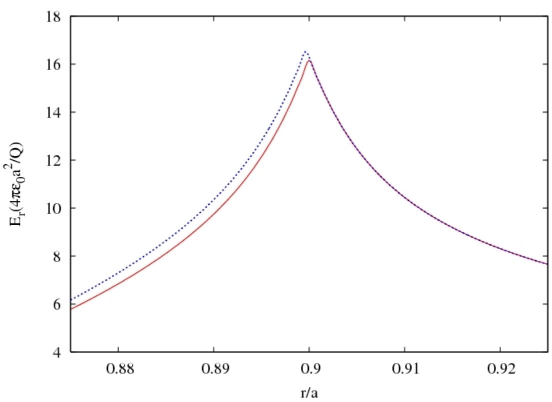

Figure 2: Normalized E_r vs. r/a for earlier times.

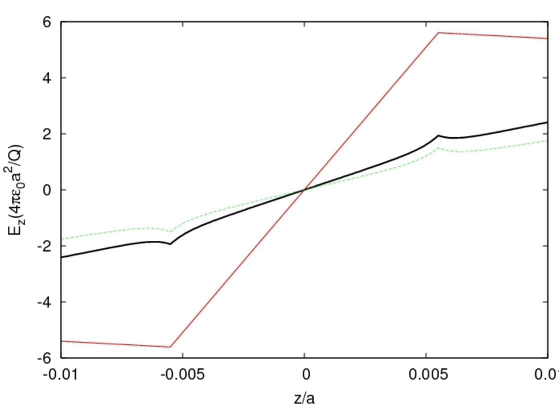

Figure 3: Normalized E_z vs. z/a for different times.

SUMMARY

We have summarized a new dispersive-free method for computing the space-charge fields of a cylindrically symmetric beam with arbitrary currents in a circular conducting pipe. The method is dispersion-free since the space-charge field components are expanded with terms that independently satisfy the field wave equation. We computed the space-charge fields for an annular bunched beam undergoing radial breathing mode oscillations and found that are method can correctly account for transient startup effects. We have also found that even for modest radial beam oscillations, the fully electromagnetic portion of the radial electric space-charge field can be important compared to the electrostatic portion.

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